AN INVESTIGATION OF ELEMENTARY GIFTED STUDENTS' MODELING COMPETENCIES THROUGH ENGINEERING-BASED MODEL ELICITING ACTIVITIES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

FİRDEVS İCLAL KARATAŞ AYDIN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
THE DEPARTMENT OF ELEMENTARY EDUCATION

SEPTEMBER 2022

Approval of the thesis:

AN INVESTIGATION OF ELEMENTARY GIFTED STUDENTS' MODELING COMPETENCIES THROUGH ENGINEERING-BASED MODEL ELICITING ACTIVITIES

submitted by **FİRDEVS İCLAL KARATAŞ AYDIN** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy in Elementary Education, the Graduate School of Social Sciences of Middle East Technical University** by,

Prof. Dr. Yaşar KONDAKÇI
Dean
Graduate School of Social Sciences

Prof. Dr. Semra SUNGUR
Head of Department
Department of Elementary Education

Prof. Dr. Mine IŞIKSAL BOSTAN
Supervisor
Department of Mathematics and Science Education

Examining Committee Members:

Prof. Dr. Erdinç ÇAKIROĞLU (Head of the Examining Committee)
Middle East Technical University
Department of Mathematics and Science Education

Prof. Dr. Mine IŞIKSAL BOSTAN (Supervisor)
Middle East Technical University
Department of Mathematics and Science Education

Prof. Dr. Gaye TEKSÖZ
Middle East Technical University
Department of Mathematics and Science Education

Assoc. Prof. Dr.Fatma ASLAN TUTAK
Boğaziçi University
Department of Mathematics and Science Education

Assist. Prof. Dr. Duygu ÖZDEMİR
İstanbul Aydın University
Department of Mathematics and Science Education

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: Firdevs İclal KARATAŞ AYDIN

Signature:

ABSTRACT

AN INVESTIGATION OF ELEMENTARY GIFTED STUDENTS' MODELING COMPETENCIES THROUGH ENGINEERING-BASED MODEL ELICITING ACTIVITIES

KARATAŞ AYDIN, Firdevs İclal
Ph.D., The Department of Elementary Education
Supervisor: Prof. Dr. Mine IŞIKSAL BOSTAN

September 2022, 337 pages

The aim of this study is to investigate the cognitive modeling competencies of elementary gifted students when they are engaged in engineering-based model eliciting activities. In this study, the case study method was employed. Participants of the study are 19 elementary gifted students between the ages of 9 and 12 who attend at the Science and Art Center in Ankara. Purposive sampling method was used to determine the participants. Six groups received three engineering-based model eliciting activities developed within the scope of this study over a period of five weeks, each activity to two groups in a classroom setting. The engineering-based model eliciting activities, video and audio recordings of group works, student worksheets, observer's field notes and group interviews were used as data collection tools in this study. The data obtained from this study were analyzed based on de Villier's (2018) classification of modeling competencies framework and the group modeling competency observation guide. In accordance with the findings of this study, the mentioned framework and guideline have been expanded. In this regard, it has been determined that the cognitive modeling competencies of elementary gifted students as a group are mostly exemplary level, and that some groups demonstrate proficient level of sub-modeling competencies in some activities. Based on the findings of this study, it can be concluded that engineering-based model eliciting activities can be used as enrichment tools that allow elementary gifted students to reveal their potential.

Keywords: Gifted Students, Mathematical Modeling, Engineering Design Process, Model Eliciting Activities, Modeling Competencies

ÖZ

ÜSTÜN YETENEKLİ İLKÖĞRETİM ÖĞRENCİLERİNİN MODELLEME YETERLİLİKLERİNİN MÜHENDİSLİK TEMELLİ MODEL OLUŞTURMA ETKİNLİKLERİ YOLUYLA İNCELENMESİ

KARATAŞ AYDIN, Firdevs İclal
Doktora, İlköğretim Bölümü
Tez Yöneticisi: Prof. Dr.Mine IŞIKSAL BOSTAN

Eylül 2022, 337 sayfa

Bu çalışmanın amacı üstün yetenekli ilköğretim öğrencilerinin mühendislik temelli model oluşturma etkinliklerine katıldıklarında gösterdikleri bilişsel modelleme yeterliliklerini incelemektir. Bu araştırmada durum çalışması yöntemi kullanılmıştır. Çalışmanın katılımcıları Ankara'da bir Bilim Sanat Merkezi'ne devam eden yaşları 9 ile 12 arasında değişen 19 üstün yetenek tanısı konulmuş ilköğretim öğrencisidir. Katılımcıların belirlenmesinde amaçlı örnekleme yöntemi kullanılmıştır. Altı gruba, bu çalışma kapsamında geliştirilen üç mühendislik temelli model ortaya çıkarma etkinliği, her bir etkinlik bir sınıf ortamında iki gruba olmak üzere beş haftalık süre boyunca uygulanmıştır. Mühendislik temelli model oluşturma etkinlikleri, grupların çalışmaları sırasında alınan video ve ses kayıtları, öğrenci çalışma kâğıtları, araştırmacının alan notları, gözlemci notları ve grup görüşmeleri bu çalışmada veri toplama araçları olarak kullanılmıştır. Bu çalışmadan elde edilen veriler, de Villiers (2018) tarafından önerilen modelleme yeterliliklerinin sınıflandırılması çerçevesi ve grup modelleme yeterliliği gözlem kılavuzu temel alınarak değerlendirilmiştir. Bahsedilen çerçeve ve kılavuz bu çalışmanın verileri doğrultusunda genişletilmiştir.

Bu doğrultuda, üstün yetenekli öğrencilerin grup olarak sergiledikleri bilişsel modelleme yeterliliklerinin büyük ölçüde örnek gösterilebilir düzeyde olduğu, bazı grupların bazı etkinliklerde bazı alt modelleme yeterliliklerini yeterli düzeyde gösterdiği belirlenmiştir. Bu çalışmanın bulgularından hareketle, mühendislik temelli model oluşturma etkinliklerinin üstün yetenekli öğrencilerin potansiyellerini ortaya koyabilmesine imkân tanıyan zenginleştirme araçları olarak kullanılabileceği söylenebilir.

Anahtar Kelimeler: Üstün Yetenekli Öğrenciler, Matematiksel Modelleme, Mühendislik Tasarım Süreci, Model Oluşturma Etkinlikleri, Modelleme Yeterlilikleri

To my husband for his endless love, patience and support

ACKNOWLEDGMENTS

First of all, I would like to express my sincere appreciation to my supervisor, Prof. Dr. Mine Işıksal-Bostan for her support and guidance, endless patience and encouragement, enlightening comments and advice throughout my doctoral study. I would like to convey my gratitude for helping to inspire me to develop as a mathematics teacher and researcher. I cannot thank her enough for her confidence in me and the courage she provided me to live these days together, as well as the support she has given me during my extremely difficult days.

I would like to extend my gratitude and sincere appreciation to the devoted members of my thesis monitoring committee, Prof. Dr. Erdinç Çakıroğlu and Assoc. Prof. Dr. Fatma Aslan-Tutak for their time, consistent support, helpful suggestions, and constructive feedback that challenged my perspective and helped strengthen my dissertation. In addition, I would like to thank the other members of my committee, Prof. Dr. Gaye Teksöz and Assist. Prof. Dr. Duygu Özdemir, for the valuable time they contributed, the insightful feedback they provided, and the suggestions they made to enhance the quality of the study.

I am grateful for Elif Özdemir, who continually supported and encouraged me throughout the present study, especially offering constructive feedback on the data analysis. I would also like to express my appreciation to the administrators and teachers at BILSEM for their assistance in completing this study.

Moreover, I would like to express my deepest gratitude to Güler Barış, who serves as the director of the graduate program. I am appreciative of all the support she has provided and for making me feel like a member of this huge family at METU.

I would like to thank my director, Prof. Dr. Mustafa Şanal and all my colleagues Mustafa Serkan, Seyfi and Hakan for their tolerance and support me through this process.

I would like to offer my most heartfelt gratitude to my family and to all of my close friends. My family has always encouraged my passion for learning and has maintained so throughout my entire life. I would want to express my gratitude to my parents, Nimet and Hasan, for instilling in me the notion that I am capable of achieving anything, as this dissertation is evidence of that. Other thanks go to my dear brother Cengiz, who always looks up to me as a model in his own life and reminds me of what I am capable of doing even when I feel like giving up. I would like to extend my gratitude to Rabia for joining me in online study sessions, as well as to my friends Eray, Meryem, Özge, and Deniz, who are as accessible as a phone even though they are physically separated from me, and to my friends Kıvanç and Huri Mine, who helped relieve some of the stress I was feeling at this site.

The greatest appreciation goes to my husband, who made a significant sacrifice to make this possible. Between work and school, I nearly disappeared, but he understood and supported me throughout. Thank you for contributing to my sleeplessness with your teas. Huge thanks to everyone who has always believed in me, supported me, accepted my absence, and patiently waited until I could once again contribute to their life. I am lucky and appreciative.

TABLE OF CONTENTS

CHAPTERS

APPENDICES

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

BILSEM	Science and Art Center, in Turkish, Bilim ve Sanat Merkezi
MEAs	Model Eliciting Activities
MoNE	Ministry of National Education
MMP	Models and Modeling Perspective
NAE	National Academy of Engineering
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
PISA	Programme for International Student Assessment.
STEM	Science, Technology, Engineering and Mathematics

CHAPTER 1

INTRODUCTION

There is no widely accepted definition of the concept of *giftedness* among researchers. Those that have been offered vary depending on the researcher's area of expertise, the focus of the research, and research trends over time (Türkman, 2020). While some researchers hold that a child's giftedness should be determined based on scores from standardized intelligence tests (Jensen, 1980; Terman, 1925), others claim that a number of relevant characteristics cannot be determined by such tests (Gagné, 2015; Renzulli, 2016; Stenberg, 2018). Eventually, the term giftedness has moved from intelligence-based definitions to a more multidimensional understanding of the concept (Winner, 2000). Gagne (2004) states that personal attributes such as motivation and temperament, along with both the environment and the interaction among these characteristics, and congenital giftedness all play an essential role in developing giftedness.

In the light of contemporary definitions and conceptualizations of giftedness, potentially gifted children should demonstrate specific characteristics. These characteristics include fast-paced learning, an extended concentration span, exceptional memory, the ability to understand complex concepts, and enhanced observational ability (Harrison, 2004). Thus, gifted children bring diverse learning characteristics to the classroom environment. For example, they can quickly solve problems and study abstract ideas transferred to diverse contexts (Gross, MacLeod, Drummond, & Merrick, 2001). Also, they can display intense curiosity, sensitivity, and desire to take on novel activities (Davis & Rimm, 2004). Hence, any program for gifted students should provide challenges and flexibility in addition to encouraging both higher-order thinking skills (van Tassel-Baska, 2003) and the development of personal characteristics such as curiosity, creativity, insight, perseverance, and imagination (Özyaprak, 2016; Türkman, 2020).

Since gifted students have such particular characteristics and unique skills, there is a need for differentiation in the curriculum (Assouline & Lupkowski-Shoplik, 2021; Ozdemir & Isiksal-Bostan, 2021; Winebrenner & Brulles, 2012). According to previous studies, these learners require specified instructional opportunities to address their academic needs (Borland, 2009; Johnson, 2000; Matthews & Foster, 2006). Nonetheless, studies have revealed that gifted students are rarely challenged in schools, particularly at the elementary level, due to teachers' inadequate and infrequent use of differentiation strategies (Reis et al., 2004; Tomlinson et al., 2003). As a result, the majority of gifted students fall behind in class, lose their motivation, and become bored (Diezmann & Watters, 2000). To address the unique needs of gifted students, teachers should use differentiation techniques that are easily implemented, positively impact student learning, and are interconnected with curricular standards of instruction for the student's grade level (Tomlinson, 2014).

Some countries have a long history of offering well-established gifted education programs, which take into account the special needs of these students (Heuser, Wang, & Shahid, 2017). However, Turkey has not kept pace with others in terms of its attention to gifted education. Since the foundation of the Republic in 1923, gifted students have been the most neglected category in the area of special education (Ataman, 1998). As the Turkish national curriculum has been developed largely - though not exclusively- according to the abilities and learning capacities of regular students, it has not met the educational needs of gifted learners (Levent & Bakioğlu, 2013). Some special education programs have, however, been prepared for these students, as their characteristics and needs require a different educational program than that used by their peers (Akgül, 2021; Tomlinson, 2014). So as to offer such an educational program, the Science and Art Centers (in Turkish, Bilim ve Sanat Merkezleri, BILSEM) were established by the Ministry of National Education in 1995. Through these centers, attempts have been made to reach more gifted students in order to stimulate economic, social, educational, and technological developments in the country. There are three main stages of student selection for BILSEM. Potentially gifted students are nominated by the classroom teacher via an observation form and then they take a group test. After that, individual intelligence tests are administered to students who achieve the required scores on the group tests. If students are able to demonstrate the required level of accomplishment at the end of these stages, they are recognized as gifted and are given the approval to enroll in BILSEM (Şahin & Zorlu, 2022). As the focus of BILSEM is to not interrupt gifted students' education in formal institutions (MoNE, 2012), but rather supplement it, BILSEM conduct only after-school activities. In sum, the overriding institutional goal of BILSEM is to provide educational opportunities for gifted students in line with their interests and enable them to use their existing capacity at the highest level (MoNE, 2012).

Despite BILSEM's attempt to provide a program of education appropriate for the gifted demographic, it seems to fall short of truly meeting the needs of the students. Kurnaz (2014) conducted a study to determine the current state, problems, and suggestions for the problems of BILSEM using workshop and meeting reports from the last five years and the opinions of thirty-two BILSEM administrators from various regions of Turkey. The study identified key challenges for the program, which seemed to derive from the lack of a teaching program framework for educational activities in BILSEM. The study went on to point out that the activities carried out in BILSEM did not correspond with the curriculum implemented in the students' regular school and were insufficient in attracting students' interest. Partially in response to this critical feedback, in 2016-2017, the MoNE prepared a teaching program framework and implemented it flexibly in BILSEM, attempting to resolve some of the problematic program components. Different implementations among BILSEM were eliminated, focus skills were established, and confusion about what teachers and students will do was resolved. Subsequently, Çetin and Doğan (2018) investigated these updates and identified issues in the scope and the implementation of the teaching program framework. In their study, it was concluded that the order, duration, and level of activities included in the teaching program framework were not appropriate, contained scientific errors, did not attract students' attention, and did not correspond with the curriculum implemented in schools. It was also found that these issues posed challenges for teachers. In a more recent study, Bildiren and Citil (2022) investigated the education of gifted students in Turkey from a historical perspective covering the period 1923-2020, and evaluated contemporary practices in gifted education at the national level. The researchers found that the implementation of the educational program is a common theme in research conducted with BILSEM teachers. The teachers emphasized that the program at BILSEM should be differentiated based on the special needs of gifted students. Based on the information given above, it can be concluded that there are some problems in BILSEM in a representative sample of Turkey, which may demonstrate that gifted students do not always receive a comprehensive education. Hence, it is necessary to develop and apply appropriate activities that target such problems to satisfy the needs of gifted students in BILSEM.

In the era of rapid innovative advancements and global challenges, the need for scientifically, mathematically, and technologically literate citizens in the 21st-century societies has generated a consensus on the necessity of improving students' knowledge and skills through STEM education (Caprile, Palmen, & Sanz, 2015). Recently, the importance of STEM education in both national and international curricula as well as policy documents has been emphasized (Akgündüz et al., 2015; MoNE, 2016, National Academy of Engineering and National Research Council, 2009). An important argument in support of STEM education is that people who work in the field of science, technology, engineering, and mathematics play a crucial role in economic growth and STEM education generates professionals and citizens equipped with 21st-century skills such as critical thinking and problem solving as well as collaboration, communication, creativity, and innovation (Akaygun & Tutak, 2016; Erduran, 2020; Yıldırım & Altun, 2014). In addition to these, STEM education provides opportunities for gifted students to nurture their interest and curiosity, develop creative solutions to complex problems, and make new discoveries (Lee, Baek, & Lee, 2013). Clinkenbeard (2007) claims that today's gifted children will substantially contribute to science, technology, as well as the business world in the future and that governments should generously invest in education programs for gifted children. For this reason, early participation in STEM integration activities is important for gifted students who are more likely to be included in the competitive mass of workers in the increasingly global world (Roman, 2012).

Model eliciting activities (MEAs), which incorporate client-driven, real-world situations, and engineering design processes in open-ended problem solving (Maiorca & Stohlmann, 2014), have the potential to serve as a means for STEM integration (Baker & Galenti, 2017; Kertil & Güler, 2016). MEAs are specifically developed to be accessible to all students, improve their modeling competencies, and produce documentation of their progression in technological, scientific, engineering, or mathematical thinking (Gainsburg, 2013; Moore, Miller, Lesh, Stohlmann, & Kim, 2013). MEAs extend problem-solving experiences to enable students to express their own ideas more deeply and improve their thinking skills (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016). In addition, MEAs can be used to reveal students' mathematical reasoning, explanation, and justification skills while engaged in the activity (Doerr & Lesh, 2011; Hamilton, Lesh, Lester, & Brilleslyper, 2008; Mentzer, Huffman, & Thayer, 2014). Students often find MEAs challenging because of the difficulty in describing real-life scenarios mathematically; yet, active participation in MEAs can be beneficial in reinforcing their mathematical and scientific knowledge and skills as well as strengthening their understanding of the world (Groshong, 2018). Such activities expand classroom experiences addresing the different learning styles and strengths that students bring to the classroom (English, 2013; Hamilton et al., 2008). In short, the nature of MEAs make them an important tool for the education of gifted students, as differentiated instruction for these students can be accomplished through their use. According to Gross and colleagues (2001), gifted students require less repetition and have a greater preference for higher levels of thinking. As a result, MEAs have the potential to meet both the demands of the challenges and the needs of gifted students.

It is important for students to see how the mathematics they are learning is applied in real-world circumstances (English, 2011). MEAs allow this by incorporating both a client and a user and thus serving as a bridge between mathematics and engineering, while also adding a new dimension to real-world issues (Mann et al., 2011). Despite the fact that the challenges are mathematical, the students are directly involved in the engineering design process (Cunningham & Hester, 2007). The engineering design process is a crucial, lifelong skill that requires students to apply and develop a variety of different types of thinking (Mann et al., 2011). More precisely, they seek information about the client's requirements, brainstorm a variety of model proposals, develop a strategy in collaboration with their group members, build and test their first model using the data given by the client, and then use additional but similar data to revise their model (Mann et al., 2011). In order to produce a successful solution to an MEA, students must ask effective questions, obtain, examine, and interpret data, and present their results. A similar set of skills is also necessary to solve problems in the engineering design process. Hence, the iterative nature and model-elicitation which are essential features of mathematical modeling process and the engineering design process make MEAs effective tools for integrating engineering (Diefes-Dux et al., 2004; Lyon & Magana, 2021). Even if the literature emphasizes the similarities of these two processes (Groshong, 2018), the main issue is that the relationship between these processes is not examined in the long term. Specifically, engineering-based MEAs are defined as the authentic engineering situations in which students are repeatedly asked to express, test, and refine or revise their current ways of thinking while attempting to generate a structurally significant product (English & Mousoulides, 2011). Through these activities, students develop and apply mathematical and scientific concepts to real-world engineering problems, as well as explaining and predicting the nature of the problems (English & Mousoulides, 2011). Hence, engineering-based MEAs provide challenging tasks for gifted students with varying degrees of complexity, breadth, and depth of understanding (Dailey, 2017). In addition, the common attributes of gifted students and successful engineers such as high-level thinking and problem-solving skills, and curiosity (Mann et al.,2011) play a significant role in promoting solutions in engineering-based MEAs. For this reason, engineering-based MEAs can be used as a vehicle to fulfill the needs of elementary gifted students in the classroom.

Due to the fact that modeling needs a variety of competencies and skills in addition to domain knowledge, it can be a cognitively challenging activity for gifted students. Engineering-based MEAs can be used as a way to reveal students' conceptual understanding and the processes they develop during their efforts to solve real-world problems (Lesh & Doerr, 2003). Accordingly, modeling competencies are described as the skills and abilities necessary to complete the modeling process appropriately, and the individual must be willing to participate in this process (Maaß, 2006). The cognitive perspective of modeling is crucial to describe, analyze, and explain what is happening in students' minds when they are engaged in the modeling process (Blum, 2011). According to Biccard and Wessels (2011), cognitive modeling competencies cover the entire modeling process. In accordance with these definitions, modeling competency is associated with the specific description of the modeling process (Jensen, 2007). The phases of the cycle are examined as competencies and can be utilized as criteria for assessments and referred to as cognitive modeling competencies.

Although mathematical modeling perspectives vary according to application fields and present definitions that highlight distinct aspects of mathematical modeling (Hıdıroğlu & Bukova-Güzel, 2016; Kaiser & Sriraman, 2006), the major characteristics of the main processes are stated. However, the specific characteristics of the sub-steps of the process are not discussed in detail, especially for mathematical modeling using the cognitive approach (Blomhøj & Jensen, 2003; Galbraith & Stillman, 2006). While mathematical modeling plays a crucial role in mathematics teaching, real modeling problems are still rarely used in mathematics lessons (Blum, 2011; Turner, 2007). Despite the fact that competencies have been defined in the literature and a certain measure and determination of these competencies have been performed, there is still more research to be conducted in this field (Maaß, 2006). Regarding gifted students, it was stated that they can use their previous knowledge by adapting it to meet the original conditions of the problem situation (Pativisan, 2006), they can think more flexibly and creatively when addressing problems than their peers (Bayazıt & Koçyiğit, 2017), and they can develop novel solutions and strategies in addition to the usual approaches (English, 2007b). Hence, it is critical to determine how gifted students reflect these special characteristics in the modeling process. Consistent with this view, teachers may need to know how to design and implement activities that aim to help students transfer their previous knowledge to real-life problems in order to reveal their competency. By offering engineering-based MEAs developed by considering the needs of gifted students, the present study would provide an answer to the problems such students and their teachers often experience. To put it plainly, it would create a learning environment where they can reflect their unique characteristics through such activities. Although the available literature suggests engineering-based MEAs for gifted students (Mann et al., 2011), there is no validated framework merging mathematical modeling and engineering competencies for how and to what extent gifted students demonstrate their competency when engaged in engineering-based MEAs. An in-depth investigation of the cognitive modeling competencies of elementary gifted students based on a theoretical framework would be an important step in addressing the problems that were discussed above. This investigation would shed light on the design of activities for the specific needs of these students and the validation of the framework for them. Thus, the current study analyzes the cognitive modeling competencies of elementary gifted students when engaged in engineering-based MEAs.

1.1. Aim of the Study and Research Questions

The aim of this dissertation study is to investigate the cognitive modeling competencies of elementary gifted students when they are engaged in engineering-based model eliciting activities. Hence, this study addresses the following research question:

1. To what extent do elementary gifted students display cognitive modelingcompetencies (internalizing/interpreting/structuring/ symbol izing/adjusting/organizing/generalizing) when they are engaged in engineering-based model eliciting activities?

1.2. Significance of the Study

The significance of this dissertation study lies in its contribution to both literature and educational practices. The primary contribution of this study to literature is on giftedness. It might provide evidence on contemporary understanding of giftedness and specific instructional practices to address their unique needs. For example, a framework which has been provided by different gifted education curriculum models such as the Parallel Curriculum Model (Tomlinson et al., 2008) and the Enrichment Triad Model (Renzulli, 1977) was used in the present study in the creation of challenging and meaningful learning experiences tailored to gifted students' unique interests and readiness. To differentiate instruction based on gifted students' needs in the 21st-century, the engineering design process provides students with a similar learning experience to the models mentioned, and is thus proposed as an instructional approach (Dailey, 2017; Mann & Mann, 2021). Beyond these time-tested foundational approaches, the current study also takes inspiration from contemporary theories (e.g. Gardner, 2011; Renzulli, 2021; Sternberg, 2019), which focus on gifted students' acquisition of 21st-century skills. In addition, a new paradigm for the conception of giftedness has been proposed to meet the needs of the 21st-century, according to which learning experiences of these students should contribute to the welfare of people through their competence, commitment to a task, and concern for others (Chowkase, 2022). Hence, the stated nature of the engineering design process may provide gifted students with suitable learning environments to practice these three pathways and validate them with research. Although the engineering design process is a recommended approach for gifted students in literature (Dailey, 2017; Mann et al., 2011; Mann & Mann, 2021), the body of empirical literature on gifted students' engagement in the engineering design process is limited, yet emerging (Han & Shim, 2019; Sen, 2018; Sen, Ay, & Kiray, 2021). In this regard, the findings of this study may contribute to the literature of gifted education by investigating gifted students' engagement when they are involved in the engineering design process.

The second contribution to literature on gifted education is the potential of contextualizing giftedness within the models and modeling perspective (MMP), a comprehensive theoretical approach founded on constructivist and sociocultural theories (Lesh & Doerr, 2003). According to this perspective, mental models are used to help students make sense of real-world circumstances (Erbaş et al., 2014; Lesh & Lehrer, 2003). On the other hand, in the conventional approach, which is the second perspective of MMP, mathematical modeling is taught in a method that assumes that students learn the information in a predetermined manner (Lesh & Zawojewski, 2007). According to the research studies, the conventional approach may deprive gifted students of the opportunity to contribute and show diversity to problem-solving and design processes, to exhibit their competence, and to increase the number of students that are interested in pursuing professions in engineering (Eisenhart & Finkel, 1998; Margolis & Fisher, 2002). Accordingly, the need to build the MMP in engineering education has been specified by the researchers (Gainsburg, 2006; Zawojewski, Hjlamarson, Bowman & Lesh, 2008). Thus, this study might provide empirical evidence of the MMP in engineering education research for elementary gifted students.

The current study's another contribution to the literature on giftedness is that it attempts to determine connections between the mathematical modeling process and the engineering design process for elementary gifted students. Groshong (2018) emphasizes the importance of determining resemblances in related processes on teaching and learning practices. These parallelisms can be useful for modeling situations that are often based on interdisciplinary knowledge and skills. The strong association between the mathematical modeling cycle and the engineering design process highlights the iterative nature of both processes (Zawojewski et al., 2008). Those processes also provide multiple entry and exit points to problems where students elicit the learning of content or process (Mann et al., 2011). However, the critical relationship has been ignored in designing modeling experiences and engaging in such processes (English, 2017), especially for elementary (Dorie, Cardella, & Svarovsky, 2014; Portsmore, Watkins, & McCormick, 2012) and gifted students (Mann et al., 2011). Hence, this study may provide empirical evidence recognizing parallelism between mathematical modeling process and the engineering design process for a certain group of students who are identified as gifted.

In addition to its possible contribution to the literature on giftedness, this study may contribute to mathematical modeling and engineering literature by merging them. The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes the need to include mathematical modeling from pre-school to high school curriculum. In accordance with this view, many countries around the world have revised their mathematics curriculum to include mathematical modeling (Common Core State Standards Mathematics [CCSI], 2014; Ministry of Education Singapore, 2007). In a similar vein, the revised elementary Turkish mathematics curriculum (MoNE, 2013; 2018) is based on the mathematical modeling approach. However, the findings of the study (Doğan, Gürbüz, Erdem, & Şahin, 2019) indicate that mathematical modeling is not reflected in any available elementary mathematics textbooks in Turkey, especially for some topics, and the models used are limited to only concrete and visual structures. Elementary mathematics teachers' lack of knowledge and materials covering the integration of mathematical modeling in the classroom are also stated in the literature (Dedebaş, 2017; Işık & Mercan, 2015). Hence, the studies to be conducted on modeling for elementary school students are necessary and important. In particular, model eliciting activities (MEAs) have the potential to bridge the gap between mathematics and engineering by putting mathematics and science practices in the real world and to enhance another aspect of the challenges related to real-world situations (Diefex-Dux, Hjalmarson, Miller, & Lesh, 2008). By integrating engineering-based MEAs in elementary classrooms, students could engage in creative and innovative real-world problem experiences involving the engineering design process and mathematical modeling process, thus building on their existing mathematical and scientific knowledge, as well as work collaboratively to solve complex problems

(English & Mousoulides, 2011). Thus, the open-ended nature of MEAs enable students to address the creation of a solution at an adequately challenging level. Such characteristics enable MEAs to meet both the competencies necessary for the engineering design process and the needs of the gifted students (Mann et al., 2011). Although there has been literature mainly focusing on using MEAs for undergraduate engineering education to advance students' creativity, motivate them to use advanced engineering knowledge and techniques, and promote conceptual thinking in these students (Moore & Diefex-Dux, 2004; Moore et al., 2014; Yıldırım, Shuman & Besterfield-Sacre, 2010), there has been limited literature on using MEAs to integrate engineering into elementary classrooms (Cunningham & Hester, 2007; English, 2007; English & Mousoulides, 2011; Sen, 2018). In this regard, the current study could make a significant contribution to the existing literature by providing evidence of elementary gifted students' engagement in engineering-based MEAs.

The other important contribution of this study might be the provision of evidence-based instructional practices of gifted students through a combination of mathematical modeling competencies and engineering competencies. To address the needs in raising global citizens in this era, the scientific community has placed emphasis on developing modeling competencies of individuals in order to cope with complex real-life problems (Biccard, 2010; Blomhøj & Jensen, 2003; Ludwig & Xu, 2010; Maaß, 2006; Sahin & Erarslan, 2017). A review of the international literature reveals that there are studies seeking to determine and develop the modeling competencies of students from different age groups (Biccard, 2010; Biccard & Wessels, 2011; Blomhøj & Jensen, 2003; Kaiser, 2007; Ludwig & Reit, 2012; Maaß, 2006). An examination of the national literature reveals an increase in studies examining the modeling competencies of students (Hıdıroğlu & Bukova Güzel, 2016; Kabar & İnan, 2018; Kocayayla, 2019; Şahin & Erarslan, 2017; Yıldırım, 2019). However, a meta-synthesis study directed by Aztekin and Şener (2015) indicated that the participants of mathematical modeling studies are generally prospective teachers, and that those studies that were conducted on students mostly involve high school students. These studies have shown that, in general, model eliciting activities improve students' modeling competencies (Biccard, 2010; Blomhoj & Jensen, 2003; Kaiser, 2007; Maaß, 2006), that the older the students are, the more successfully they completed the modeling process (Henning & Keune, 2007; Ludwig & Xu, 2010), and that students struggle in the competence of making an assumption (Yıldırım, 2019) as well as in verification and justification (Kabar & İnan, 2018; Kaiser, 2007; Maaß, 2006; Tekin-Dede & Yılmaz, 2015). In addition, Koç (2020) found similar findings in her examination of theses and dissertations over the past two decades. A search of the literature revealed that few studies conducted with elementary students. There is still considerable work to be done in this field, even though the literature has already stated some competencies and some of these competencies have been measured and determined. According to Maaß (2006), there is scant comprehensive research on modeling competencies and the limitations that accompany them. Still, the useful components of the admittedly limited studies have been identified and the association between mathematical modeling competencies and engineering competencies has been investigated (de Villiers, 2018; Huffman, 2015). In this way, crucial engineering competencies that can co-develop with mathematical modeling competencies through MEA in the study by de Villiers (2018) have been mapped. In this regard, the current study can make a significant contribution to addressing the gap by determining the modeling competencies of gifted elementary school students and which competencies are revealed at what level. It has the potential to enrich and adapt the framework for elementary gifted students as a result of the validation of the framework in various settings and with various student groups. Interdisciplinary in nature, this study might provide major contributions to the literature in STEM and gifted education.

The educational practices are another area in which the present study could provide a potentially valuable contribution. This study describes possible contributions to educational settings from the point of view of students, teachers, and educational resource designers. At the practical level, MEAs could be a vehicle for integrating engineering and for incorporating aspects of 21st-century challenges into mathematics education (English, 2017). These connections can be established for two major reasons: (1) students use interdisciplinary knowledge for real-world problems in MEAs and (2) the competencies required to conduct a modeling process overlap with 21st-century skills to a considerable extent (Maass, Geiger, Ariza, & Goos, 2019). Engagement in MEAs could enable intra-and inter-group collaborations to be used as important intervention and scaffolding techniques (English & Mousoulides, 2011). The present research seeks to monitor the effectiveness of each of these interventions on students' progress in constructing and strengthening their models. Hence, the findings of this study may be useful for future research on elementary gifted students' engagement in MEAs. The determination of their competencies assists teachers in the development and adjustment of engineering-based MEAs, as well as in providing greater support for students' progress. The findings of this study may thus be utilized in developing teaching methods and materials that facilitate students' development of modeling competencies in future classrooms and BILSEMs. Additionally, this study may be helpful for curriculum design in that the findings promote the development of more effective and supportive educational or special programs for elementary gifted students

1.3. Definition of Important Terms

Gifted Students: Gifted students refer to the individuals who show high performance in intelligence, creativity, leadership, motivation, or special academic fields compared to their same-age peers (MoNE, 2012). In this study, the students who attended BILSEM were recognized as gifted students.

Elementary Students: Elementary students are defined as students who attend kindergarten through grade 6. In the present study, elementary students refer to students between the ages of 9 and 12.

Models: Models are conceptual systems that are stated using external notation systems and are used for the purposes of constructing, describing, or explaining the actions of other systems. It is feasible to create models of other systems by putting together elements, relationships, operations, and rules regulating interactions (Lesh & Doerr, 2003). In this study, the model includes the conceptual systems in students' minds that students utilize to understand and interpret the given problem situation, as well as the external representation systems for these conceptual systems (ideas, representations, rules, and materials).

Mathematical Models: The conceptual systems that are concerned with the underlying structural properties of the relevant systems is defined as mathematical models (Lesh & Doerr, 2003). In this study, the mathematical models consist of a variety of representations, operations, and relations rather than a single one in order to assist in the comprehension of real-world situations.

Mathematical Modeling Process: The mathematical modeling process is described as the process by which students engage in a model eliciting activity by progressing through the phases of a modeling cycle. The modeling cycle includes the following phases: understanding the problem, simplifying the problem, mathematizing, working mathematically, interpreting the problem, validating, and reporting the solution process. In this study, modeling refers to the process through which students create their own models of a problem rather than applying a known model (Lesh & Doerr, 2003). In addition, mathematical modeling is defined as the process of developing shareable, adjustable, and reusable conceptual tools or mathematical models for describing, predicting, and regulating real-world circumstances (Lesh & Doerr, 2003).

Mathematical Modeling Competencies: Mathematical modeling competencies refer to skills and abilities required to carry out modeling processes in an appropriate and goal-oriented way, as well as the willingness to put these skills and abilities into practice (Maaß, 2006). In this study, mathematical modeling competencies are merged with engineering competencies and employed as modeling competencies.

Model-Eliciting Activities (MEAs): Model eliciting activities referred to thought revealing problems that ask students to develop a technique, explanation, prediction, description, or solution to a problem for a specific client in a given situation. They are developed based on six design principles (Lesh et al., 2000), which include model construction, reality, self-assessment, model documentation, model shareability, reusability, and effective prototype (Diesfes-Dux, Hjalmarson, Miller, & Lesh, 2008).

Engineering-Based Model Eliciting Activities: Engineering-based model eliciting activities are open-ended, real-world, client-driven, and complex situations which focus on the engineering design process and the development of higher-order understandings that lead to solutions (Diefes-dux et al., 2004; Mousoulides & English, 2011). The activities promote a future-oriented approach to learning by providing opportunities for students to elicit their own mathematical and scientific concepts as they analyze and solve the problem (Lesh & Zawojewski, 2007; Zawojewski et al. 2008). In the current study, engineering-based model eliciting activities refer to three activities developed within the scope of this study; namely, Bridge Construction, Mars Lunarcrete, and Dr. Ahmet’s Will.

Engineering Design Process: The engineering design process is defined as asking about the details of the problem and constraints, imagining many different ideas, and planning, creating, improving, and documenting (Moore & Richards, 2012).

Engineering Competencies: Engineering competencies are the knowledge, skills, and abilities required to perform the work of an engineer (de Villiers, 2018).

Cognitive Modeling Competencies: Cognitive modeling competencies are defined as the abilities to comprehend a real-world problem and develop a model based on it, to create a model from the real-world model, to solve mathematical problems within the mathematical model, to interpret mathematical results in the context of real-world situations, and to validate the solution (Maaß, 2006). For this study, the mapping approach is used to recognize what the cognitive modeling competencies mean and establish a method for recognizing and identifying these competencies in students' work. This technique reveals the relationship between engineering competencies and mathematical modeling competencies to respond to the search for cognitive modeling competencies to be investigated (de Villiers, 2018).

CHAPTER 2

LITERATURE REVIEW

The purpose of this study is to investigate the cognitive modeling competencies of elementary gifted students when they are engaged in engineering-based model eliciting activities. Accordingly, this chapter is divided into seven sections. The first section presents the conceptions and definitions of giftedness, the characteristics and instructional needs of these students. The second section addresses models and modeling, approaches to mathematical modeling, and the mathematical modeling process. The following section states the conceptual framework drawn from models and the modeling perspective. The fourth section presents the definitions of engineering design process and relevant literature. The next section addresses model eliciting activities and engineering-based model eliciting activities. The sixth section presents the definitions of modeling competency and illustrates the literature on elementary students and mathematical modeling competency. The last section presents the conclusions based on the literature review which guided this study.

2.1. Definitions and Conceptions of Giftedness

Terman (1925), a pioneer in the field of giftedness research, defined giftedness as having a general intellectual ability in the top one percent of the Stanford-Binet Intelligence Scale. While some researchers, including Terman (1925), argued that giftedness is determined by an intellectual test score (Jensen, 1980; Morris, 1977), others argued that these tests do not capture a wide range of characteristics (Gagne, 2015; Renzulli, 2016; Stenberg, 2018; Torrance, 1974). Correspondingly, the concept of giftedness has continued to be criticized by researchers (Türkman, 2020). Over time, the focus of research has shifted away from the concepts of genius and intelligence toward the concepts of field-specific ability (Van-Tassel Baska, 2005) and creativity (Gagné, 2005).

Accordingly, there have been proposals for a more comprehensive definition of giftedness that includes several dimensions of cognitive functioning, including creativity. Another prominent pioneer in this field, Guilford (1950) brought the concept of creativity to the forefront of giftedness research. Guilford (1967) defined creativity as divergent thinking, which consists of four components: fluency (quantity of ideas), flexibility (quantity of types of ideas/categories), originality (uniqueness), and elaboration (the number of details). In addition, many recent definitions of creativity emphasize the importance of usefulness as a fundamental component of creativity, defining it as the extent to which an idea or product produces a beneficial answer or solution within a particular setting (Dow, 2017; Runco & Jaeger, 2012). Following Guilford’s study, creativity has been a major component of most theories of giftedness.

Similarly, Torrance (1974) emphasized divergent thinking as a requirement for creativity and designed Torrance Tests of Creative Thinking (TTCT). The TTCT has not been just about measuring creativity, but about using it as a tool to gain a deeper understanding of the factors that promote and nurture creativity in general (Kim, 2006). Other leading theories of giftedness also consider creativity, including the Differentiated Model of Giftedness and Talent (DMGT) of Gagné (2005), the Three-Ring Conception of Renzulli (1986, 2005), and the Wisdom, Intelligence, Creativity, Synthesized (WICS) Model of Stenberg (2005). To put it more explicitly, Gagné (2005) defined gifts as intrinsic abilities (or aptitudes) across many domains (intellectual, creative, socio-affective, and sensorimotor), and talents as exhibited mastery of these abilities. In the Three-Ring Conception of Renzulli (1986, 2005), giftedness emerged through the interaction of above-average ability, creativity, and task commitment, and each of these characteristics contributes significantly to the development of gifted behavior. According to Stenberg’s (2005) WICS model, giftedness is defined as a combination of wisdom, intelligence, and creativity. Taken together, these studies support the notion that each definition has incorporated a new dimension to giftedness in order to generate better representations of the gifted demographic and its diversity (Turkman, 2020). As many studies on the conceptions of giftedness are currently available, the most widely recognized theories are detailed below.

One of the important conceptions of giftedness is constructed on Gagné (1985)'s definition of giftedness, which is the existence of innate abilities or aptitudes that are untrained, spontaneously expressed, and widely recognized in at least one ability domain as being outstanding. In subsequent research, Gagné (2010) refined the model and highlighted three catalysts: the intrapersonal, the environment, and chance. Natural abilities (gifts) are transformed into a carefully cultivated ability (talent). through a developmental process. Catalysts have an effect on the development process, either promoting or impeding development. The intrapersonal catalyst comprises attributes such as physical appearance, motivation, and personality. In addition, the environmental catalyst is the environment that contains individuals, provisions, and events (Gagné, 2005, 2010). Chance has an impact on both the intrapersonal catalyst and the environmental catalyst, as well as on the developing process itself. The degree of chance can be reduced to some extent, for example through giving all students a high standard of education in every region of a country, however, certain components of chance will still play a role in a child's development.

Another important conception of giftedness is constructed on Sternberg's triarchic theory of intelligence, which involves analytical (componential), creative (experiential), and practical (contextual) facets (Sternberg, 1986). Each of the three facets represents a different sub-theory of intelligence. Stenberg (1986) asserted that the interaction between these sub-theories is critical to a thorough understanding of intelligence. According to this theory, the three components of giftedness are the individual's information processing ability, one's previous experiences with a particular activity or problem, and one's relationship with the external world. Sternberg (1986) defined intelligence as the intentional adaptation to, structuring of, and selection of real-world situations that are significant to one's life. In addition, he argued that individuals who are recognized as gifted in one culture or context may not be regarded in the same way in another culture or setting.

Similar to Stenberg's theory, Gardner's notion of giftedness is based on the theory of intelligence. Gardner proposed that an individual possesses eight abilities or intelligences: spatial, linguistic, logical-mathematical, bodily-kinesthetic, musical, interpersonal, intrapersonal, and naturalistic (Gardner, 1983). In subsequent research, he suggested that moral, spiritual, and existential intelligence may also be included in this list as ‘candidate’ intelligences (Gardner, 2011). He pointed out that there are many different combinations of intelligence, and by drawing on these combinations, individuals are better equipped to solve problems or create products that are appreciated in one or more cultural settings. Gardner (1983) defined gifted individuals as ones who progress quickly through a domain of knowledge because of strengths in their intelligence and opportunities in the environment to improve them.

One of the comprehensive theories of giftedness is Renzulli’s (1978) three-ring definition. Its difference from traditional theories based on Intelligence Quotient (IQ) tests (Jensen, 1980; Terman, 1925) and Gardner’s (1999) Multiple Intelligence Theory lies in its focus on diverse facets of giftedness. The conceptual framework of the three-ring definition is about the relationship among three interconnecting clusters of characteristics (above average ability, task commitment, and creativity) in the specific domains of individual performance (Renzulli, 2005, 2012). These characteristics constitute the major dimensions of creative productivity, and it is the interaction between these characteristics that produce the favorable conditions for a creative and productive process to proceed (Renzulli, 2012). More precisely, above-average ability refers to both general ability and specific ability. The ability to process information, to integrate experiences in a way that leads to appropriate and flexible responses to new situations, and to engage in abstract thought are all examples of general abilities. On the other hand, specific ability is associated with the manner in which individuals express themselves in everyday life. Intellectual, academic, artistic, visual, and performing arts, leadership, and psychomotor domains are only a few examples of specific abilities. The other dimension of giftedness is task commitment, which is related to an individual’s ability to be motivated in a certain field or to complete a particular work. Task commitment can be defined as a combination of characteristics such as perseverance, persistence, continuous effort, dedication, self-confidence, belief in one’s ability to perform significant work, and action directed toward one’s area of interest. The last dimension of the three-ring definition is creativity. Creativity can be understood as the creation of a novel, distinct idea or product, and it is enabled by the individual’s distinctive way of thinking and the use of novel methods (Renzulli, 2012). Creativity is also identified by characteristics such as the willingness to take risks when necessary, the ability to communicate clearly and understandably, the ability to be original in what they produce, the ability to predict results easily, and the ability to solve problems (Karabey & Yürümezoğlu, 2015).

According to Renzulli (2005), the research has focused on two types of giftedness: schoolhouse giftedness and creative-productive giftedness. Schoolhouse giftedness is defined as the ability to perform well in school and is easily measured by cognitive ability tests, such as IQ tests. This type is generally used to select students for entrance into special programs. On the other hand, creative-productive giftedness is a product-oriented approach, which implies that individuals who possess this type of giftedness are producers rather than consumers of original knowledge, materials, or products. Renzulli (2005) argued that education programs addressing the second type of giftedness should be distinct from regular school programs. When designing and developing appropriate educational programs, three components of Renzulli's (2005) giftedness conceptualization should be considered. This means that a wide range of educational opportunities and services are required by individuals who exhibit or are capable of establishing interaction among the three clusters. These opportunities and services are not typically offered by regular instructional programs. According to this perspective, it is crucial to educate all children in problem-solving skills and procedures. Additionally, it highlights the importance of creating motivational learning experiences based on real-life problem solving (Mammadov, 2012).

Following Renzulli's (1978) theory, Sternberg and Zhang (1995) proposed a pentagonal implicit theory. This theory specifies five conditions, each of which is individually essential and collectively adequate for an individual to be recognized as gifted. There are five criteria an individual must meet: (a) excellence, (b) rarity, (c) productivity, (d) demonstrability, and (e) value. Firstly, the excellence criterion states that the individual is exceptional to peers in some dimension or collection of dimensions. Although the definition of 'extremely high' varies according to context, the gifted individual is always considered as having an abundance of something, whether it is creativity, wisdom, or another ability or construct. Secondly, the rarity criterion indicates that the individual must possess a high level of an attribute that is unusual when compared to their peers. Thirdly, the productivity criterion asserts that the dimension(s) on which an individual is deemed exceptional must result in or have the potential to result in productivity. Fourthly, the demonstrability criterion maintains that the exceptionality of individuals on the dimension(s) that define giftedness be provable through one or more valid assessments. Lastly, the value criterion specifies that an individual must demonstrate excellent performance in a dimension valued by one's society. Hence, individuals who meet all five of this theory's requirements are determined as gifted.

Moreover, in recent studies, Hallahan, Kauffman, and Pullen (2009) described giftedness as a combination of cognitive superiority -not necessarily genius-, creativity, and motivation of sufficient magnitude to distinguish the child from the vast majority of age peers and enable them to make a significant contribution to society. Crammond (2004) also suggested that a rigorous definition of giftedness is not required. She used the example of scientists, who had to be able to identify the universe before they could research its nature. On the other hand, Webber (2011) asserted that while schools provide different education programs for gifted students, they must have some working definition of giftedness to be able to decide on student selections and qualifications for these programs. She further discussed Crammond's theory that all students must participate in a completely individualized education plan which is beneficial but impractical for schools where some categorization for students is needed in order to provide them education services.

According to a more recent study, Chowkase (2022) proposed that the term giftedness should be revised in order to better serve the demands of the 21st-century population. He supposed the idea that any conception of giftedness is solely based on one's own interest is inadequate. Therefore, concern for others became an integral element of his conceptualization. According to the three C's theory of giftedness, gifted behavior is characterized by the interaction of three clusters of human characteristics: competence in one's action, commitment to the task, and concern for others as shown in Figure 2.1 (Chowkase, 2022). The first two components of this conception are derived from Renzulli's (1978) components three-ring model for giftedness , which describes giftedness as an interaction of above-average ability, task commitment, and creativity. According to the three C's theory of giftedness, the above-average ability is associated with competence. Aside from that, creativity is the result of both creative competence and task commitment. However, one distinguishing feature of the three C's notion is the inclusion of concern for others as an intrinsic aspect of giftedness as opposed to other conceptions of giftedness. It is in this context that the person is positioned within their social and global setting, which reflects the true nature of human life. To summarize the components of the three C's theory of giftedness, competence is referred to the collection of abilities and skills that are required to efficiently carry out a task. Task commitment is described as an individual's motivational desire to complete a task. Concern for others is characterized as perceiving the needs and challenges of others, establishing a sense of belonging with them, experiencing the emotional desire to assist them, and cultivating the motivation to act on that perception (Chowkase, 2022). In line with these views, this theory sheds light on the concept of giftedness in the current study and gifted individuals are the ones who, through their competence, task commitment, and concern for others, demonstrate a promise to contribute to the welfare of others.

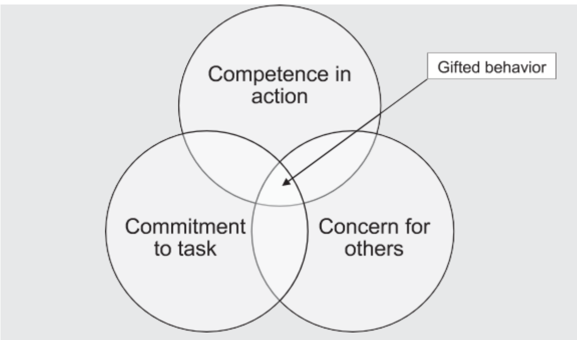


Figure 2.1. Three C's Conception of Giftedness (Chowkase, 2022, p.2)

The majority of proposed theories of giftedness have focused on individual characteristics such as intelligence, creativity, and motivation. Some have added environmental elements that either support or hinder the definition of giftedness. Studies have concluded that giftedness has been defined in many ways, with different suggestions (Chowkase, 2022; Gardner, 2011; Renzulli, 2012; Sternberg & Zhang, 1995) and researchers have not agreed on a single definition. The Marland Report, published by the U.S. Commissioner of Education, Sydney Marland in 1972, provides one of the most comprehensive definitions of giftedness:

> Gifted children are those identified by professionally qualified people who by virtue of outstanding abilities are capable of high performance. These children who require differentiated educational programs and/or services beyond those normally provided by the regular school program in order to realize their contribution to self and society. Children capable of high performance include

> those with demonstrated achievement and/or potential in any of the following areas, singly or in combination: general intellectual ability, specific academic aptitude, creative or productive thinking, leadership ability, visual and performing arts, psychomotor ability (p. 2).

Correspondingly, gifted children are defined for the present study as individuals who show high performance in intelligence, creativity, leadership, motivation or special academic fields compared to their same-age peers (MoNE, 2012; National Association for Gifted Children, 2005). In light of these, different theories highlight different aspects, but cognitive abilities, achievement, motivation, and, in some cases, involvement with a certain area are considered as crucial in many of these views. However, they all propose that gifted children should receive an education tailored to their unique characteristics. Thus, the following section of the literature review presents the characteristics of gifted students.

2.1.1. Characteristics of Gifted Students

In accordance with the definitions of giftedness, the cognitive and affective characteristics of gifted students are different from their same-age peers (Kücükoglu, 2014). In this regard, students who have the potential to be gifted are expected to exhibit certain characteristics, while others can be acquired through instruction. Thus, the key point that emerges is that there is a hidden potential with a genetic component of disposition, on the one hand, and observable performance, achievement, or skill, on the other (Singer, Sheffield, Freiman, & Brandl, 2016). Although each gifted student has unique characteristics, researchers have identified numerous cognitive and affective characteristics that are common to many gifted students (Clark, 2008; Hoh, 2008).

With respect to their cognitive aspects, gifted students are capable of rapidly encoding complex information in a comprehension process, concentrating for extended durations, and absorbing and retaining a considerable amount of information (Harrison, 2004). In addition, these students are curious and have a wide range of interests, some of which are intense (Clark, 2008). Gifted students frequently have a strong desire to learn more about their own interests (Hoh, 2008). In other words, they are persistent, goal-oriented, and deeply interested in the issues they care about. Moreover, they prefer to engage in complex and challenging activities (Sayı &

Yurtseven, 2021). They require a certain amount of autonomy in their learning environments (Van Tassel-Baska, 2003). Also, they engage in deep learning because of their self-motivation to learn (Davis & Rimm, 2004). They recognize uncommon connections between fields or concepts (Wellisch & Brown, 2013). They are capable of transferring knowledge and applying it to novel situations (Vogelaar & Resing, 2018). Additionally, gifted students come up with original ideas and solutions to problems (Betts & Neihart, 2017). They develop their own ways of thinking about problems and ideas that are unique to them (Wellisch & Brown, 2013). They possess accelerated and flexible thought processes (Clark, 2008). For instance, they complete the problem-solving process faster than predicted (Cooper, 2000). Similarly, Sak (2014) stated that gifted students act on the basis of facts and evidence throughout the problem-solving process and that rationality serves as the foundation for their views during the decision-making process.

Besides, they learn things at an earlier age than their peers (Bates & Munday, 2005). Gifted students exhibit a number of distinguishing characteristics, among which are the development of abstract thinking ability and advanced reasoning ability (Davaslıgil, 2002; Johnson, 2000). Extraordinary imagination, advanced language development, and verbal ability are other characteristics that may be observed in gifted students (Miedijensky, 2018). According to VanTassel-Baska (2003), gifted students have three key characteristics. These are (a) precocity, (b) intensity, which refers to the ability to focus on activities that interest them intensely, and (c) complexity, which refers to the ability to engage in higher-order thinking, deal with complex concepts, and appreciate challenging activities.

Given the subject-specific nature of mathematics, mathematical giftedness requires a combination of specialized mathematical skills as well as personal characteristics (Ozdemir & Isiksal Bostan, 2021; Singer et al., 2016). In particular, mathematically gifted students are those who look at the world from mathematical views (Krutetskii, 1976). Mathematically gifted students can identify relationships among topics, concepts, and ideas without receiving formal instruction (Rotigel & Fello, 2004). According to Greenes (1981), when compared to a general group of students studying mathematics, mathematically gifted students demonstrated the ability to: spontaneously form problems, flexibly handle data, demonstrate mental agility through idea fluency, organize data, interpret data with originality, transfer ideas, and generalize. Hence, mathematically gifted students need to study mathematics “in greater depth, making more connections and generalizations than others” (Sheffield, 1994, p. 15).

In addition to the cognitive characteristics that general giftedness and mathematical giftedness, there are affective characteristics that may be apparent in gifted students (Clark, 2008; Renzulli, 2004). Strong sensitivity to the feelings of others, emotional depth and intensity, and a high level of moral judgment are all characteristics that may be observed in this group (Davis, Rimm, & Siegle, 2014). Other common characteristics include perfectionism, high self-awareness, high expectations of self and others, and a sense of justice (Clark, 2008; Sak, 2010). Indeed, they have feelings and thoughts regarding social problems. Thus, the role of gifted students is crucial in bringing solution suggestions in areas where society requires assistance and in drawing inferences based on the society’s value judgments (Sak, 2014). Many studies have demonstrated that gifted students are proficient communicators, and particularly adept at interacting with others on their own mental level (Ogurlu, 2010; Saranlu & Metin, 2012). The aforementioned cognitive and affective characteristics result in a variety of needs that should be addressed by teachers in the classroom setting. That is to say, these characteristics can be transformed into the educational needs of gifted students. In this regard, the following part of the section presents the needs of gifted students in order to offer them effective instruction.

2.1.2. Instructional Needs of Gifted Students

A large body of research supports the notion that gifted students need specialized instruction to meet their cognitive and affective needs (Borland, 2009; Johnson, 2000; Matthews & Foster, 2006; Ozdemir, 2016; Peterson, 2009). However, the available literature has revealed that gifted students experience a lack of challenge in schools, particularly at the elementary level, because of teachers’ ineffective and infrequent use of differentiation strategies (Reis et al., 2004; Tomlinson et al., 2003). Consequently, many gifted students fall behind in class, lose their motivation, and become bored (Diezmann & Watters, 2000; Taber, 2014).

According to the equity principle, these students, like all other students, should have the right to an education that enables them to maximize their potential and further develop their skills (Subban, 2006). To achieve this, differentiated instruction has emerged, a teaching philosophy founded on the notion that students learn best when their teachers take into consideration their specific interests, learning styles, and readiness (Tomlinson, 2005). The concept of differentiated instruction was primarily based on Vygotsky's (1978) Constructivist Learning Theory, Gardner's (2011) Multiple Intelligences Theory, and Tomlinson's (2009) Learning Profiles Theory, which were combined with a wide range of research on how diverse students learn (Tomlinson & Allan, 2000). Correspondingly, many different gifted education curriculum models have been proposed, including the Parallel Curriculum Model (Tomlinson et al., 2009), Enrichment Triad Model (Renzulli, 1977), and Purdue Three-Stage Model (Moon, Kolloff, Robinson, Dixon & Feldhusen, 2009), which provide the framework for developing challenging and meaningful learning experiences for gifted students while taking their interests and readiness into consideration. Enrichment is one of the most common ways to differentiate instruction for gifted students among these models (Tomlinson, 2005). It provides students with a variety of learning opportunities, such as materials to support the development of higher-level thinking skills, as well as deep and extended learning activities that are not included in the regular school curriculum (Van Tassel-Baska & Brown, 2007).

As a way to enhance learning, enrichment delivers a specialized educational experience by adapting the components of content, process, product, and learning environment (Tomlinson, 2005; VanTassel-Baska & Stambaugh, 2006). First of all, enrichment in content involves advanced and deep content with a variety of fields and topics beyond the prescribed curriculum (Schiever & Maker, 2003). It should appeal to the interests of gifted students, provide opportunities to engage in conceptual exploration, and direct them to think on more complex ideas (Taber, 2014). Secondly, enrichment in process emphasizes integrating teaching methods and embedded activities in order to promote higher-order thinking skills such as creativity and problem solving (Reis & Renzulli, 2009). In this way, students can take on the role of active investigators, applying skills such as analyzing, synthesizing, and evaluating information while also exploring new ways of thinking (Taber, 2014). This means that an emphasis should be placed on open-ended questions and tasks, as well as problem solving that requires critical thinking and provides time for discovery and exploration. Consequently, challenging activities promote the development of self-directed learning by enhancing students’ cognitive abilities and motivation (de Corte, 2013; Ozdemir, 2016). Thirdly, enrichment in product involves authentic products that are primarily intended to have a desired effect on a specific audience. It emerges when the teacher modifies the way in which students demonstrate their understanding and mastery of the subjects (Tomlinson, 2005). Lastly, enrichment in the learning environment for these learners should enable them to display their abilities, reveal their creativity, present their products in real-world settings (Kurnaz, 2014), establish connections between different subjects (Ozdemir, 2016), and enrich the subject by probing the details of the topic (Renzulli, Rizna & Smith, 2002). It should also be flexible and open to new ideas; foster investigations, questions, and discussions; and promote student independence and responsibility (Van-Tassel-Baska & Hubbard, 2016).

Along with the education required to reveal and develop their unique skills, gifted students also need differentiated instruction, challenging activities, and resources that engage their attention, curiosity, and creativity (Assouline & Lupkowski-Shoplik, 2021; Gül, 2021; Mammadov, 2019; Ozdemir & Isiksal-Bostan, 2021; Şengil-Akar, 2017). In addition, Lee and colleagues (2021) conducted a systematics review to investigate the existing research on gifted classroom environments that foster creative process skills in elementary and secondary school students. The research revealed key themes, such as the integration of creative process skills, adaptive environments, reflective classroom culture, and implementation challenges. In particular, Aygun (2010) determined the needs of gifted students regarding middle school mathematics programs. The findings revealed that the applications of depth and enrichment should be integrated into the mathematics education of gifted students. Additionally, it was stated that gifted students should be given the opportunity to develop their creativity, abstract thinking skills, reasoning skills, and problem-solving skills. In order to accomplish this, it was required to develop original materials and activities that were appropriate for the characteristics of gifted students. Although BILSEM offers after-school program opportunities to meet these specific needs of students, the studies demonstrated some problems related to teaching practices (Bildiren & Citil, 2021) and appropriate learning activities (Cetin & Dogan, 2018; Kurnaz, 2014). Besides, gifted students grow bored with activities that are similar to the instructional activities in their schools and are focused on knowledge exclusively (Sarı & Ogulmus, 2012). Hence, in these centers, it is important to offer activities that can reveal the unique skills of gifted students.

This section of the chapter employed the aforementioned studies to shed light on the conception of giftedness and the characteristics and instructional needs of gifted students. To sum up, this section provided accessible literature about the specific group on which the study was conducted. Model-eliciting activities (MEAs) and the modeling process may provide opportunities for gifted students to practice their unique skills. Hence, the following section provides relevant literature on modeling, which is another aspect of the current study.

2.2. Models and Modeling

A large and growing body of literature has investigated the terms model, modeling, mathematical model, and mathematical modeling to clarify the meanings of the terms (Erbaş et al., 2014; Kaiser, Blomhoj, & Sriraman, 2006; Lehrer & Schauble, 2007; Lesh & Doerr, 2003). Models are defined as mental conceptual systems that contain different structures such as rules, operations, and relations. These structures are handled during the process of constructing, describing, and explaining complicated systems and are then translated to the external world through various representations (Lesh & Doerr, 2003). Mental models are defined as conceptual systems in the mind, which are all mental tools that students employ during the modeling process (Lehrer & Schauble, 2007). These can be concepts, viewpoints, norms, or instruments employed while attempting to understand real life (Lehrer & Schauble, 2007). In other words, models are representations of the state of a complex system that are transferred to the external world in a different form through mental blending of the situations affecting the complex system.

On the other hand, modeling is the process of describing, explaining, or creating situations related to a problem, arranging problem situations in the mind, and using and creating different schemes and models (Lesh & Doerr, 2003). A more general definition of modeling is the process of generating different representations of real-life situations (Erbaş et al., 2014). Consequently, while modeling describes a process, a model represents the product that arises from the process (Sriraman, 2006).

Considering the definitions of the terms model and modeling, mathematical model and modeling is defined as an approach to make complex systems mathematically meaningful using mathematical language (Lesh & Doerr, 2003). Lesh and Doerr (2003), one of the pioneers of the mathematical modeling approach in mathematics education, used the concept of model eliciting, which includes both mathematical model and modeling concepts. In the current study, the term model eliciting is used in the same meaning.

Lesh and Doerr (2003) define mathematical models as conceptual systems developed by students to explain, describe, interpret, and represent a real-life situation mathematically. Mathematical models include a variety of representations, operations, and relations of a part of real-life situations using mathematical language for a specific purpose (Lehrer & Schauble, 2007). In other words, mathematical models are external representations of mental processes that have been translated to mathematical form and can be used to assist in the interpretation and solution of real-world problems (Lesh & Doerr, 2003). To sum up, mathematical models are external mathematical representations that blend the mental structures of the real-life situations that students want to solve, the mathematical knowledge they decide to use to produce solutions, and the mental structures of other concepts that affect the situation.

Mathematical modeling is defined as transforming the problem into a mathematical form in order to find solutions to real-life problems (Lesh & Doerr, 2003). According to Lehrer and Schauble (2007), mathematical modeling is a collection of mathematical entities and their relationships that students utilize to represent a part of a real-world situation. Gravemeijer (2002) defines mathematical modeling as the process of expressing real-life situations with the symbolic language of mathematics in order to make sense of real-life situations. The studies presented thus far provide evidence that mathematical modeling is the act of analyzing a real-world problem using mathematical techniques and translating it to the mathematical world (Borromeo Ferri, 2006; Erbaş et al., 2014; Maaß, 2006).

Cirillo and colleagues (2016) highlight the following five items as the common properties of mathematical modeling. The first is related to the nature of the problems. Authentic, ill-defined and complex real-world problems provide no single solution and hence present an opportunity for creative interpretation of problems. The second property is that mathematical modeling is used to investigate and explain real-world phenomena and make predictions about the future behavior of a real-world system. The next is that the modeler is expected to be creative when developing assumptions, predictions, and conclusions in mathematical modeling. The fourth property is based on the iterative nature of mathematical modeling. The final property is the fact that the mathematical modeler is able to investigate various strategies to solve the problem because no single, clear, definite approach or answer exists (Cirillo et al, 2016).

Together, these definitions indicate that the modeling process is influenced by mental models, and the models are external representations of these models. Additionally, the definitions contain two significant elements. The first is an emphasis on the relationship between the real world and the mathematical world, while the second is a process-oriented approach to mathematical modeling. Mathematical modeling is described in this study as the process of generating a mathematical solution to real-world situations using mental conceptual structures. Following a description of models and modeling as well as mathematical models and modeling, the following part presents approaches to mathematical modeling.

2.2.1. Approaches to Mathematical Modeling

Mathematical modeling has received significant attention in the mathematics education literature over the last two decades (Erbas et al., 2014; Galbraith, 2012). Despite considerable attention, investigations in this field have revealed that there is not a common understanding of modeling among researchers (Kaiser & Sriraman, 2006). Additionally, the terms mathematical models and modeling have been defined variously in research studies on mathematics education depending on the research goal and perspective (Ferri, 2013). While many researchers agree that mathematical modeling has its origins in the studies of Piaget, Vygotsky, and pragmatists, other researchers have demonstrated that ethno-mathematics and socio-cultural influences have had a significant impact on their study (Kaiser & Sriraman, 2006; Lesh & Doerr, 2003). These influences result in subtle as well as fundamental differences between the various approaches and concepts of mathematical modeling. Kaiser and Sriraman (2006) classified modeling approaches into six categories in their study: realistic or applied modeling, epistemological or theoretical modeling, contextual modeling, educational modeling, socio-critical modeling, and cognitive modeling.

The realistic or applied modeling approach aims to apply the mathematical information students have acquired to address real-world problems. It utilizes mathematics, particularly in engineering and technology-related problems. In other words, it relates to the application of mathematics to real-world problems. As a result of these applications, the approach aims to raise students who have developed mathematical modeling competency and can overcome real-world problems. Since the primary criteria for students' learning are based on their ability to solve real-world problems, this approach puts an emphasis on the subject area of mathematics application and views modeling as an interdisciplinary problem-solving activity (Blomhøj, 2009). Pollak (1979), a promoter of this approach, emphasizes the importance of keeping problems helpful, meaningful, and as close to real-world circumstances as possible. He also defines modeling as the process through which mathematics interacts with the world outside of mathematics.

The epistemological or theoretical modeling approach recognizes mathematical modeling as a lens through which broad theories about mathematics teaching and learning can be established. The primary goal of this approach is to develop mathematical understanding. Since this perspective does not require a model of a real-world problem or situation, it is distinct from other perspectives. Importantly, modeling is not restricted to the mathematization of non-mathematics problems; it can be applied to any activity in mathematics. Freudenthal (2006), a leading representative of theoretical modeling as well as the originator of 'Realistic Mathematics Education', asserts that mathematics begins with real-world problems that are mathematized, and that mathematics is then constructed using a formal system.

Contextual modeling or a model-eliciting approach assumes that students acquire mathematical concepts more meaningfully and solve verbal problems in an appropriate environment. According to Lesh and Doerr (2003), one of the leading proponents of the contextual modeling approach, one of the most important goals of mathematics education is to enable students to develop mental structures in which they can analyze the events they experience. Rather than concentrating on the modeling process itself, this approach concentrates on the development of model eliciting activities. This approach is driven by six principles: “the reality principle, the model construction principle, the self-evaluation principle, the construct documentation principle, the construct generalization principle, and the simplicity principle” (Lesh & Doerr, 2003). By incorporating these ideas into teaching and learning, the modeling approach is not limited to solving a single problem, but can be extended to establish a system of generalizable relationships.

The educational modeling approach is concerned with the integration of models and modeling into mathematics instruction. The purpose of educational modeling is not only to develop mathematical modeling competencies, but also to facilitate the learning of mathematics. Hence, it emphasizes both ‘modeling as content’ and ‘modeling as vehicle’. In the modeling as content approach, mathematical modeling is regarded as a fundamental competency, and the goal of teaching mathematics is to provide students with the ability to use mathematical modeling to address real-world problems in mathematics and other fields (Blomhøj & Jensen, 2007; Galbraith, 2012; Julie & Mudaly, 2007; Niss et al., 2007). On the contrary, in the modeling as a vehicle approach, mathematical modeling is regarded as a mechanism for assisting students in their attempts to create and develop their primitive mathematical knowledge and models. It is believed that the model eliciting activities (MEAs) enable students to develop conceptual understanding through mathematical modeling problems. Two examples of this approach are the Models and Modeling Perspective (Lesh & Doerr, 2003) and Realistic Mathematics Education (Gravemeijer & Stephan, 2002).

Based on the educational modeling approach, Blum and Niss (1989) justify the inclusion of mathematical modeling in mathematics curriculum under six categories. These are: pragmatic, formative, cultural, critical, instrumental, and psychological. The pragmatic justification is that students seek solutions to difficult and diverse situations that they may experience in daily life through mathematical modeling. The formative justification claims that MEAs provide students with a broad perspective and self-confidence. While engaged in MEAs, students develop a creative and exploratory view of problems by approaching them from a variety of perspectives.

According to the cultural justification, the aim is to create conditions suitable for the development of a mathematical culture in students through MEAs. In addition, modeling is the process of representing mathematics in accordance with its significance in students' life and their understanding of mathematics. The primary goal of the critical justification, for its part, is to give students a critical perspective of mathematics by conveying that there are various applications of mathematics to different situations experienced in daily life. In the instrumental justification, mathematical modeling is a tool for students to use mathematical concepts in daily life. Finally, the psychological justification supports an increase in students' interest in mathematics, their belief in their ability to succeed, and their motivation for the lesson via the use of MEAs (Blum & Niss, 1989).

Another approach, socio-critical modeling, is related to the socio-cultural dimensions of mathematics. This approach aims to reveal the reflective and critical discourses of the students about the real-life situation that they analyze while engaged in the mathematical modeling process. Student discourse refers to mathematical models as non-neutral representations of reality that elicit the acquisition of mathematical concepts and the development of modeling competencies through the modeling procedures. These modeling processes assist students in understanding a current social situation, enabling them to develop into critical, engaged citizens (Barbosa, 2006). In this sense, Blomhøj (2004) stated the importance of modeling as a mathematics learning process established in students' reality. In particular, he emphasized that mathematical modeling is a way to describe and understand students' daily life experiences, and that students who embed mathematics in their culture make a connection between their daily life experiences and mathematics (Barbarosa, 2006). This approach highlights the critical role of facing social demands as well as explaining real situations in mathematical modeling.

In the cognitive modeling approach, the main goal is to understand the mental processes and structures that occur in the modeling process. It also aims to explain what cognitive structures and processes occur in students' minds during the solution of MEAs. From the cognitive modeling approach, one of the most essential goals is to identify and examine the cognitive functions that students employ while engaging in MEAs. Cognitive structures that emerge while performing mathematical modeling activities serve as a guide in creating suitable environments for the development of mathematical thinking skills (Bukova-Güzel, 2016).

The diverse approaches on mathematical modeling suggest a variety of implications for mathematics education and learning that are not mutually exclusive but that do differ from one another (Kaiser, Sriraman, Blomhaj & Garcia, 2007). All of these approaches are supportive of the move toward mathematical modeling, and the strong connections between them are highlighted. Hence, it is critical to situate this study within or among existing theoretical and conceptual frameworks and to provide a justification for them. Due to the fact that this study demonstrates educational, psychological, and subject-related purposes, it can be regarded as an example of an ‘integrative perspective’ (Kaiser & Sriraman, 2006). This study integrates and advances ‘contextual’, ‘educational’, and ‘cognitive’ approaches on mathematical modeling (Kaiser & Sriraman, 2006). A contextual approach emphasizes the importance of MEAs from meaningful contexts, whereas an educational approach emphasizes the integration of mathematical modeling within mathematics instruction (Blomhøj, 2006). In the conceptualization of modeling competencies, cognitive and educational approaches are merged for this study. The empirical component of the study is primarily concerned with a cognitive approach. This study focuses on group routes and competencies through the use of visible external representations (Borromeo-Ferri, 2006) of group modeling sessions, while also emphasizing the meaningful integration of modeling as a significant means of learning mathematics in mathematics education.

The mathematical modeling approaches mentioned above demonstrate that the field of mathematical modeling education has a rich and varied history. Despite the fact that the approaches differ in many significant ways, they still share a common view that modeling involves a process. The following part provides different mathematical modeling processes in the literature.

2.2.2. Mathematical Modeling Process

Mathematical modeling is a mental process that involves solving a real-world problem mathematically and then creating, interpreting, and evaluating the solution in relation to the real world (Maaß & Gurlitt, 2011). There have been multiple approaches to the modeling process developed and utilized for a variety of reasons in the literature (Blum & Leiβ, 2007; Galbraith, 2012; Kaiser et al., 2006; Lesh & Doerr, 2003; Lu & Kaiser, 2021). The preliminary investigations of the modeling process indicate that linear studies are used to connect the real world with the mathematical world (Kapur, 1982; Pollak, 1979). In further investigations, the general consensus among researchers came to be that modeling is the application of mathematics in real-world situations in order to develop solutions to real-world problems, and that it is a cyclic process that includes stages such as describing a real-life problem situation, solving it by creating a model with mathematical methods, interpreting the solution and evaluating the model in the real world (Blomhøj & Jensen, 2006; Borromeo Ferri, 2006; Hıdıroğlu, 2012; Lu & Kaiser, 2021; Mousoulides, 2007). Kaiser and colleagues (2006) identified six distinct applications for the use of the modeling cycle as an analytical tool. These are: a) analyzing the modeling process in order to select and design appropriate modeling problems, as well as to understand and validate the process; b) identifying the major components of modeling competency; c) analyzing students' modeling work in general in order to determine their difficulties; d) assisting students' modeling work and related metacognition; e) planning modeling courses or projects; and f) defining and analyzing the curricular elements in mathematics teaching. In addition to the various uses of the modeling process, the differences in the definitions are generally due to the elaboration of the process and the components that are emphasized. Some of the modeling processes are detailed in the following.

Lesh and Doerr (2003) investigated the mathematical modeling process in the four steps shown in Figure 2.2., defining it as the relationship between the real world and the mathematical world. The initial phase, description, establishes a mapping between the real world and model world. The real-world problem situation is thoroughly defined and the given information is analyzed mathematically. As a result, information that is mathematically meaningless is eliminated, and the situation becomes more understandable. In the manipulation phase, mathematical relationships are constructed between the structures to generate solutions relevant to the original problem situation. The third phase, translation or prediction, involves the adaptation of the obtained model to the real world and mathematically evaluating the meaningfulness of the solution back into the real world. The final phase, verification, evaluates the relevance and usefulness of the produced model in the real-world.

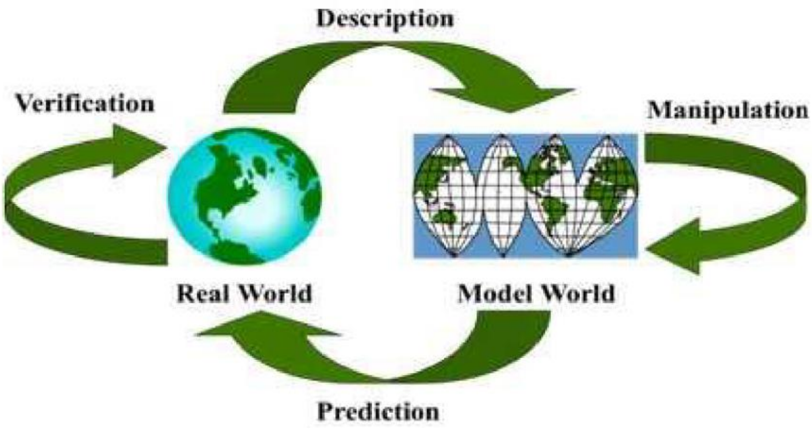


Figure 2.2. Modeling cycle (Lesh & Doerr, 2003, p. 17)

According to Lesh and Doerr (2003), there is no strict application of the steps in the mathematical modeling process. The process is flexible, and reversals frequently occur between steps. It should also be performed many times in order to produce models that can be assessed, rearranged and shared. Several studies indicate that the modeling cycle should be repeated many times in order for the ambiguous and complicated mental tools, assumptions, and models that emerged initially to become more precise and solution-oriented (Lesh & Carmona, 2003; Zawojewski & Lesh, 2003).

Blum and Leiß (2007) introduced the ‘situation model’ as an intermediate stage in the DISUM (Didactical intervention modes for mathematics instruction oriented towards self-regulation and directed by tasks) project, which focused on the modeling process. The term “situation model”, often referred to as the problem model, is defined depending on the experiences of the students about the context, regardless of how it is defined in the problem text. In other words, the situation as it exists in the context and the situation as represented by the student’s experiences are different. Thus, in the mathematical modeling process, the student’s description of the context is referred to as the situation model. The modeling process proposed by Blum and Leiß (2007) consists of seven phases as seen in Figure 2.2. These phases of the modeling process are described as in the following.

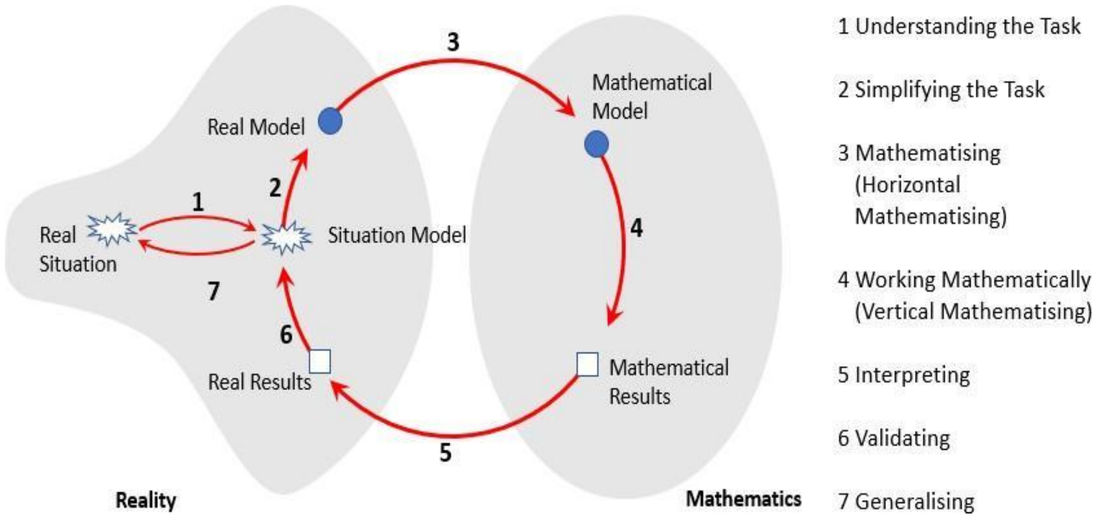


Figure 2.3 - The modelling process as adapted from Blum and Leiβ (2007, p. 225)

Understanding the task. In this stage, students are confronted with unstructured real-world problem situations in which neither the aim nor the mathematical structure is stated directly (Zbiek & Conner, 2006). The construction of mental representations of the situation described in the problem are their primary purpose (Borromeo Ferri, 2006). This procedure results in the creation of a situation model. This process begins in the real world when the student realizes that there is a problem and that it is necessary to find a solution to that problem (de Villiers, 2018). In order to make sense of a new problem, students begin by using their current knowledge and previous experience (Chick & Stacey, 2013). By questioning, researching, brainstorming and clarifying specific details in the problem during their active exploration of a real-world situation, the students reveal additional details regarding the situation that were not previously evident to them (de Villiers, 2018).

Simplifying the task. At this phase, students are expected to distinguish between the knowledge required for the solution and, if necessary, to fill in the gaps with data obtained from their own life experiences. The understanding of the problem is the requirement of this stage, that is, the mental representation of the real-world situation. This phase entails developing reasonable and efficient assumptions in order to further simplify and comprehend a real-world problem (de Villiers, 2018). Depending on the nature of the problem, a need for extra-mathematical knowledge may emerge (Borromeo Ferri, 2006).

Mathematizing. At this phase, verbally stated assumptions and beliefs regarding the solution of a problem are converted to the mathematical language. Students are expected to present their ideas using mathematical expressions, symbols, and drawings. Schematizing, forming formulas, and creating visual representations of problems are all common concerns for students as they learn to identify and describe specific mathematical concepts in a broader context. They also learn to identify and recognize isomorphic aspects in various problems, and so translate real-world problems into mathematical problems (de Villiers, 2018).

Working mathematically. This phase is referred to as mathematical analysis because students use the mathematical model to find a solution and then put their findings to the test (de Villiers, 2018).

Interpreting. Aspects of this stage involve interpreting and justifying mathematical conclusions as well as explaining, conveying, and critically evaluating the model and its constraints (de Villiers, 2018). Within the mathematical system, reorganization occurs. Additional parameters and properties can be discovered through the analysis, manipulation and interpretation of the mathematical entity. When students examine and reflect on their solutions, attempting to reconcile them with the underlying issues, the appropriateness of their solutions is evaluated (de Villiers, 2018).

Validating. At this stage, it is determined whether the produced model is applicable to the real-world situation (Blum & Leiβ, 2007). If the context requires that a revision and validation of the model is needed, the model can be moved back to the real-world situation (Blomhøj & Jensen, 2003). An inappropriate model at this stage is one that is unsuitable for making predictions or acting in real-world situations (Lesh & Zawojewski, 2007). It is necessary to repeat the entire cyclic process of formulating, testing, and revising the trial solution whenever the student faces a problem during the verification phase (de Villiers, 2018).

Generalizing. At the final stage of the modeling cycle, students are given the opportunity to modify their own model, or a different model that they have previously examined in relation to a new situation (Lesh & Zawojewski, 2007). The mathematical model has been separated from its original context and created as a reusable and shareable method for interpreting various contextual problems (de Villiers, 2018).

On the other hand, Borromeo Ferri (2006) adopted the modeling process of Blum and Leiβ (2007), and used the term “mental representation” instead of “situation model”. She analyzed various modeling cycles as part of the COM2 project (Cognitive psychological analysis of modeling processes in mathematics lessons), with a particular emphasis on the first three stages of the Blum and Leiß (2007) modeling cycle. She classified modeling cycles into four distinct types using the real state (RS), mental representation of the situation (MRS), situation model (SM), and mathematical model (MM) processes in these cycles. The terms situation model (SM) and mental representation of the situation (MRS) are used interchangeably in this classification. According to this classification, the first group is comprised of cycles in which both the real model and the state model are used separately, the second group is comprised of cycles in which the real model and state model are used together, the third group comprises cycles in which the real model and the state model are used together in the same way, and the fourth group is comprised of direct transitions from the real state to the mathematical model.

In a recent study, Lu and Kaiser (2021) enriched the modeling process by integrating aspects of creativity. Due to the lack of standard approaches for addressing real-world problems and the context-bound nature of each step of the modeling process, creativity is required throughout the modeling cycle (Wessels, 2014). Hence, components of creativity are included in the above mentioned commonly used stages of the modeling process (Kaiser & Stender, 2013) as shown in Figure 2.3.

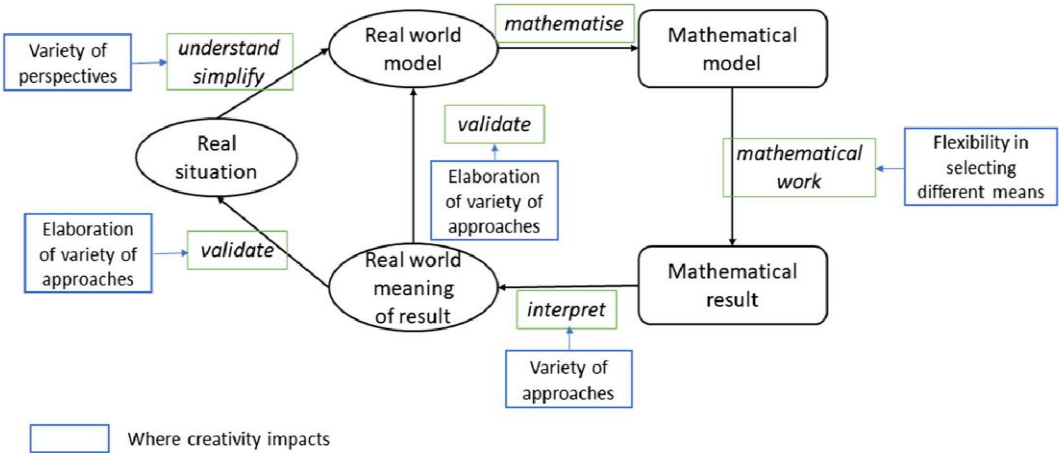


Figure 2.4. The modeling cycle enriched by aspects of creativity (Lu & Kaiser, 2021, p.292)

Following an examination of several studies attempting to explain the mathematical modeling process, a general consensus has emerged that there is no strict procedure for accomplishing a certain goal during the mathematical modeling process (Blum & Niss, 1991; Borrmoeo-Ferri, 2006; Crouch & Haines, 2004; Lesh & Doerr, 2003). Another common point is that the process of mathematical modeling is cyclical in nature (Zbiek & Conner, 2006). As a broad definition, mathematical modeling can be defined as the process by which interactions between the real world and mathematical world occur. The definitions provided vary in terms of the steps involved in the interactions between these two worlds. While some research provides a more straightforward explanation of these phases, others go into great depth to explain them. Indeed, in some studies, the sub-processes that are encountered as one progresses through the steps are specified in depth, as are the transitions between sub-processes. Progress, or the ability to move from one step in the mathematical modeling process to the next, requires successfully overcoming cognitive obstacles (Blum, 2007). In the literature, these competencies are referred to as mathematical modeling competencies. Taking into consideration the mathematical modeling process steps that Blum and Leiβ (2007) outlined, they also defined the mathematical modeling competencies that provide the transition between these stages, as well as the sub-competences of these competencies. The mathematical modeling process outlined by Blum and Leiβ (2007) was used in this study, as the mathematical modeling competencies and sub-competencies necessary for the mathematical modeling process to be completed successfully are defined in detail. The phases of the cycle are examined as competencies, which can be employed as assessment criteria or rephrased as modeling competencies (Maaß, 2006). Taken together, different modeling processes are nurtured from various theoretical backgrounds. Thus, the models and modeling perspective which serve as the theoretical approach of this study are presented in the following section.

2.3. Models and Modeling Perspective (MMP)

Due to its complex nature, there is no single theory that can adequately describe modeling. Various theoretical approaches shed light on different stages of the modeling process. This diversity provides the opportunity to understand and critically evaluate learning processes from different perspectives (Cobb, 2007). According to

Sriraman and Lesh (2006), models and modeling perspective (MMP) originated from concept development research rather than problem solving research. Thus, it is essential to understand the theoretical foundations of modeling. Among the theoretical perspectives, Social Constructivism is the starting point to explain modeling (Lesh & Doerr, 2003). The theory outlines how learning occurs in student groups. Since the emphasis is on the activity in these groups, activity theory is considered (Lesh & Doerr, 2003).

Within the MMP, models are conceptual tools that are used to represent real-world situations mathematically, and modeling is the process through which a model is adjusted or constructed to solve the problem (Lesh & Doerr, 2003). On the other hand, the modeling process is an interactive learning cycle in which students work in groups to 1) examine the situation, 2) define the problem to be solved and the variables involved, 3) develop a model, 4) test the model, 5) interpret the results, 6) validate the applicability of the model to the original situation, and 7) apply the model to other similar situations to determine its usefulness (Kang & Noh, 2012).

Furthermore, students not only reflect their own ideas but also express their ideas in a way that group members can assess, reject, or accept since MMP promotes collaborative learning in the modeling process. Overall, each group member constructs their ideas based on collaboration (Lesh et al., 2003). Since learning occurs in the social environment of student interaction, the MMP promotes and advocates unstructured, collaborative learning (Vygotsky, 1978). The MMP places a strong emphasis on collaborative learning, and it is based on the developmental theories of Piaget (1926) and Vygotsky (1978), which state that face-to-face work on open-ended tasks or projects with multiple possible paths leading to multiple acceptable solutions encourages cognitive development (Springer et al., 1999). According to this principle, MMP recognizes that 1) it is critical for students to not only argue and discuss their opinions, but also to share their ideas and perspectives with each other working collaboratively, that 2) students can reveal their inadequate reasoning as a result of their discussions, and that 3) resolving these disagreements improves their understanding (Springer et al., 1999). Accordingly, MEAs facilitate these forms of problem-solving activities by enabling students to collaborate with others, test, reflect and re-enact their ideas. As part of the social and communal creation of knowledge

(Vygotsky, 1978), MEAs encourage students to both develop their own knowledge while engaging with their environment and other people, as well as actively participate in the process of constructing knowledge for their learning community (Tangney et al., 2001). Students work together to achieve a common goal by merging their knowledge and abilities in order to strengthen and broaden their skills. This increases their motivation and stimulates their interest in problem-solving activities (Zawojewski, Lesh, & English, 2003). Additionally, it enables students to develop and design more sophisticated and powerful solutions through the use of mathematical representations and inscriptions such as graphs, tables, and diagrams to assist their cognitive processes and reasoning. To promote collaborative learning, researchers must ensure that the design of an MEA elicits students' involvement throughout the modeling cycle that results in the construction of a model (Lesh et al., 2000). Consequently, MMP theories and concepts used in this study assume that students construct models in order to generate meaningful interpretations of real-world problems collaboratively (Lesh & Doerr, 2003; Vygotsky, 1978). Students' backgrounds and experiences have an effect on their interpretation, which they communicate through different media.

Furthermore, MMP enables the integration of the engineering design process, which is apparent in each of the three criteria for effective engineering education (NAE & NRC, 2009). According to these principles, K-12 engineering education should (1) emphasize the engineering design process, (2) integrate important and developmentally appropriate mathematics, science, and technology knowledge and skills and (3) promote engineering "habits of mind". Taking all these principles together, the engineering design process as a pedagogical approach should provide problem solving environments that are iterative, open-ended (with multiple solutions), meaningful contexts for the acquisition of scientific, mathematical, and technological concepts, and a stimulant for the skills required for the 21century such as collaboration, system thinking, and communication. In addition to similarities between the mathematical modeling process and the engineering design process, engineering-based MEAs and their related products reflect all of these perspectives because of their less restricted and more complex nature than MEAs. Accordingly, the following section addresses the engineering design process, another aspect of the current investigation.

2.4. Engineering Design Process

The engineering design process has been offered as an instructional approach to address the problems in the fields of science, technology, engineering, and mathematics (STEM). Accordingly, the development of design processes has been a crucial component of elementary school engineering education (English, King & Smeed, 2017). There have been several distinct interpretations of the engineering design process in the literature (Arik & Topcu, 2020). Although some researchers (Dym, Agogino, Eris, Frey, & Leifer, 2005; Rogers & Wallace, 2000) supported the idea that young learners do not have the skills to design and plan approaches to solve engineering-based problems, others (Dorie, Cardella, & Navoa Svarovsky, 2014; Lachapelle & Cunningham, 2014) supported the opposite idea. However, similar stages have been conceptualized for a variety of engineering design interpretations (Guzey et al., 2017; Johnston et al., 2019; NAE & NRC, 2009).

Engineering design for young learners is generally defined as an iterative process with different phases, including (a) identifying the problems by stating the goal and limitations, (b) generating ideas and assessing which is the best option to meet the requirement of the problem, (c) designing and constructing, (d) testing and checking goal attainment, (e) redesigning and reconstructing, considering reflection on the first design and (f) reflecting and interpreting the entire design process (Engineering is Elementary, 2017; English et al., 2017; Moore et al., 2014). Throughout the iterative design process, students have the opportunity to design and test their solutions in order to arrive at the best potential solution (Bryan et al, 2015). The justification in the design process is an effective mechanism to elicit students' development, and hence, facilitates conceptual development (Park et al., 2018). Welch and Lim (2000) examined the strategic thinking of novice designers. They found that untrained seventh-grade students developed solutions to a design-and-make task in ways that were notably distinct from the solutions specified by numerous textbooks and theories concerning the process of learning to design. A novice designer sequenced the sub-processes of designing quite differently than the models that are prescribed, generated multiple possible solutions and chose the most effective one, made considerable use of three-dimensional modeling and used less two-dimensional modeling than is suggested by textbooks, and continually evaluated their design proposal even from the initial instants of the design-and-make process.

Additionally, students require not just the skills necessary to integrate STEM disciplines, but also the prior knowledge necessary to apply that information to the engineering design process (Fan & Yug, 2017). Students' tendency to solve innovative and creative problems with multiple ideas and approaches is considered to provide a rich basis for developing engineering design-based activity, which effectively integrates STEM disciplines at early ages (English et al., 2017; Moore et al., 2014). Hence, the iterative nature of the engineering design process offers gifted students the challenges they need and a differentiated learning environment (Mann et al., 2011). From this point of view, the current study attempts to provide an engineering design-based learning environment to plan, construct and reflect their prototypes on their experiences.

In addition, Lucas and Hanson (2016) investigated the distinct ways that engineers think and perform. The researchers generated the engineering habits of mind descriptions including systems thinking, problem finding, creative problem solving, adapting, visualizing, and improving (Lucas & Hanson, 2016). Furthermore, engineering habits of mind have been suggested in policy documents to enable students in K-12 to draw authentic relationships between what they are learning and engineering practices. Six fundamental engineering habits of mind include systems thinking, creativity, optimism, teamwork, communication, and ethics (Householder &Hailey, 2012; Katehi et al., 2009; National Academy of Engineering [NAE], 2010). Divergent thinking is another habit of mind that should be considered when monitoring engineering design in a classroom (Cooperrider, 2008).

Considering this is a key principle of K-12 engineering education, there are a limited number of research studies investigating the skills displayed by students in the engineering design process (Wheeler et al., 2019). The studies conducted by English and colleagues can provide examples of integrating the engineering design process in elementary and middle schools (English, King, & Smeed, 2017; English & King, 2018; King & English, 2016). In a three-year longitudinal research study (English, King & Smeed, 2017), sixth-grade students addressed an engineering-based problem on earthquakes that included integrated STEM learning. The students used the engineering design process and STEM subject knowledge to plan, sketch, and ultimately construct an earthquake-resistant structure while considering a variety of restrictions. They redesigned their structures after they were tested in order to develop a better structure. Based on their design process framework, researchers evaluated the students' skills in planning, making annotated sketches, and transforming these into three-dimensional models. According to the findings, group problem solving required iterative transitions between design phases, regular review of the problem scope, understanding the goals of the problem, boundaries, and constraints, and an appreciation for design sketches as guiding constructions. Moreover, students were observed considering multiple problem components concurrently, demonstrating their ability to handle the complexity of the task. Similarly, English and King (2018) conducted the study with sixth grade students to construct a paper bridge that could withstand an optimal load. Based on their planning, it appears that students were able to justify their suggested bridge type(s), which frequently contained a combination of types, by referencing their STEM knowledge and understandings. Besides, students' design sketches indicated an awareness of the restrictions of the problem, an understanding of fundamental engineering principles, and the application of mathematics and science knowledge to the problem. Students' reflections on their actions aided them in improving the design of their bridges. These studies indicate that engineering design-based activities provide an interdisciplinary learning environment for elementary students.

In particular, Sen (2018) determined the STEM skills used by gifted students in integrated STEM activities focused on engineering design. The findings of that study revealed that gifted students use reasoning, problem solving, association, engineering, innovation, creativity, communication and cooperation, and life and career skills when participating in such activities. In addition, the recent study conducted by Sen, Ay, and Kiray (2021) sought to identify the computational thinking skills of gifted students in integrated STEM activities based on the engineering design process. The students demonstrated active use of critical thinking skills when providing explanations, making associations, questioning, providing justifications, solving problems, thinking creatively, making generalizations, and evaluating the effectiveness of the solution. In view of all that has been mentioned so far, the literature not only promotes engineering based activities for elementary students but also the specific group of students, i.e. gifted ones. In line with the purpose of the current study, the following section describes MEAs, more precisely engineering-based MEAs, and their principles.

2.5. Model-Eliciting Activities (MEAs)

Model-eliciting activities (MEAs) are unstructured, realistic, client-driven problems that require the development or adjustment of a mathematical model for a particular situation (Lesh & Doerr, 2003). MMP on activity design serves as a basis for the development of an MEA (Lesh et al., 2000). Lesh and Doerr (2003) advocated using the concept of MEAs, which includes the use of models and modeling concepts, rather than the concept of modeling problems. Similar terminology is used throughout the current study based on the MMP. MEAs are problem-solving activities in which students make inferences from meaningful real-life situations, expand these inferences by mathematizing, and re-organize these mathematized structures in the process (Lesh & Lehrer, 2003). Students are encouraged to work collaboratively on MEAs, which promote deeper and conceptual understanding when constructing models (Lesh & Doerr, 2003). In summary, MEAs are the mathematization of real-life problem situations by students working in small groups. Students work in groups to provide a description, method, or approach, and their solutions to the task reveal explicitly how they think about the given problem (Zawojewski, Lesh, & English, 2003).

The cyclic nature of MEAs enable students to repeatedly reveal, test, and improve their ways of thinking. In contrast to traditional story problems which require the short and only one exact answer, the products generated by students during MEAs include conceptual tools that can be shared, generalized, modified, and reused for the construction, description, explanation, and prediction of other situations (Lesh & Zawojewski, 2007). There is a shift in emphasis from teaching methods and abilities to enable students to develop their own models that they use and iteratively revise to understand a problem. This model elicitation and multi-cycle revision approach are the basis of MEA design (Hamilton et al., 2008).

According to English (2003), learning environments that are designed with MEAs provide rich learning experiences for students. Providing authentic situations, allowing students to explore the situation autonomously, allowing students to develop their ability to interpret, and providing an environment for students to share their models are all aspects of rich learning environments (English, 2003). In addition, MEAs can be used to reveal students' reasoning, explanations, and justifications about mathematics while engaged in the activity (Doerr & Lesh, 2011; Hamilton et al., 2008; Mentzer et al., 2014). MEAs extend problem-solving experiences to allow students to express their own ideas more deeply and improve their thinking (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016). MEAs share common features (Lesh & Yoon, 2007): (a) they are relevant to life and academically interesting; (b) they require students to express their thinking; (c) they promote discussions and feedback needed for model revisions; (d) they result in the development of a conceptual tool not just simplistic answers; (e) they support model and process sharing for validation and self-critiquing; and (f) they include opportunities for students to use their models and conceptual tools in other situations.

Chamberlin and Moon (2005) described the implementation stages of MEAs as (a) reading passage, (b) readiness questions, (c) data section, and (d) problem solving and presentation of a solution. In the reading passage stage, the aim is to introduce the context of the activity to students. A one-page text is delivered to the students first, in the form of an article or letter from a customer (Chamberlin & Coxbill, 2012). Students reading the text are asked readiness questions related to the text in order to warm up the students before moving on to a more challenging problem (Chamberlin & Moon, 2005). The third stage of the MEAs is the data section, during which the problem statement is identified and the data necessary to address the problem are gathered (Zawojewsky, Lesh, & English, 2003). At this stage, students are expected to develop a solution or method by asking them to create their own data or by presenting a ready-made data table, a context-related problem of the client in the text. In the problem solving and presentation of solution stage, students solve the problem and share the solutions with other members of the classroom (Chamberlin & Moon, 2005).

MEAs were designed for two purposes by researchers. In the former, MEAs were used to encourage students to develop mathematical models in order to solve complex problems, similar to what applied mathematicians do in the real world (Lesh & Doerr, 2003). In the latter, MEAs were used to facilitate students' exploration of mathematical thinking and conceptualization, a task endorsed by the National Council of Mathematics Teachers (NCTM, 2000) and leading mathematics educators (Wood, Merkel, & Uerkwitz, 1996). In addition to these two purposes, it has been revealed that another purpose of MEAs is to define mathematically gifted students and develop their creativity and competencies (Chamberlin & Moon, 2005; Coxbill, Chamberlin, & Weatherford, 2013; Mann et al., 2011; Wessels, 2014).

In line with the first purpose of MEAs, Hankeln (2020) made a comparison between students' modeling process in Germany and France through a frequently used MEA. This article presents the findings of a study that investigates how high school students in Germany and France approach a specific mathematical modeling problem and the challenges they face in doing so. The way in which students respond to a real-world situation, as well as the level of accuracy they strive for, are the two primary distinguishing factors. The students from France were not familiar with the fact that the context had a more significant part in the processes that led to their solutions than it did simply in motivating them. The German students frequently employed ways to reveal the mathematical information, but it appeared that they were more accustomed to a context that was drawn from the real world. They were more willing than the students from France to consider a simplification of the situation, and they developed mathematical models that were not as sophisticated. These discrepancies among nations could be explained by the different approaches that France and Germany employ to the teaching and learning of mathematical modeling.

In parallel to the recent purpose of MEAs, Şengil-Akar (2017) examined the mathematical creativity of middle school gifted students through MEAs, both individually and in groups. Individual mathematical creativity was examined regarding fluency, flexibility, making connections, and progressivity. On the other hand, collective mathematical creativity was assessed based on the accuracy, quality, generalizability, and originality of the gifted students' models. The findings of the study revealed that gifted students demonstrated varying levels of mathematical creativity in different MEAs. Additionally, it was concluded that gifted students explored new mathematical structures, rules and new information as they constructed unknown mathematical information in an interactive process. Gifted students were able to generate more and diverse solutions, think fluently and flexibly, generate more sophisticated solutions and make more associations among variables in some MEAs than in others. When engaged in MEAs as a group, they produced high quality and original products (Şengil-Akar, 2017).

MEAs are specifically designed to be accessible to all students, to improve students' modeling competencies and to produce documentation of students' progression in technological, scientific, engineering, or mathematical thinking (Gainsburg, 2013; Moore et al., 2013). Although they are accessible to all students, MEAs are designed for gifted students in line with the purpose of this study. Throughout this study, MEAs have been defined as activities that allow gifted students to go through the entire mathematical modeling process and to expand or make sense of their previous learning by mathematizing real-world problems. In the following section, engineering-based MEAs are discussed in detail.

2.5.1. An Engineering-based Model Eliciting Activity

Model eliciting activities (MEAs) are one of the most widely used methods for incorporating modeling into the engineering context (Dux et al., 2004; Lyon & Magana, 2021). Engineering-based MEAs applied in elementary classrooms are authentic and open-ended situations in which a client requests that a group of workers create a product (mathematical/scientific/engineering model) to address the given situation (English & Mousoulides, 2011). While the above-mentioned goals of MEAs in the previous section are specified for middle school mathematics education, they are equally applicable for engineering education (Diefes-Dux et al., 2008). These activities are also not intended to be a complete curriculum, but rather supplement the content of the lesson. The emphasis placed on the process for solving the problem is an essential aspect of all MEAs designed for engineering education.

MEAs are designed to help students create effective, shareable, and adaptable models or solutions (Lesh, 2010). The engineering design process, including, expressing, testing, and correcting students' solutions is incorporated into these activities. Such activities expand classroom experiences addressing their different learning styles and strengths that students bring to the classroom (English et al., 2013; Hamilton et al., 2008). Engineering-based MEAs are designed to extend problem-solving experiences to allow students to express their ideas more deeply and improve their technological, scientific, engineering, or mathematical thinking skills (Lyon & Magana, 2021).

Dasgupta (2019) investigated how students use a specific type of improvable model as well as the potential of this model to serve as a scaffold for an engineering design activity. Students employed this model as a means of productively engaging with five disciplinary practices, which included: decomposing the design challenge into individual design parameters; considering the relationship between design parameters; reasoning through multiple design parameters and making trade-offs; weighing multiple solutions; and using design heuristics for innovation and design optimization. The mentioned model facilitated students' productive engagement with a variety of disciplinary activities, including informed decision-making and systematic exploration.

In particular, there are six principles for guiding MEA development (Lesh et al., 2000). The MEA guiding principles for constructing models as part of the design process (Diefes-Dux et al., 2008) were based on mathematics' MEA principles (Lesh et al., 2000). Engineers use mathematical models to identify patterns, optimize outcomes, and make predictions (Cole et al., 2013). Engineering-based MEAs are less constricted and more complex than their multifaceted mathematical MEA counterparts (Diefes-Dux et al., 2008). Table 2.1 summarizes the MEA guiding principles for mathematics and engineering education for easier comparison (Diefes-Dux et al., 2008).

Table 2.1. Principles guiding the design of engineering-based model eliciting activities (pp. 21-22)

Principle	Mathematics education principles	Engineering education principles
Model construction	Students should use representations to create models.	Student teams use representations to construct models or design items.
Reality	Students should be able to make sense of the real-life situation based on their own knowledge and experiences.	Students should consider their experiences, the constraints of the real-life problem situation, and the needs of the client.
Self-assessment	Students should monitor their progress, clarify assumptions and look for obstacles, deficiencies, and alternative approaches.	Students should critique decisions and monitor progress and model effectiveness in meeting its purpose and the criteria of the client.

Table 2.1. (continued)

Model documentation	Students should document their thoughts about key parts of the modeling process.	Students should document (e.g. spreadsheet or computer program) the team's interactions.
Model shareability and reusability	Students should test the solution for usefulness, readability, justifiability, comprehensibility, and generalizability.	Students should evaluate the model's usability by others and generalizability for use in similar contexts, i.e. locally generalizable.
Effective prototype (Simplicity)	The model should be simplified but not to the point that it ceases to be useful or significant.	The model should serve as a template for the design of other models, i.e. globally generalizable.

Based on the above stated principles, engineering-based MEAs were developed for the current study. In particular, engineering-based MEAs are used as research tools that aim to elicit the thoughts of gifted students for the solution during implementation. Correspondingly, the following section presents the modeling competencies and studies on modeling competencies.

2.6. Modeling Competencies

The PISA 2021 mathematics framework investigates mathematical literacy by making multiple connections to mathematical modeling. Turkey's mathematical literacy competency level was identified as Level-2 (PISA, 2018). Students at this stage can realize and interpret situations that do not require more than what is seen at first glance. They also do not have the competence to develop models for complex situations by determining constraints and assumptions and to work with these models. They are not able to choose, analyze, and determine appropriate problem-solving methods for coping with the complicated issues connected with these models and evaluate these methods. Students at this level lack broad and well-structured thinking and reasoning skills enabling them to work strategically using appropriate representations, symbolic and formal definitions, and perspectives on these problems. They lack the ability to reflect on their own actions and formulations or to establish a link between their own interpretations and inferences based on their reasoning (PISA, 2018). Turkey fell behind the rest of the developed world in terms of students' competencies in mathematics, science, and problem-solving (PISA, 2012; 2018). Beyond simply falling behind, students' PISA ranking scores in group problem solving reveal that

Turkey placed near the bottom of the list (47 out of 51 countries, 2015 PISA). To deal with the concerns that students in Turkey lack crucial cognitive competencies in science and mathematics, one of the initiatives that the government put forward is to introduce students at an early age to model eliciting activities (English & Mousoulides, 2011; Şahin & Erarslan, 2016). Indeed, previous studies (Lesh & Sriraman, 2005; Blum, Galbraith & Niss, 2007) emphasized the development of students' modeling competencies as one of the crucial aims of mathematics and science education.

The development of modeling competency is considered to be an essential component of mathematics instruction and modeling competencies are even considered to be crucial for mathematical literacy (Blum et al., 2007; OECD, 2003). In order to perform MEAs appropriately, students should understand the real-life problem, express it mathematically and reach a mathematical solution. The mathematical solution needs to be analyzed, interpreted, and validated in the context of real-life situations. Students are expected to have particular skills and competencies in order to successfully complete these procedures. Hence, a thorough understanding of modeling competencies and skills is closely related to the definition of the modeling process (Maaß, 2006; Kaiser, 2007).

Several studies demonstrate that knowledge alone is not sufficient for successful modeling, and students should choose to use their knowledge while also controlling and supervising the process (Blomhoj, 2011; Kaiser, 2007; Maaß, 2006; Niss, 2003). Mathematical modeling competency refers to the ability to carry out the activities necessary for the development and investigation of mathematical models (Maaß, 2006). On the other hand, modeling skills are technical abilities such as understanding a real-world situation, creating a model, and performing mathematical operations on the model that must be possessed in order to complete any modeling process (Henning & Keune, 2007). Within this approach, modeling competencies include both modeling skills and the desire to display these abilities in pursuit of a purpose (Kaiser, 2007).

The definition of modeling competency developed by Maaß (2006) is used in this study. Her definition of modeling competencies was "skills and abilities to conduct modeling processes effectively and [which] are goal oriented, as well as the willingness to put such skills and abilities to use in practice" (p.117). In particular, modeling competencies are based on actions that occur during transitions between the real world and the mathematical world. In the modeling process, she identified sub-competencies to be understanding the real problem, developing a reality-based model, constructing a mathematical model from the real model, solving mathematical problems within the mathematical model, and interpreting the results in a real situation to validate the solution (Maaß, 2006). When engaging in MEAs as a group, it is critical to reveal the modeling competencies of the group as opposed to the individual (Vorhölter, 2018). Hence, the current study aimed to investigate students' modeling competencies as a group.

A framework for modeling competencies was developed in accordance with the findings of Blum and Kaiser (1997), who stated that cognitive modeling competencies are composed of five key competencies (Maaß, 2006). According to Maaß (2006), cognitive modeling competencies are necessary modeling competencies for the completion of a modeling process; additionally, metacognitive, emotional, and social competencies emerge. However, this study focuses primarily on the cognitive competencies required to complete the modeling process. In other words, students' cognitive competencies are determined and evaluated based on the given MEAs. Cognitive modeling competencies and sub-competences consisting of five stages are presented in the table below (Maaß, 2006).

Table 2.2. Cognitive modeling competencies (Maaß, 2006, p.116)

Category	Competency
Understanding	• to make assumptions for the problem and simplify the situation. • to recognize quantities that influence the situation, to name them and to identify key variables. • to construct relations between the variables. • to look for available information and to differentiate between relevant and irrelevant information.
Mathematizing	• to mathematize relevant quantities and their relations. • to simplify relevant quantities and their relations if necessary and to reduce their number and complexity. • to choose appropriate mathematical notations and to represent situations graphically.

Table 2.2. (continued)

Working mathematically	• to use heuristic strategies such as division of the problem into part problems, establishing relations to similar or analog problems, rephrasing the problem, viewing the problem in a different form, varying the quantities or the available data etc. • to use mathematical knowledge to solve the problem.
Interpreting	• to interpret mathematical results in extra-mathematical contexts. • to generalize solutions that were developed for a special situation. • to view solutions to a problem by using appropriate mathematical language and/or to communicate about the solutions.
Validating	• to critically check and reflect on found solutions. • to review some parts of the model or again go through the modeling process if solutions do not fit the situation. • to reflect on other ways of solving the problem or if solutions can be developed differently. • to generally question the model.

The mathematical modeling competencies mentioned above, as well as their sub-competences, are required competencies for overcoming obstacles encountered during the mathematical modeling process (Grünewald, 2012). MEAs can be used to reveal the above-mentioned sub-competencies (Lesh et al., 2000).

The literature discusses two distinct ways of developing MEAs and competencies: holistic and atomistic. In the holistic approach, all stages of the modeling cycle and modeling competencies are carried out concurrently. On the other hand, in the atomistic approach, the learning environment is organized at different times, focusing on the development of certain steps and competencies (Blomhøj & Jensen, 2003; Güç, 2015). The modeling competencies of students were investigated in this study through the use of the holistic approach. The next part of the section addresses the studies on modeling competencies.

2.6.1. Studies on Modeling Competencies

The majority of research on mathematical modeling competencies comprises theoretical and applied studies, with the goal of determining, evaluating, and developing modeling competencies (Biccard, 2010; Blomhoj & Jensen, 2003; Ludwig & Xu, 2010; Maaß, 2006; Sahin & Erarslan, 2017). These studies were primarily conducted at the high school and undergraduate levels, and it was concluded that there were limited studies determining modeling competencies at the elementary and middle school levels (Aztekin & Taşpınar-Sener, 2015; Koc, 2020). The studies indicated that students' competencies in understanding and mathematizing, which are generally the first steps of the modeling process, were at a high level, whereas the interpreting and validating competencies, the last steps of the modeling process, were at a low level (Biccard & Wessels, 2011; Kabar & İnan, 2018; Maaß, 2006; Yıldırım, 2019). The examples of the studies conducted to determine elementary students' competencies are shown in the following.

Chan, Ng, Widjaja, and Cynhia (2012) analyzed the mathematical modeling competencies of two groups of 11-years-old students who were given their first opportunity to complete a modeling activity. This study demonstrated that novice modelers are capable of completing modeling activities with varied degrees of competence. According to a criterion developed by researchers, the students' level of competency falls between scale levels 1 and 2. The findings of the study indicated that students struggled to formulate mathematical problems from real-world problems by making assumptions.

İnan (2018) conducted a case study with six seventh grade students who participated in MEAs. Three MEAs, called the Weather Problem, Car Problem, and Lawn Mowing Problem, were used to collect data in the mathematics applications course. The data was gathered from two selected groups through purposive sampling using audio recordings of student solutions, solution papers for modeling problems, focus group interviews, and observation notes. According to the findings of the research, it was revealed that students from both groups offered mathematical ideas in the modeling problem without internalizing the problem situation, and they tended to reach the conclusion as quickly as possible. In addition, it was indicated that the students had difficulty in explaining their ideas in general, and they were not able to satisfactorily justify what they did during the process and evaluate it as a group.

Sahin (2019) determined the cognitive modeling competencies of fourth-grade students through MEAs and examined the factors that could affect these competencies. Ten different MEAs were applied for ten weeks to students who had no prior modeling experience. Based on the modeling competencies approach of Blum and Kaiser (1997), the extent to which the students exhibited their cognitive modeling competencies was determined using observation guide. The findings of the research revealed that fourth-grade students can display their cognitive modeling competencies and sub-competences at different levels. They were unable to perform the sub-competency of reflecting on alternative ways of solving the problem or determining whether alternative solutions could be developed. Moreover, the context of MEAs, mathematical content, social interaction, students' personal experiences, cooperation with the group, group presentations, basic language skills, attitude towards mathematics, ability to use mathematical language, cognitive difficulties, and multiple-choice assessment methods were found to be factors revealing students' modeling competencies.

In their study with ten middle school students, Hıdıroglu, Tekin-Dede, Kula, and Bukova-Guzel (2014) investigated the students' modeling process when engaged in the Comet Problem. The research determined that students encountered difficulties as they progressed through the modeling steps, that the difficulties encountered during the process of understanding the problem naturally affected subsequent steps, and that they encountered difficulties determining and using the variables. However, they encountered difficulty during the mathematization phase since they lacked competence in establishing interdisciplinary connections. In addition, it was revealed that the lack of mathematical knowledge caused students to have difficulties in the modeling process. The reason for the common omission of the interpretation step is that students are accustomed to a result-oriented problem-solving process. A strong emphasis was placed on the importance of the processes of interpretation and validation in the modeling process, and it was suggested that studies be conducted to improve the competency of students in these steps.

In the study that provides the current study with the concept of modeling competency, de Villiers (2018) aimed to investigate the development of engineering and mathematical modeling competencies through mathematical modeling. The mathematical modeling competencies were defined and mapped to the relevant engineering competencies using the existing literature in the scope of the study. In the design-based research, twelve first-year engineering technician students volunteered to take part in a one-semester course on mathematical modeling. They worked in small groups on model-eliciting activities which required the construction of models to explain, analyze, and solve real-world situations. The findings indicated that the students showed progress in all of the competency categories, with the most significant improvements occurring in the generalizing (cognitive) and management (meta-cognitive) competencies. In addition, the development of mathematical ideas and higher-order thinking occurred in an interactive way, and the characteristics of being deeply involved in model-eliciting activities allowed for the stimulation of reflective activities.

Furthermore, the recent systematics review study of Cevikbas, Kaiser and Schukajlow (2021) reflected current discussions on mathematical modeling competencies. The research provided evidence of a great diversity in approaches for assessing modeling competencies, despite the fact that a strong emphasis was placed on non-standardized assessments. Concerning the design and provision of training courses for the development of modeling competencies, the majority of studies discussed training strategies for modeling courses. Overall, the current literature analysis highlighted the need for further theoretical work on conceptualizing mathematical modeling competencies, while also showing the variety of empirical approaches that have been established and their implementation at various educational levels. Hence, the current study considers the need in the research by specifically focusing on the modeling competencies of gifted students in engineering-based MEAs.

2.7. Summary of Literature

The concept and definitions of giftedness have evolved from an emphasis on intelligence to a multidimensional approach. Thus, the focus of research has turned away from the concepts of genius and intelligence (Jensen, 1980; Morris, 1977; Terman, 1925) and towards the concepts of field-specific ability and creativity (Gardner, 2011; Guilford, 1950; Renzulli, 2005; Sternberg, 2005; Sternberg & Zhang, 1995; Torrance, 1974). In a more recent study, Chowkase (2022) advocated that the concept of giftedness should be updated with concern for others to meet the needs of the 21st-century. This theory sheds light on the concept of giftedness in the current study, and gifted individuals are those who display a potential to contribute to the welfare of others via their competence, task commitment, and concern for others. In light of these, gifted students have different cognitive and affective characteristics from their same-age peers (Clark, 2008; Davis, Rim & Siegle, 2014; Hoh, 2008; Küçükoğlu, 2014; Renzulli, 2004; Sak, 2014; VanTassel-Baska, 2003). Hence, these special characteristics of gifted students result in a variety of needs that should be addressed in the classroom environment (Borland, 2009; Johnson, 2000; Matthews & Foster, 2006; Ozdemir, 2016; Peterson, 2009). If these needs are not met, they get bored, lose their motivation, and fall behind in class (Diezmann & Watters, 2000; Taber, 2014). In order to overcome such problems, differentiated instruction should be used to provide challenging and meaningful learning experiences for gifted students (Renzulli,1977; Moon et al., 2009; Tomlinson, 2005; Tomlinson et al., 2009). These experiences should provide flexibility and complexity, encourage both higher order thinking skills (Aygün, 2010; Özdemir, 2016; Reis & Renzulli, 2009; Taber, 2014; Van-Tassel-Baska & Hubbard, 2016) and personal attributes such as curiosity, creativity and insight (Assouline &Lupkowski-Shoplik, 2021; Gül, 2021; Mammadov, 2019; Özdemir & Işıksal-Bostan, 2021; Şengil-Akar, 2017).

In particular, the complex, highly realistic, authentic, and open-ended nature of model-eliciting activities (MEAs) underpin their potential as effective educational tools for gifted students (Chamberlin, 2005; Şengil-Akar, 2017). Since less repetition and more preference of higher level thinking activities are necessary for gifted students (Gross et al., 2001), model-eliciting activities can be used to differentiate instruction for such students. Beyond that, incorporating engineering design processes into these activities, engineering-based MEAs provide more suitable instructional tools for gifted students of a less restricted and more complex nature than mathematical MEA counterparts (Diefes-Dux et al., 2008). It enables students to express their ideas more deeply and improve their technological, scientific, engineering, or mathematical thinking skills (Lyon & Magana, 2021).

On the other hand, the engineering design process is proposed as an instructional approach to differentiate instruction based on gifted students' needs in the 21st-century (Dailey, 2017; Mann & Mann, 2016). It provides a problem solving environment through an inquiry-driven process where students use their interdisciplinary knowledge about real-life events and iterative cycles, expressing, testing, and revising their product, while verbalizing and justifying their progression in technological, scientific, engineering, or mathematical thinking (English, 2019; Gainsburg, 2013; Moore et al., 2013).

Engineering-based MEAs can be used to elicit students’ conceptual understanding and the processes they develop while attempting to solve real-world problems (Lesh & Doerr, 2003). These are cognitively challenging activities for gifted students due to the fact that they demand a number of competencies and skills in addition to domain knowledge. Hence, the determination of modeling competencies, which are required skills and abilities, to willingly complete the modeling process (Maaß, 2006) is crucial for gifted students. On the other hand, the available literature indicates that there are challenges with certain sub-competencies especially at the last steps of the modeling process, and students do not advance to the expected degree of competence (Biccard & Wessels, 2011; Kabar & İnan, 2018; Maaß, 2006; Yıldırım, 2019). In line with these, it is important to determine how gifted students display their unique characteristics in the modeling process. Thus, this study aims to determine the cognitive modeling competencies of gifted students when they are engaged in engineering-based MEAs.

CHAPTER 3

METHODOLOGY

The purpose of this study is to investigate the cognitive modeling competencies of elementary gifted students when they are engaged in engineering-based model eliciting activities (MEAs). Another focus of this study is to examine the extent to which elementary gifted students in groups displayed such cognitive modeling competencies and their related sub-modeling competencies when they are engaged in engineering-based MEAs.

Accordingly, this chapter presents an explanation and justification for the method and procedures that were used in the current study. This includes details regarding the participants, research setting, data sources, data collection methods, and data analysis. The method employed in this research made it possible to address the research questions aimed at reaching the above-mentioned purpose. Accordingly, this chapter is divided into eight sections: design of the study, research setting, participants of the study, data collection procedure, data analysis, the trustworthiness of the study, researcher background and role in the study, and ethical considerations, respectively.

3.1. Design of the Study

In accordance with the above-mentioned purpose, this study requires to analyze the cognitive modeling competencies of elementary gifted students while engaging in engineering-based MEAs as a group. In light of this, qualitative research was employed since this study mainly focused on gaining in-depth information about what actually occurs in this particular setting. Creswell (2015) states that qualitative research methods are appropriate when complex problems need to be investigated in order to “obtain a deep understanding” of a phenomenon being studied (p. 19). In this regard, qualitative researchers focus on the participants’ voices, their experiences, and interpretations of those experiences (Meriiam, 2009). More comprehensively, Denzin and Lincoln (2005) defined it as:

> Qualitative research is multimethod in focus, involving an interpretive, naturalistic approach to its subject matter. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them. Qualitative research involves the studied use and collection of a variety of empirical materials-case study, personal experience, introspective, life story, interview, observational, historical, interactional, and visual texts- that describe routine and problematic moments and meanings in individuals' lives (p.2).

Based on this approach, qualitative research was applicable for this study because elementary gifted students studied as a group in their natural context and in terms of the meanings they brought to engineering-based MEAs. According to Creswell (2012), a qualitative research design can be used in studies in which the variables that are affecting a situation are not fully known. This type of research design can be used to reveal and investigate these variables through in-depth data collection and analysis methods (Yıldırım & Şimşek, 2004). Using qualitative research methods, it is possible to investigate in more detail the mechanisms underlying the occurrence of certain behaviors, the functioning of particular processes, and the relationships that exist among specific variables. When all of the mentioned definitions and characteristics of the qualitative research method are considered, it is found as compatible with the current study, which aims to obtain detailed information about how engineering-based MEAs function in the natural environment of elementary gifted students.

As one of the approaches of qualitative research, a case study was used as a research method (Creswell, 2012; Merriam, 2009; Stake, 2005; Yin, 2009) to address the research question in this study. Since the aim of this study requires to investigate experiences of elementary gifted students when they are engaged in engineering-based MEAs regarding cognitive modeling competencies, the case study research was adopted for the current study. According to Merriam (2009), case study is "an intensive, holistic description and analysis of a single entity, phenomenon, or social unit" (p.46). Similarly, Yin (2009) describes case study as "empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context" (p. 18). Hence, this type of research enables researchers to investigate a bounded system or multiple bounded systems over a period of time, through detailed, in-depth data collection involving multiple resources of information, and then reports a case description and case-based themes (Cresswell, 2007). Hence, one of the most critical aspects in the case studies is to define the case being studied. A case is a specific, complicated, recognized, and integrated system, in which the case itself serves as the subject of the study rather than the procedure (Stake, 2005). The case should provide the researcher with access to appropriate study participants who can address or contribute to research questions (Crowe et al., 2011). In light of this, the case study researchers defined the case within a bounded system in order to understand how the parts operate together in the system (Merriam, 2009; Stake, 2005; Yin, 2009). To put it differently, the boundaries of the system enable the researcher to determine what the case is and what is not. There are different ways to bound a case, some of which are as follows: (a) by time and place (Creswell, 2003); (b) by time and activity (Stake, 2005); and (c) by definition and context (Miles & Huberman, 1994). Another critical aspect in the case study is to determine the unit of analysis. An entity that you intend to have something to say about at the conclusion of the investigation is referred to as a unit of analysis, and this entity is considered to be the main focus of the study (Yin, 2009).

In particular, in the current study, the case is defined as engineering-based MEAs which were developed by the researcher. The case is bounded by group experiences of elementary gifted students who attended BILSEM, the context (BILSEM) and application procedure of the activities and the use of think-aloud procedure to elicit elementary gifted students' cognitive modeling competencies. In addition, cognitive modeling competencies that elementary gifted students displayed when they are engaged in engineering-based MEAs served as the unit of analysis for the present study. In order to keep the focus and scope of the study, the researcher is required to concentrate on the specific details of a case and to conduct an analysis of its context (Merriam, 2009). In this regard, it is necessary to analyze the cognitive modeling competencies of elementary gifted students that emerge in the engagement of engineering-based MEAs as a group, examine them holistically and reveal them in detail. For this reason, each activity was addressed separately for each group, but as a holistic situation in itself. This facilitates recognizing cases as holistic entities, each of which consists of parts and acts within the context of their settings (Stake, 2005).

There are various classifications of case study designs depending on characteristics such as the number of case, the researcher's interest, and the purpose of the researcher (Merriam, 2009; Stake, 2005; Yin, 2009). Yin (2009) identifies three types of case studies: exploratory, descriptive, and explanatory. First, exploratory case studies are designed to explore any phenomenon in the data that acts as a point of interest for the researcher. Second, descriptive case studies are designed to describe the natural phenomena observed in the relevant data. Third, explanatory case studies are designed to provide an explanation for the observed phenomena in the data by looking at it from a variety of perspectives.

Moreover, Stake (2005) classifies three categories of case studies according to the goal of the researcher conducting the study as intrinsic, instrumental and collective. In an intrinsic case study, the primary aim of the researcher is to gain a better understanding of a particular case. In an instrumental case study, the issues are still the primary focus, and the case itself provides insight into a specific event or phenomena or refines a theory (Stake, 2005). If the issue or question to be addressed in a case study is more significant to the researcher than the case itself, the instrumental case study method is recommended. In other words, the instrumental case study is seen advantageous for investigating individual's perceptions in the context of a particular action or activity (Creswell, 2012; Stake, 2005). The aim of an instrumental case study is to understand and describe the processes associated with a theory or method (Stake, 1995). On the other hand, it is not the purpose of instrumental case studies to generalize or address a typical scenario; rather, the purpose of these studies is to gain in-depth knowledge about a subject or theme, or to throw light on a particular situation (Creswell, 2012; Mills, Durepos & Wiebe, 2009; Stake, 2005; Yin, 2009). According to Gordin (2006), instrumental case studies are able to be conducted in order to achieve an in-depth understanding of a certain topic by analyzing one or more cases. In the context of an instrumental case study, Stake (2005) claimed that there is more than one setting that may be used to collect data on the relevant research question in order to answer the research question. When multiple instrumental cases are investigated, the study is referred to as collective case study (2005). In collective case study, multiple instrumentally bounded cases are selected to obtain a deeper understanding of the phenomenon than a single case provides (Mills et al., 2009). The use of a multiple-case design enables the examination of processes and outcomes across multiple cases, the identification of the ways in which individual cases might be affected by different environments, and the identification of the specific conditions under which a finding might occur. It also might be helpful to develop broader classifications of how the particular conditions might be related to one another. All of this makes the findings more convincing than those from a single case and displays the issues across a wider range of circumstances than can be provided by a single case alone (Mills et al., 2009).

In accordance with the goal of this study, the design of collective case study in which the exploration of multiple instrumental cases was employed. The use of collective case studies requires an in-depth investigation of a number of individual case studies as instruments. Figure 3.1 demonstrates how the current study may be modelled.

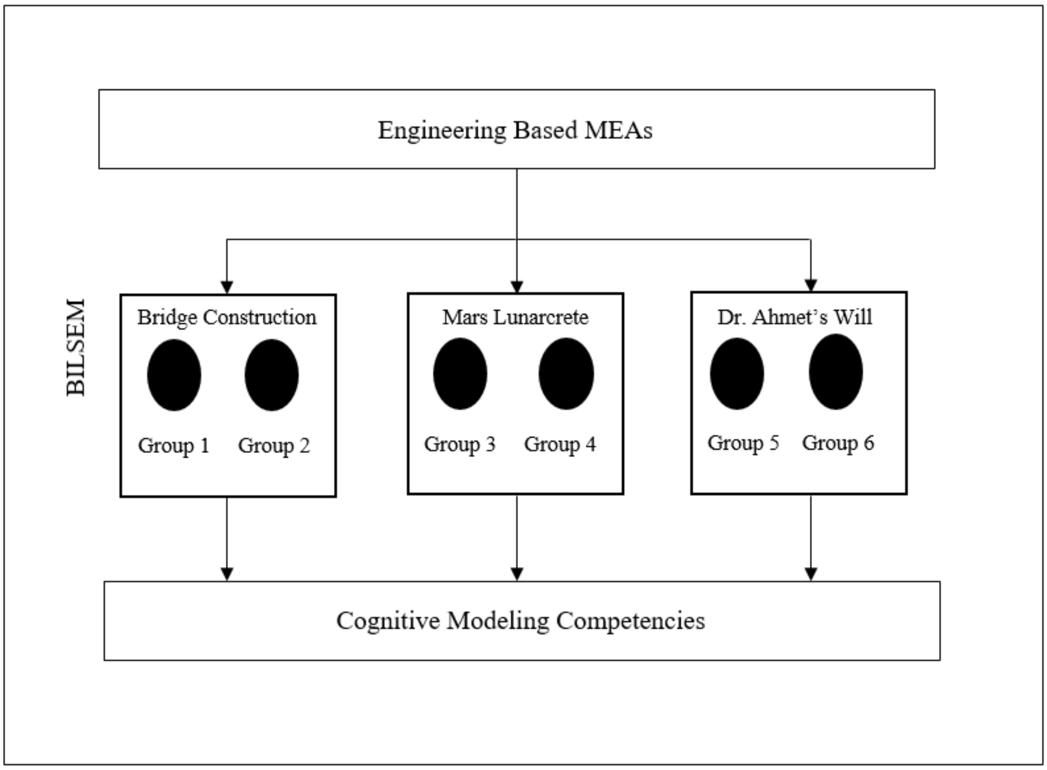


Figure 3.1. Collective case study in this research

In this study, collective case study approach was used since the aim was to determine the level of elementary gifted students' modeling competencies rather than the use of the activities for elementary gifted students. Accordingly, engineering-based MEAs had the potential to be instrumental value. To analyze findings across similar cases, a researcher may select cases with similar characteristics (Mills et al., 2009). Within the scope of this study, each of the three engineering-based MEAs, namely Bridge

Construction, Mars Lunarcerete and Dr. Ahmet’s Will was considered as an instrumental case with similar characteristics and evaluated holistically. In order to investigate elementary gifted students’ modeling competencies while involving in the design process, these specific activities used as a tool. Thus, the cases play a secondary and supportive role to “help our understanding of something else” (Stake, 1994, p.237) in this study. Specifically, the process, content and structure of the activities are all constitute the details of the case. In the Figure 3.1, each engineering-based MEAs is considered as an individual case. Each single bounded case’s data are carefully scrutinized and structured into a thorough description of a singular, holistic entity, modeling competency. As a result, one of the key goals of collective case study is to get a deeper knowledge of the method and processes, in order to enable the methods and processes to be adjusted or applied in other settings. Before explaining the specific approach utilized in this case study, it is essential to provide the researchers with a summary of the organizational structure of the programs at BILSEM and activities so they can comprehend how the activities function as a whole.

3.2. Research Setting

The current study was conducted with elementary gifted students who attended one of the BILSEMs in Ankara. BILSEMs are institutions where gifted individuals receive education in order to develop their abilities, skills, competencies, and potentials in the disciplines of science and art, based on their educational needs according to the fields in which they have been identified (MoNE, 2019). If an individual is assessed as having a potential in at least one of three distinct skill areas - general mental, visual arts, or music- they are eligible to get training and participate in activities offered at the centers.

The students who will enroll in these centers are chosen through a multi-stage evaluation process. Firstly, teachers in regular primary school classrooms use an observation form in order to identify children who are talented in general mental ability, visual arts or music. Then, the selected students take the group intelligence test. Successful students, as determined by the results of the group intelligence test, proceed to take the individual intelligence test. Following the completion of the individual intelligence tests, the placement committee carry out their final evaluation of the students. Students who have an IQ of 130 or higher are considered for placement at BILSEM (MoNE, 2007). The committee makes the ultimate decision after analyzing the results of group and individual intelligence tests administered to students, as well as the capacity of a center located within a province (Şahin, 2015).

Educational activities are conducted by BILSEM as after school activities without interrupting gifted students' education in formal institutions (MoNE, 2019). Thus, BILSEM provides educational opportunities for gifted students in line with their interests and enable them to use their existing capacity at the highest level (MoNE, 2019). Gifted students have the option of receiving education in BILSEMs either once or twice a week, outside of regular school hours on weekdays. In some BILSEMs, gifted students also have the option of receiving full-time education on Saturdays. These centers offer a form of pull-out enrichment program. The programs implemented in these centers do not have national objectives. The program is modified for each student so that it caters to their own interests, needs, and pace of learning. The teaching method is the primary distinction between the programs offered by the centers and those offered by the general education program. Higher levels of thinking, an open-ended environment, opportunities for discovery and creativity, freedom of choice, and collaborative work constitute the main components of the process in these centers (Sak, 2010).

The completion of five programs; orientation program, the support training program, the individual talent recognition program, the special talent development program, and the project production and management program, is required for progression in the programs (MoNE, 2019). In the first stage of the program, students and their parents are informed about physical environment, the mission and vision of the BILSEMs and the educational model which is followed in these centers. Gifted students and their parents who enroll at BILSEM for the first time are required to participate in an orientation program that lasts no longer than two months and forty lesson hours. At the end of this phase, students determined gifted with general mental ability are transferred to the support training program, whilst students determined gifted with visual arts or music are transferred to the special talent development program (MoNE, 2019). The second stage of the program is called the support training program. The support training program involves the activities of the education program focused on combining the most essential skills that students must acquire with all subjects or disciplines. The goals of this program are to develop skills such as collaboration, communication, the ability to learn, teamwork, scientific research, entrepreneurship, problem solving, critical and creative thinking, effective decision making, technological literacy, efficient use of resources, and social responsibility. Gifted students who are qualified to enroll BILSEM in the second and third grades participate in the support training program for a total of two academic years, including the orientation program. The students will receive the majority of their instruction during the first year from their regular classroom teachers. For higher grade levels, the program is applied for at least one academic year, including the orientation program. (MoNE, 2019). In the third stage of the program, which is called the individual talent recognition program, the students are given the opportunity to discover their skills by participating in activities from a variety of fields. The primary goal of this program to introduce students who have been determined gifted with general mental ability to a wide range of academic fields. The duration of the individual talent recognition program attended by students within the scope of this study is planned and implemented across three academic years so that each student is exposed to all disciplines. Within this program, the students continue their study in a minimum of two and a maximum of four different subjects. After completing this program, the teachers' board evaluates students using the multiple assessment approach. Students are guided to specialized talent development program areas based on the results of this evaluation (MoNE, 2019). In the fourth stage, the special talent development program is for students to acquire advanced knowledge, abilities, and behavior in a discipline while also considering inter-disciplinary connections. The activities in this program include production-oriented studies with the goal of obtaining in-depth and advanced knowledge, skills, and behaviors. Maximum of two subjects are offered to students in the special talent recognition program. This program is comprised of two academic years for individuals gifted in general mental ability and four academic years for students gifted in visual arts or music (MoNE, 2019). In the final stage of the program, called the project production and management program, consultation is provided to students in order to enhance their knowledge and experience of the project preparation and development processes (MoNE, 2009). Each academic year, the students enrolling in this program are required to complete at least one individual project. Gifted students continue to participate in the project production and management program until they graduate from high school.

To be more specific, the research was conducted in the science laboratory at the BILSEM, which served as the research setting for the study. This setting was chosen because gifted students had access to a variety of science and mathematics-related resources. The seating arrangement in this laboratory is in the form of clusters. In the following part of the section, specific information about the participants of the study is offered.

3.3. Participants of the Study

The participants of the current study are nineteen elementary gifted students who enrolled in the individual talent recognition program at one of the BILSEMs in Ankara. In qualitative research, the primary goal is to determine the participants and sites where the researcher gather information on the research questions in the most effective way possible to understand the central phenomenon (Creswell, 2012). According to Sankar-DeLeeuw (2004), when studying a sub-population of gifted students, it is difficult to select samples of gifted students from a large population. A random selection method can be impossible. Hence, this study employed purposive and convenience sampling. A purposive sampling is a non-probability sample selected based on population characteristics and the purpose of the research (Patton, 2014). The real significance and effectiveness of purposive sampling derives from its emphasis on the selection of information-rich, specific situations to collect in-depth data (Patton, 2014). The researcher used purposive sampling to select the participants because the researcher obtains the more knowledgeable information about the modeling competencies when they are engaged in engineering-based MEAs from that group. Another reason to select this sampling method is related to the participants' characteristics since the aim of the individual talent recognition program at BILSEM that enables gifted students to reveal their skills in activities from a variety of fields overlapped with the engineering-based MEAs possible contribution in literature. In addition, as a convenience sampling, the researcher selected the most convenient site among the BILSEMs in Ankara to obtain data.

The present study was carried out within the scope of the summer school programs carried out in BILSEM. First of all, the administrators, teachers, parents and students were initially informed on the goal and scope of this study. Then, students who volunteered to participate in this study applied to this program of the summer school. After receiving parental consent, the participant groups of the study were formed. Considering their schedule in the summer school program, administers at BILSEM separated participants into two groups based on the grade levels they have completed. In order to keep the identity of the participants confidential, each participant was assigned a number, and each group was assigned a letter. The detailed information about the participants of the study is shown in Table 3.1.

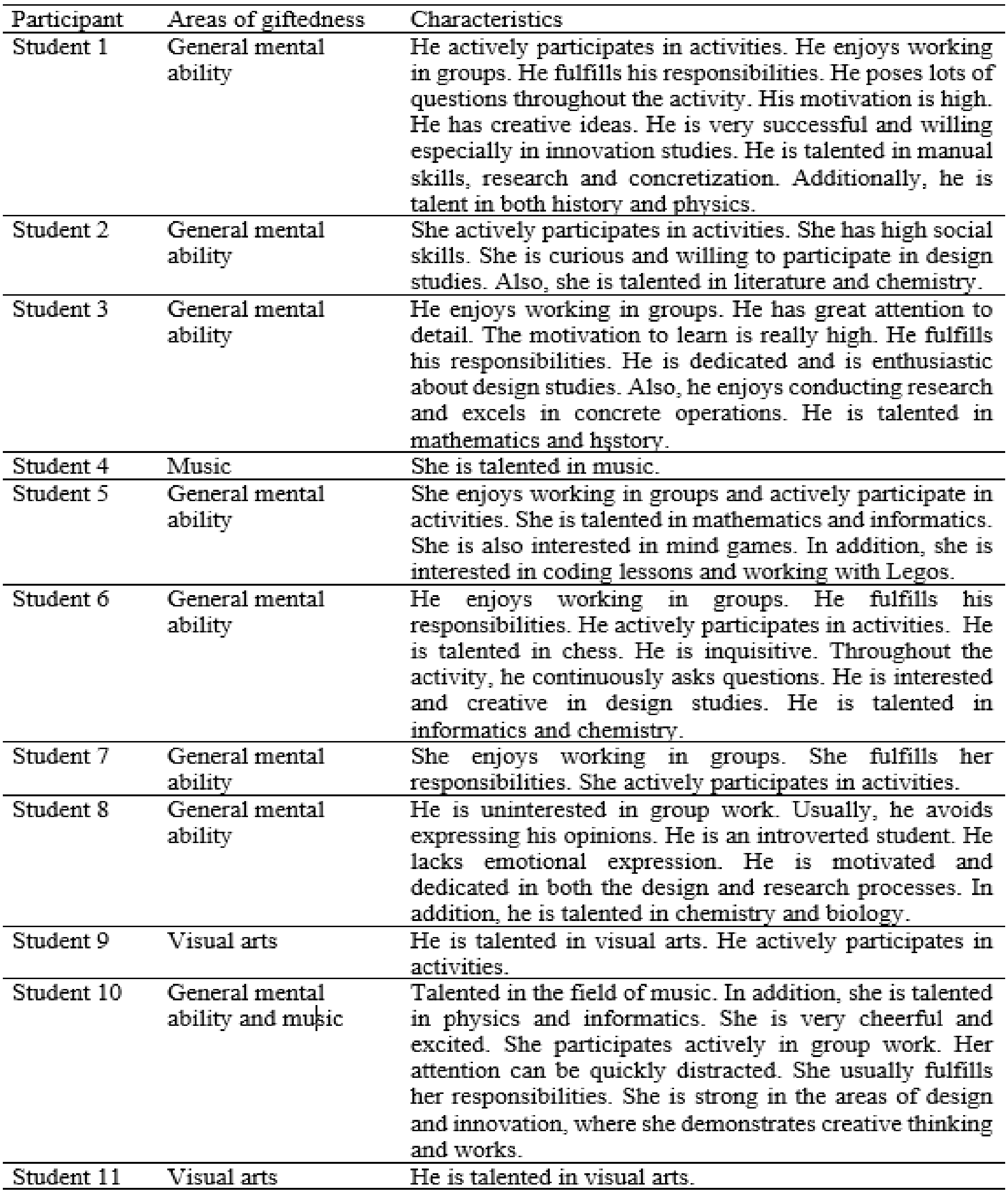
Table 3.1. Demographic information about the participants

Group	Participant	Gender	Type of regular school	Grade Level	Age	Duration of attending BILSEM (year)
Group A	Student 1	Male	State	5	10	1
	Student 2	Female	Private	5	10	2
	Student 3	Male	Private	6	12	2
Group B	Student 4	Female	Private	6	11	2
	Student 5	Female	State	5	10	2
	Student 6	Male	Private	5	10	3
Group C	Student 7	Female	Private	6	11	3
	Student 8	Male	State	5	10	2
	Student 9	Male	State	6	11	2
Group D	Student 10	Female	Private	6	10	3
	Student 11	Male	State	5	10	1
	Student 12	Male	State	5	10	1
Group E	Student 13	Male	State	4	9	1
	Student 14	Male	Private	4	9	1
	Student 15	Male	State	4	9	1
Group F	Student 16	Female	Private	4	10	2
	Student 17	Male	State	4	9	2
	Student 18	Male	State	4	9	2
	Student 19	Male	State	4	9	1

As shown in the Table 3.1, most of the participants are male (13) and the others are female (6). In addition, seven of them are fourth graders, seven of them are fifth graders and five of them are sixth graders. Among them, most of them (11) are attended state school while remaining eight of them attended private school in regular education. Moreover, six participants are nine years old, nine are ten years old, three are eleven years old, and one is twelve years old. When the duration of attending

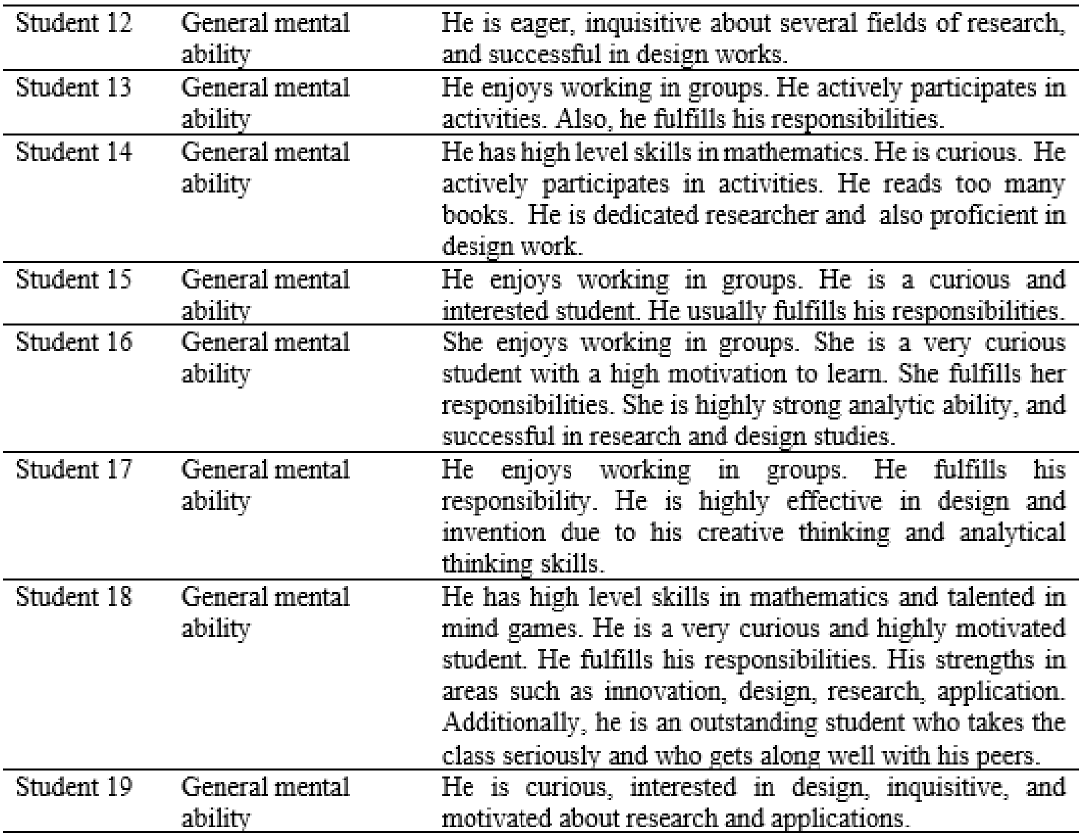
BILSEM is considered, seven of the participants have attended for a period of one year, nine participants for a period of two years, and three participants for a period of three years. In addition to some demographic information, the individual characteristics of the participants were presented in the table 3.2. The individual characteristics of the participants in the study were determined by interviewing with BILSEM teachers.

Table 3.2. Individual characteristics of the participants



Participant	Areas of giftedness	Characteristics
Student 1	General mental ability	He actively participates in activities. He enjoys working in groups. He fulfills his responsibilities. He poses lots of questions throughout the activity. His motivation is high. He has creative ideas. He is very successful and willing especially in innovation studies. He is talented in manual skills, research and concretization. Additionally, he is talent in both history and physics.
Student 2	General mental ability	She actively participates in activities. She has high social skills. She is curious and willing to participate in design studies. Also, she is talented in literature and chemistry.
Student 3	General mental ability	He enjoys working in groups. He has great attention to detail. The motivation to learn is really high. He fulfills his responsibilities. He is dedicated and is enthusiastic about design studies. Also, he enjoys conducting research and excels in concrete operations. He is talented in mathematics and hşstory.
Student 4	Music	She is talented in music.
Student 5	General mental ability	She enjoys working in groups and actively participate in activities. She is talented in mathematics and informatics. She is also interested in mind games. In addition, she is interested in coding lessons and working with Legos.
Student 6	General mental ability	He enjoys working in groups. He fulfills his responsibilities. He actively participates in activities. He is talented in chess. He is inquisitive. Throughout the activity, he continuously asks questions. He is interested and creative in design studies. He is talented in informatics and chemistry.
Student 7	General mental ability	She enjoys working in groups. She fulfills her responsibilities. She actively participates in activities.
Student 8	General mental ability	He is uninterested in group work. Usually, he avoids expressing his opinions. He is an introverted student. He lacks emotional expression. He is motivated and dedicated in both the design and research processes. In addition, he is talented in chemistry and biology.
Student 9	Visual arts	He is talented in visual arts. He actively participates in activities.
Student 10	General mental ability and music	Talented in the field of music. In addition, she is talented in physics and informatics. She is very cheerful and excited. She participates actively in group work. Her attention can be quickly distracted. She usually fulfills her responsibilities. She is strong in the areas of design and innovation, where she demonstrates creative thinking and works.
Student 11	Visual arts	He is talented in visual arts.

Table 3.2. (continued)



Student 12	General mental ability	He is eager, inquisitive about several fields of research, and successful in design works.
Student 13	General mental ability	He enjoys working in groups. He actively participates in activities. Also, he fulfills his responsibilities.
Student 14	General mental ability	He has high level skills in mathematics. He is curious. He actively participates in activities. He reads too many books. He is dedicated researcher and also proficient in design work.
Student 15	General mental ability	He enjoys working in groups. He is a curious and interested student. He usually fulfills his responsibilities.
Student 16	General mental ability	She enjoys working in groups. She is a very curious student with a high motivation to learn. She fulfills her responsibilities. She is highly strong analytic ability, and successful in research and design studies.
Student 17	General mental ability	He enjoys working in groups. He fulfills his responsibility. He is highly effective in design and invention due to his creative thinking and analytical thinking skills.
Student 18	General mental ability	He has high level skills in mathematics and talented in mind games. He is a very curious and highly motivated student. He fulfills his responsibilities. His strengths in areas such as innovation, design, research, application. Additionally, he is an outstanding student who takes the class seriously and who gets along well with his peers.
Student 19	General mental ability	He is curious, interested in design, inquisitive, and motivated about research and applications.

The above table provided participants' areas of giftedness and some individual characteristics of participants. The participants' areas of giftedness show the areas which are recognized as gifted in order to attend BILSEM. In this table, the areas of giftedness are those in which students are identified to attend BILSEM.

In particular, the participants are not familiar with mathematical modelling and engineering design problems. Thus, engineering-based MEAs was applied this sample purposefully. Although the current mathematics curriculum emphasized the mathematical modelling, students do not have an opportunity to involve in model eliciting activities in their regular classroom environments. On the other hand, the activities in BILSEM enable them to communicate with their peers and work in groups.

3.4. Data Collection Tools

The data sources of this study are videotapes of gifted students' modelling experiences and interviews, their worksheets, researcher field notes and observation notes of the teacher. When conducting research, all implementation process was recorded through video or voice recorder. The following is an explanation of how the process is actualized through the use of engineering-based MEAs.

3.4.1. Engineering-based MEAs

When preparing activities for elementary gifted students, the characteristics of differentiated activities that cater to the unique needs of these students were identified (Ozdemir, 2016). As a result, engineering-based MEAs were determined to be appropriate activities due to the fact that they are multidisciplinary, open-ended, complicated, and authentic (Lesh & Doerr, 2003). To put it more precisely, such activities are challenging, interesting and requires higher level of thinking to satisfy the instructional needs of gifted students (Ozdemir, 2016).

Accordingly, all three engineering-based MEAs used in the current study were developed based on the six principles for designing MEA in mathematics and engineering education (Diesfes-Dux et al., 2008) and the background knowledge of this specific group, elementary gifted students. In accordance with the goal of the present study, engineering-based MEAs were developed in response to the need for gifted students to provide evidence of their modeling competencies in order to successfully complete the modeling process. To put it plainly, when developing these activities, it was expected that seven modeling competencies (internalizing, interpreting, structuring, symbolizing, adjusting, organizing, and generalizing) would be demonstrated, because as the stages required to complete the modeling process proposed by Blum and Leiß (2007) are related to modeling competencies. In the scope of this study, *Bridge Consruction*, *Dr. Ahmet's Will* and *Mars Lunarcrete* activities were used. The purpose of using three distinct engineering-based MEAs is to conduct an analysis of cognitive modeling competencies in a variety of activities that are designed to prepare participants for various forms of engineering, which are civil, architectural and data mining engineering. In other words, the Bridge Construction and Mars Lunarcrete activities involve building a prototype using civil and architectural engineering. The former is a situation frequently encountered in daily life, while the latter is related to space. On the other hand, Dr. Ahmet’s Will activity requires developing procedure, not the product. Hence, the researcher aims to investigate elementary gifted students’ cognitive modelling competencies when they are engaged in different forms of engineering-based MEAs.

In addition, it was concluded that gifted students are interested in the events occurring in their surrounding area and across the world, as well as the issues on the agenda (Taber, 2014). The context of the problems was chosen by BILSEM teachers in mathematics and scientific education, as well as specialists in science and mathematics education, taking into consideration gifted students’ individual areas of interest. For instance, it was expected that, *send your name to Mars* project, which was on the agenda at the time, the *Mars Lunarcrete* activity would capture the interest of the students. In addition, the context of the activities including *Bridge Construction* and *Dr. Ahmet’s Will* includes regional and global challenges that students may encounter frequently in their daily lives.

Engineering-based MEAs were implemented by the researcher. In addition, the science teacher was also present in the classroom as an observer. Before the implementation, students solved apple pie problem (Schukajlow, Leiss, Pekrun, Blum, Müller, & Messner, 2012) individually. In this situation, Sevinç’s mother requested her to purchase three kg of apples and return. Sevinç has two options from which she must choose the most logical one. Option 1: At the grocery close to your home a half kilogram of apples costs 1 Turkish Lira. Option 2: 1 kilogram of apples costs 1.5 Turkish Lira at a market a short distance from your home. Due to the distance to the market, she must take the bus. Then, the interview conducted with each participant. In the interview, some questions such as why you choose this option, what you consider when choosing it, what you take into account any more factors were posed in order to obtain the details of the solution. Based on the data from their modelling process and interview, students were separated into two groups by the researcher and science and mathematics teachers.

The participants worked in mixed-ability groups of three or four and spent five 45-minute sessions on the engineering-based model eliciting activities. In the first session, elementary gifted students studied the newspaper article and the readiness questions for *Bridge Construction* and *Mars Lunarcrete* activities. For *Dr. Ahmet's Will* activity, the students watched the video prepared by Turkey for the 2012 Olympic candidate countries and replied the readiness questions. The *Bridge Construction* and *Mars Lunarcrete* problem were presented to the students and they draw design sketches. They determined which material they use for the construction of their bridge or construction of their structure. In the *Dr. Ahmet's Will* activity, the problem situation was presented and they determined which variables they use to specify the best location for relatives. In the next two sessions, they constructed their prototypes or models and wrote letters to committee, cooperation or lawyer, describing and documenting their prototypes or models. In the last two sessions, students revised their prototypes or models for new criteria and present their models to the whole class for discussion. Internet was available for them to make easier their explorations during the development of their models. All of these procedures are summarized in Table 3.3.

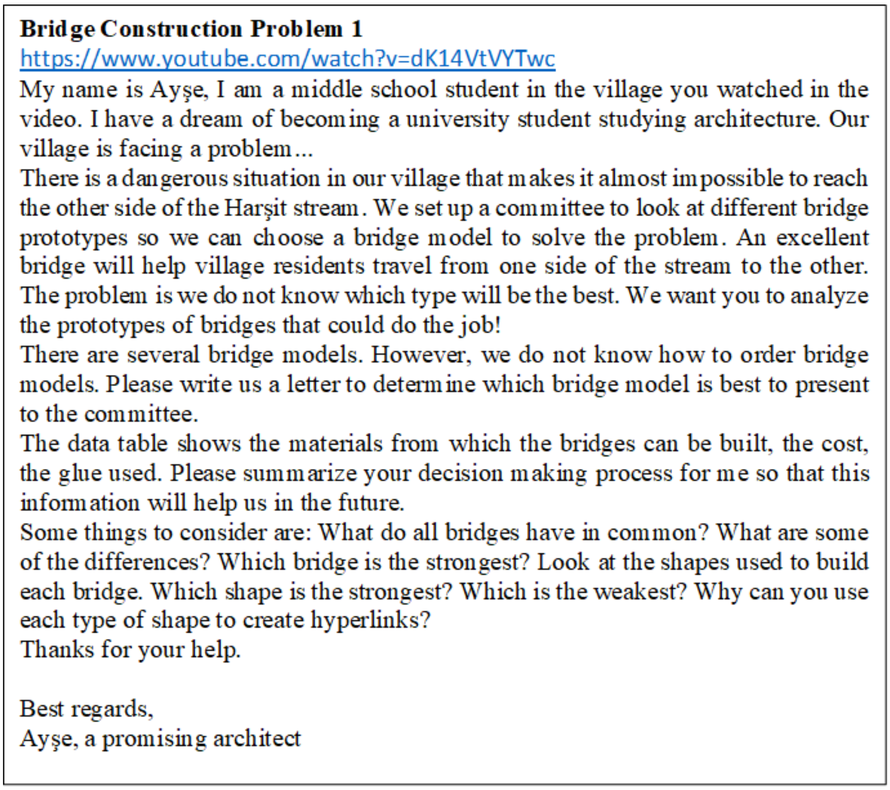
Table 3.3. The study's progression plan and accompanying data collection tool

Sessions	Bridge Construction Activity	Mars Lunarcrete Activity	Dr. Ahmet's Will Activity
Week 1	• Warm-up task: Newspaper article (The dramatic story of the Brooklyn bridge) • Readiness questions • The news video • Bridge Construction problem sheet 1 and data set 1 • Drawing sheet • Focus-group interview	• Warm-up task: Newspaper article (Lunarcrete construction in Moon) • Readiness questions • Mars Lunarcrete Problem sheet 1 and data set 1 • Drawing sheet • Focus-group interview	• Warm-up task: Video prepared by Turkey for Tokyo 2012 Olympic candidate counties • Readiness questions • The letter from Dr. Ahmet's to his relatives • Dr.Ahmet's Will problem sheet 1 and data set 1 • Focus-group interview
Week 2	• First letter to committee • Focus-group interview	• First letter to corporation • Focus-group interview	• First letter to lawyer • Focus-group interview
Week 3	• Bridge Construction problem sheet 2 and data set 2 • Focus-group interview	• Mars Lunarcrete problem sheet 2 and data set 2 • Focus-group interview	• Dr. Ahmet's Will problem sheet 2 and data set 2 • Focus-group interview
Week 4	• Second letter to committee • Focus-group interview	• Second letter to corporation • Focus-group interview	• Second letter to lawyer • Focus-group interview
Week 5	• Group presentation to whole class • Whole class discussion • Focus-group interview	• Group presentation to whole class • Whole class discussion • Focus-group interview	• Group presentation to whole class • Whole class discussion • Focus-group interview

In the next sections, the detailed information about each engineering-based MEAs are provided.

3.4.1.1. Bridge Construction Activity

Bridge Consruction is a widespread activity in literature for students (e.g. Carroll, 1997; English, Hudson, & Daves, 2012; English & King, 2018). However, the news video about the request of the peasants to construction of the bridge instead of tumbledown, 60-year-old wooden suspension bridge which provides transportation among three villages in the Black Sea Region of Turkey was related to real-life problem as shown in Figure 3.2. Consequently, in the present study, the researcher adapted the activity for the Turkish context.



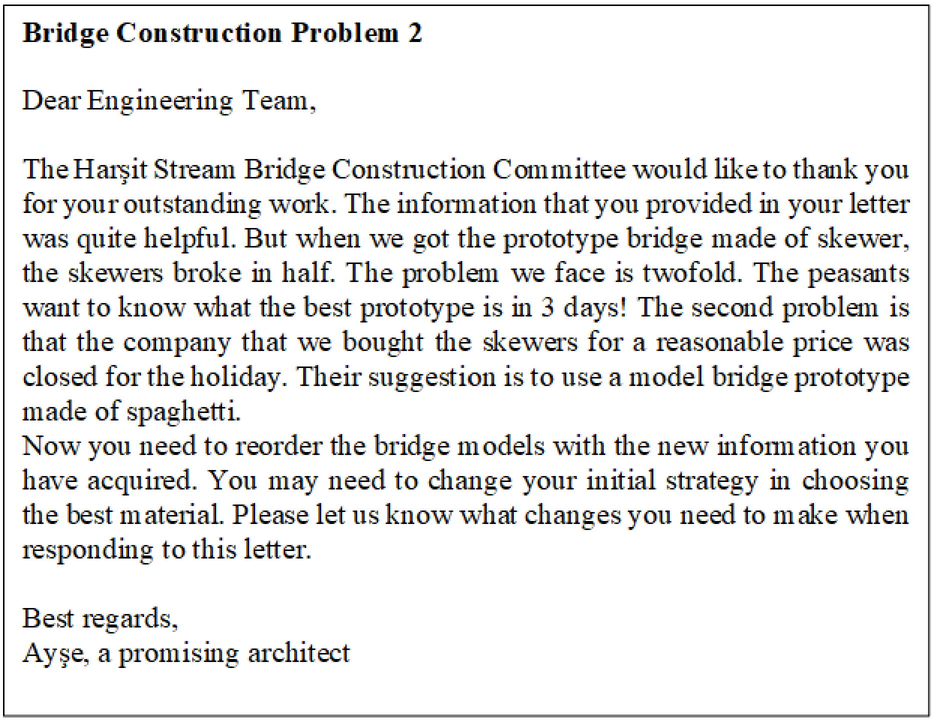
Bridge Construction Problem 1
https://www.youtube.com/watch?v=dK14VtVYTwc
My name is Ayşe, I am a middle school student in the village you watched in the video. I have a dream of becoming a university student studying architecture. Our village is facing a problem...
There is a dangerous situation in our village that makes it almost impossible to reach the other side of the Harşit stream. We set up a committee to look at different bridge prototypes so we can choose a bridge model to solve the problem. An excellent bridge will help village residents travel from one side of the stream to the other. The problem is we do not know which type will be the best. We want you to analyze the prototypes of bridges that could do the job!
There are several bridge models. However, we do not know how to order bridge models. Please write us a letter to determine which bridge model is best to present to the committee.
The data table shows the materials from which the bridges can be built, the cost, the glue used. Please summarize your decision making process for me so that this information will help us in the future.
Some things to consider are: What do all bridges have in common? What are some of the differences? Which bridge is the strongest? Look at the shapes used to build each bridge. Which shape is the strongest? Which is the weakest? Why can you use each type of shape to create hyperlinks?
Thanks for your help.

Best regards,
Ayşe, a promising architect

Figure 3.2. Bridge Construction Problem 1

As stated in Figure 3.2., the data table is provided to show the materials from which the bridges can be built, the cost, the glue used. In particular, as shown in Table 3.3, this activity involves: (a) a warm up task including a newspaper article about the construction and dramatic story of the Brooklyn Bridge to introduce the context of the model-eliciting activity. (b) Readiness questions to be answered related to the article.

Students defined vocabulary words for bridge construction. After whole class discussion, different types of bridges (truss, arch and beam) were introduced. (c) The news video about the request of the peasants to construction of the bridge instead of tumbledown, 60-year-old wooden suspension bridge which provides transportation among three villages in the Black Sea Region of Turkey. (d) The problem to be solved with the data. Students are asked to use the information provided and any other resources they may find useful to develop bridge prototype in order to help the committee make the best type of the bridge. After completing the activity, students write a letter to committee, documenting the method they use to develop their model. After they construct their first bridge and write first letter to committee, the second problem situation as shown in Figure 3.3 was presented to gifted students.



Bridge Construction Problem 2

Dear Engineering Team,

The Harşit Stream Bridge Construction Committee would like to thank you for your outstanding work. The information that you provided in your letter was quite helpful. But when we got the prototype bridge made of skewer, the skewers broke in half. The problem we face is twofold. The peasants want to know what the best prototype is in 3 days! The second problem is that the company that we bought the skewers for a reasonable price was closed for the holiday. Their suggestion is to use a model bridge prototype made of spaghetti.

Now you need to reorder the bridge models with the new information you have acquired. You may need to change your initial strategy in choosing the best material. Please let us know what changes you need to make when responding to this letter.

Best regards,
Ayşe, a promising architect

Figure 3.3. Bridge Construction Problem 2

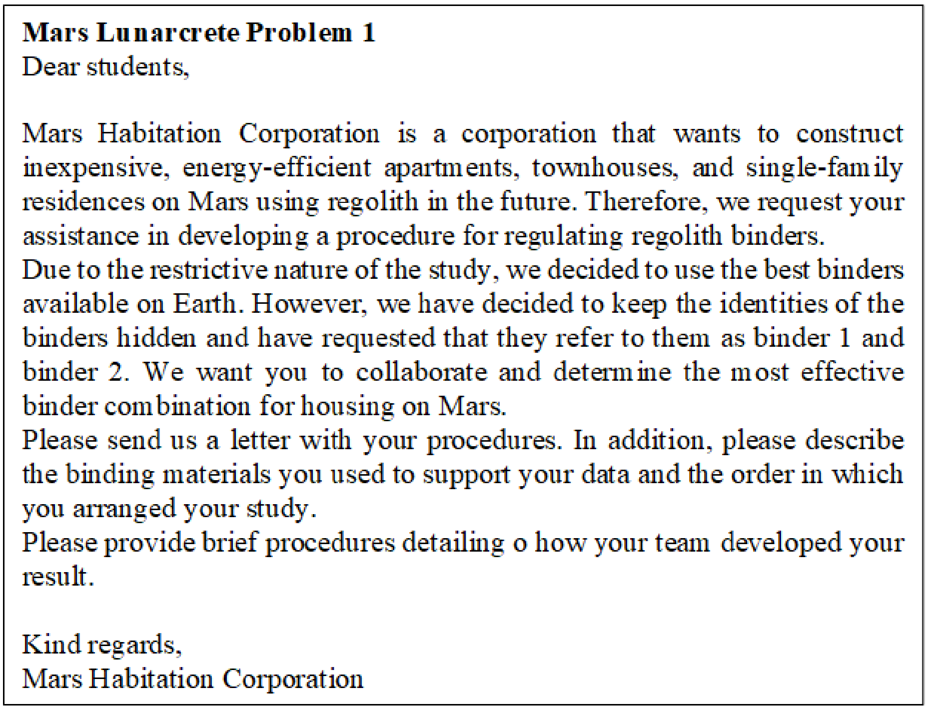
The students reviewed their first prototype in line with the second problem and, if necessary, built a second prototype. After completing the prototype, they wrote a letter to the committee explaining the procedures they used, comparing their first and second prototypes. At the end of the process, elementary gifted students present their models as a group to the whole class for discussion. That is to say, both groups gave each other feedback on the prototypes that they had developed. Considering all of these, Bridge

Construction activity addresses six model-eliciting principles for constructing models in mathematics and engineering education as stated in literature review chapter (Diesfes-Dux et al., 2008). The detailed information regarding Mars Lunarcrete activity is provided in the following part of this section.

3.4.1.2. Mars Lunarcrete Activity

The activity was based on the work of CPALMS (2017b), the official source of standards information and course descriptions for the State of Florida. CPALMS was established by the Florida State University's Florida Center for Research in Science, Technology, Engineering, and Mathematics (FRC-STEM). In order to develop an activity associated with Mars, the researcher developed the context of this activity by using the literature (CPALMS, 2017b).

In this activity, students would create a model that works to determine the best regolith for settlement in Mars and participate in NASA's building design competition for colonies settling in Mars. Students would contact a company requesting their services. Students would examine the work done to create their own lunarcrete, work on this issue and create their own lunarcrete as shown in Figure 3.4.



Mars Lunarcrete Problem 1

Dear students,

Mars Habitation Corporation is a corporation that wants to construct inexpensive, energy-efficient apartments, townhouses, and single-family residences on Mars using regolith in the future. Therefore, we request your assistance in developing a procedure for regulating regolith binders.
Due to the restrictive nature of the study, we decided to use the best binders available on Earth. However, we have decided to keep the identities of the binders hidden and have requested that they refer to them as binder 1 and binder 2. We want you to collaborate and determine the most effective binder combination for housing on Mars.
Please send us a letter with your procedures. In addition, please describe the binding materials you used to support your data and the order in which you arranged your study.
Please provide brief procedures detailing o how your team developed your result.

Kind regards,
Mars Habitation Corporation

Figure 3.4. Mars Lunarcrete Problem 1

As shown in the above table 3.3 and Appendix H, this activity involves: (a) a warm up task including a newspaper article about the Lunarcrete construction in Moon to introduce the context of the model-eliciting activity. (b) Readiness questions to be answered related to the article. Students replied the questions such as what lunarcrete is, how it is constructed and what regolith is. (c) The problem situation related to determining best regolith solution to construct structure on Mars. (d) The data was provided with density, wind resistance, friability and cost. The ratio of regolith and binders were also presented. (e) After writing the letter for their procedure, the new situation was presented as shown in Figure 3.5.

Mars Lunarcrete Problem 2

Dear students,

Mars Habitation Corporation has found binding agent x in its pursuit of an adhesive of higher quality. We have provided all information required to do the analysis. In addition to this, we request that you shorten setting time to reduce the cost.

Please send us a letter with your procedures. Additionally, please describe the order of the binding agents and the research used to support your results. Please describe in detail how your team arrived at your conclusion.

Kind regards,
Mars Habitation Corporation

Figure 3.5. Mars Lunarcrete Problem 2

In addition to their initial request, the corporation asked students to shorten setting time to reduce the cost. After their final decision, they wrote the letters which summarize all the decisions they made and then discuss their models with other groups' members. Considering all of the processes used in this activity, it is evident that it follows to the six essential model-eliciting principles for mathematics and engineering education (Diesfes-Dux et al., 2008). For model-construction principle, students create a model that works to determine the best regolith for settlement in Mars by using multiple representations. For reality principle, students think about their previous experiences, the boundaries of the real-life problem situation which is new trending topic, and the demands of the client when attempting to solve Mars Lunarcrete problem. For self-assessment principle, Mars Lunarcrete activity gives students the opportunity to analyze their progress, clarify their assumptions, and seek for obstacles, problems, and alternative strategies while working in groups. For model documentation principle, students document their model by writing a letter to the corporation. For model shareability and reusability prenciple, students evaluate the model's applicability and generalizability. For effective prototype principle, multiple feedback methods are provided to encourage students to rethink their models and to explore the strengths and weaknesses of their models as well as other models in the whole class discussion environment. In the next part of the section, the information about Dr. Ahmet's Will activity is provided.

3.4.1.3. Dr. Ahmet's Will Activity

For this activity, the researcher searched for a context that can be associated with the context of the video prepared by Turkey for the 2012 Olympic candidate countries. Similarly, the O'Hare Airport Problem (Zawojewski & Lesh, 1999) and On-Time Arrival (Chamberlin & Chamberlin, 2001) were used in the literature to determine the best way. The phrase best was chosen deliberately since it is ambiguous and encourages creativity as in the mentioned studies. Another reason for using this type of activity is to highlight that engineering design is more than just prototyping; it also entails the creation of procedures based on large amounts of data. In this regard, the researcher adapted *Dr. Ahmet's Will* activity context in CPALMS (2017a). In this study, it was preferred to use the units that are used frequently in the world. For instance, the price of a flight is indicated in dollars, the air temperature is expressed in Fahrenheit degrees, the altitude is expressed in feet, and the rainfall is expressed in inches. The purpose of this was to observe how students engage with these unfamiliar units. For this activity, challenging problem situation was formed with many variables. Students would determine the best location for reading Dr. Ahmet's will. Students interpret data sets including temperature, air pollution, travel costs, flight times and health issues in order to rank five global locations for relatives of Dr. Ahmet as shown in Figure 3.6.

Dr. Ahmet's Will Problem 1

Dear Relatives,

I am lawyer Bora, the representative of Dr. Ahmet Öztürk's last will and testament. I am sorry to inform you that Uncle Ahmet passed away last Saturday. As you know, Uncle Ahmet was a strange person. Uncle Ahmet loved to travel the world doing research and collecting unusual plant specimens. As a result of his studies, he became a distinguished pharmaceutical researcher for Amazon Medication. Uncle Ahmet was highly concerned about his belongings. Therefore, it should not come as a surprise to anyone that his last will and testament contains unusual directions to follow.

First of all, the reading of his will is scheduled to take place on April 1, which is also Uncle Ahmet's birthday. Second, in Uncle Ahmet's last will and testament, he specified that his family should get together in a specific location so that his will could be read.

In fact, the condition in his will; if a member of the family does not attend, all of his wealth will be distributed to the pharmaceutical foundation. This means that everyone; Cousin Ozan, Cousin Ezgi, Aunt Belma, Aunt Hatice, Uncle Nedim and Cousin Mert must be present during the reading of the will. The third condition states that the reading of the will should take place preferably in the spring rain. While Uncle Ahmet traveled the world, he spent most of his time in the tropical rainforests and learned to love the rain.

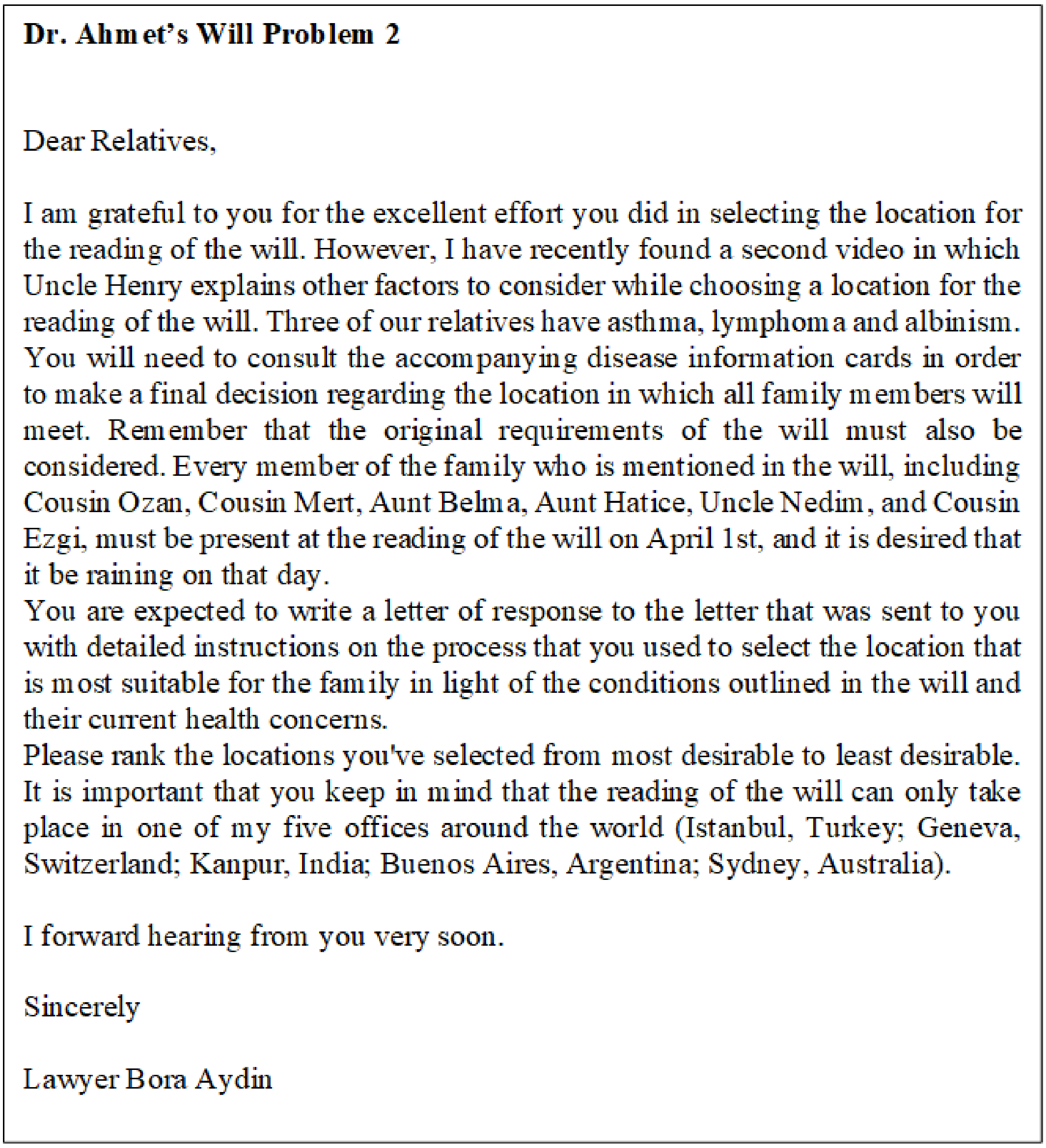
The uncle's last requirement is for the six relatives to determine the best location according to the requirements of the will. You should choose the most suitable location for all family members to travel. As his lawyer, I am ready to meet in any of five global locations: Istanbul, Turkey; Geneva, Switzerland; Kanpur, India; Buenos Aires, Argentina; Sydney, Australia. Please write back to me and list the places that all six relatives find the most ideal to meet on April 1st. Please include a detailed description of how you rank the locations from best to worst. Do not forget to include the reasons why you chose your ranking.

Please note that it also includes a direct video message from Uncle Ahmet.
I look forward to hearing from you.

Sincerely
Lawyer Bora Aydin

Figure 3.6. Dr. Ahmet's Will Problem 1

As shown in the above table 3.3 and Appendix F, this activity involves: (a) a warm up task including a video prepared by Turkey for 2012 Tokyo Olympic candidate countries to introduce the context of the model-eliciting activity. (b) Readiness questions to be answered related to the video. They answered questions such as what factors do you take into account when choosing a place, do you consider all factors, and whether some factors are more important than others. (c) The letter from Dr. Ahmet's to his relatives to determine the best places in which given in the data set. (d) The data sets which include temperature, air pollution, travel costs, flight times for five global locations and character cards for relatives were provided. (e) determine the best location and wrote their procedure to lawyer. Then, the second problem situation as shown in Figure 3.7 was presented.



Dr. Ahmet’s Will Problem 2

Dear Relatives,

I am grateful to you for the excellent effort you did in selecting the location for the reading of the will. However, I have recently found a second video in which Uncle Henry explains other factors to consider while choosing a location for the reading of the will. Three of our relatives have asthma, lymphoma and albinism. You will need to consult the accompanying disease information cards in order to make a final decision regarding the location in which all family members will meet. Remember that the original requirements of the will must also be considered. Every member of the family who is mentioned in the will, including Cousin Ozan, Cousin Mert, Aunt Belma, Aunt Hatice, Uncle Nedim, and Cousin Ezgi, must be present at the reading of the will on April 1st, and it is desired that it be raining on that day.

You are expected to write a letter of response to the letter that was sent to you with detailed instructions on the process that you used to select the location that is most suitable for the family in light of the conditions outlined in the will and their current health concerns.

Please rank the locations you've selected from most desirable to least desirable. It is important that you keep in mind that the reading of the will can only take place in one of my five offices around the world (Istanbul, Turkey; Geneva, Switzerland; Kanpur, India; Buenos Aires, Argentina; Sydney, Australia).

I forward hearing from you very soon.

Sincerely

Lawyer Bora Aydin

Figure 3.7. Dr. Ahmet’s Will Problem 2

For the second situation, three of the relatives had health problems, albinism, asthma and lymphoma. Similar to the procedures for other engineering-based MEAs, elementary gifted students reviewed their decision and wrote second letter to the lawyer for considering both situations. Finally, they discussed their models with the other group. Taken all of the aforementioned into account, this activity also conforms to the six essential principles of model eliciting activity for mathematics and engineering education (Diesfes-Dux et al., 2008) as seen in the Bridge Construction and Mars Lunarcrete activities. As stated above, all the processes of elementary gifted students when they are engaged in engineering-based MEAs as a group was recorded for data analysis. In addition, the worksheets they used and their letters are used as data collection tool. The following section address the information about the interviews.

3.4.2. Interviews

Interview is one of the most common data collection tool in qualitative studies (Bogdan & Biklen, 2007). A qualitative interview enables researchers to pose general, open-ended questions to participants and record their responses (Creswell, 2012). Since elementary gifted students are engaged in engineering-based MEAs as a group and this aims to investigate their modeling competencies as a group, focus group interview was conducted in the present study. A focus group interview is referred to as the process of collecting data by conducting interviews with individuals who are participating in a focus group about a certain topic (Merriam, 2009). This form of interview is used to collect a shared understanding from a number of different participants as well as to get the perspectives of specific individuals (Creswell, 2012). Focus group interview is a helpful tool to use when the interaction between interviewees is likely to give the best information and collaborate with each other (Frankel & Wallen, 2006). Throughout the process, at the end of each session, I conducted group interview with students. She asked them to explain what they made through the section. She directed why and how questions to elicit their thinking in the design process. The participants listen to one another's responses to the questions while seated together in a group setting. After hearing the various responses, they frequently provide additional commentary in addition to what they had first said. They can either agree with one another or disagree with one another; consensus is not required nor desired. The purpose of this method is to reveal students' actual feelings towards a topic by putting them in a social setting in which they may listen to the perspectives of others and assess how those perspectives compare to their own (Frankel &Wallen, 2006). Hence, engaging three to four groups of participants helps the researcher validate the data and determine whether the findings are influenced by specific group dynamic (Mills et al., 2009). In the following, there are some sample questions asked the groups at the end of the first section of Bridge Construction activity. For example; a) Could you tell us what you plan to construct your bridge in your drawing? (b) Which type of bridge will you construct? (c) Which materials will you use? (d) What was the most important thing you considered to construct the bridge? (e) Which shapes will you use and why? (f) How will you make your bridge strong enough? (g) What will be the cost of your bridge? were asked the group of students.

In addition, the sample questions which is asked at the end of the third section are as follows: How did you change your model after the second situation? Why did you revise your model or make a new model? How did you decide this? Do you think this model of yours will be suitable for the new situation?

Finally, the following are examples of some of the questions that were asked during the group discussions: When you compare both of your models, what are their strengths and weaknesses? Which one do you think is better? Do you think this model can be used for other situations? If available, how? What would you change if you made a model again? In the following part of the section, the researcher field notes are described in detail.

3.4.3. Researcher Field Notes

Field notes are defined as researchers' written detailed written explanations of their personal observations related to data collection (Frankel & Wallen, 2006). When conducting a qualitative study in which the researcher also acts as the participant-observer, it is important to collect field notes that are precise, accurate, and comprehensive in order to produce credible findings (Bogdan & Biklen, 2007). Accordingly, the researcher in the current study acted as a participant-observer and properly documented her observations immediately after each section. In particular, the researcher would take descriptive notes whenever there was an interesting discussion, an unexpected situation throughout elementary gifted students' engagement in engineering-based MEAs as a group, and the special characteristics of the participant.

3.4.4. Observation Notes

Observation is a form of data gathering that enables the researcher to access the data first-hand in describing and explaining the behaviors that are occurring in any setting or institution in detail (Creswell, 2012). The role of observation might change depending on how the researcher chooses to gather data in order to gain a better understanding of the key phenomenon with the participants in a given context (Frankel &Wallen, 2006). A nonparticipant observer is an observer who visits a place and records notes without taking part in the activities being carried out by the participants (Creswell, 2012; Merriam, 2009). In the current study, science teacher at BILSEM acts as nonparticipant observer. He has thirteen years of experience as a teacher and is a Phd candidate in science education. He took some notes based on his observation and shared with the researcher based on observation protocol (Appendix E). He is Due to the interdisciplinary nature of this study, such notes help the researcher to reflect the science aspects of the nonparticipant observer.

3.5. Data Collection Procedure

Data collection procure in this study included preparation of activities, the pilot study and actual study implementation. Before beginning the implementation of the pilot study, the relevant legal permissions were obtained from the METU Ethics Committee as well as the Ministry of National Education. The context of the activities was developed by the researcher by examining the relevant literature. Prior to the pilot application, the drafts of the activities were presented to science, mathematics, and technology teachers in BILSEM to provide their feedback. In the process of taking expert opinions, the researcher was in the research environment when there were science and mathematics classrooms, and spent time with the teachers and students in order for students and teachers to accept the researcher in the environment and not to feel alienated. In addition to the teachers in BILSEM, applying expert opinion from a variety of fields such as science, technology and mathematics education and engineer, the final form of the activities was implemented gifted students who attended mathematics and science classrooms in the individual talent recognition programs at BILSEM. Through this approach, the researcher was able to gain insight into the behaviors of gifted students before to the actual study as well as monitor their education process. During this process, materials such as video cameras and sound recorders that were kept in the classrooms without recording in order not to affect the natural behavior of the students. Table 3.4 includes a representation of a time schedule that provides a summary of the data collection and analysis.

Table 3.4. Time schedule for data collection and analysis

Date	Data collection and analysis procedure
February 2017- April 2018	Development of engineering-based MEAs
May 2018- February 2019	Expert opinions about engineering-based MEAs and implementation procedures
April 2019-May 2019	Pilot study
May 2019- June 2019	Analysis of the pilot study
July 2019-August 2019	Actual implementation of the study
August 2019- December 2021	Data analysis of actual study

As presented in Table 3.4., the first step of data collection was the development of engineering-based MEAs. Considering the special characteristics of gifted students and the purpose of this study, expert opinions were taken from specialists and teachers from a variety of fields as mentioned. In line with expert opinions, the activities were finalized and a pilot study was carried out. After the analysis of the pilot study, the final form of the activities was implemented in the summer school at BILSEM. Finally, the data from the actual study was analyzed and reported.

Think aloud protocol was used to capture students' thought processes on videotape when they involve in engineering-based MEAs. That protocol is used in many fields including engineering and technology to elicit student design process and provides an in depth understanding of the processes students use to solve engineering design problems (Bursic & Atman 1997, Christiaans and Dorst 1992; Mentzer,2014). To conduct verbal protocol, three step approach (recording, transcription/segmenting and coding into categories is proposed by the researchers (Ericson & Simon,1993). According to Ericson and Simon (1993), it enables to elicit the sequence of information that is heeded by the student without altering the cognitive processes, while other kinds of verbal reports may change these processes. The pilot study phases of data collection procedure were provided in the following.

3.6. Pilot Study

The pilot study was carried out with groups of 2, 3 and 4 students in the mathematics classroom. Each activity was carried out with a different class group with seven students.The participants were elementary gifted students who attended individual talent recognition programs for both mathematics and science classrooms. The pilot study was conducted in two 40-minute sections during the block class hour. During the pilot implementation process, the mathematics teacher of the class was also in the classroom as an observer.

Since the researcher participated as a participant observer throughout the pilot study and actually applied the activities, she recorded her observations everyday after each activity. Following the end of the implementation, those who had taken part in the pilot implementation were invited to a discussion on the process. The following are some of the topics that were covered in this discussion: the students' feedback on the activities and their suggestions for improvement. There were some changes after the pilot study conducted. Because of the interconnected nature of the activities, it was decided that the mathematics teacher and the science teacher should work together to carry out the activities. Since the researcher was already mathematics teacher, the science teacher took place in the actual study. Due to the lack of materials in the mathematics classroom, the science laboratory, where easy access to both mathematics and science-related materials can be provided, was determined as a research area. All of these are the suggestions of the mathematics teachers. According to the observations of the researcher, the time was insufficient for the students to realize their actual potential. In her notes after the pilot study, she suggested four sections. However, in the actual study, based on the phases of engineering design process, she planned five sections as planning, constructing, testing, reconstructing and reflecting. Another observation is that groups should consist of 3 or 4 students. When compared to the other groups, the two-person groups had a more difficult time going through this process. In the pilot study, they were assigned to the desired groups. However, this situation prevented forming groups heterogeneously. Because of this, in the actual study, the heteregenous groups was formed by researcher and their teachers considering many criteria such as gender, grade level and special characteristics in group work. After observing that some of the groups in the pilot study tended towards sketching, it was determined that students should be asked to draw their designs for prototypes.

Considering student requests in the context of the activities, a variety of materials were used in the actual application. In line with the suggestions of the students, spaghetti, silicone and tape were included in the Bridge Construction activity as materials. When it was observed that the materials they intended to use in the activities were very different, the researcher planned the first lesson as planning and determining the materials. Therefore, she considered being able to provide the materials they required for the other session. Another view of the students was that illnesses of relatives in Dr. Ahmet's Will activity were not given. They claimed that since there was some personal information about the relatives in the activity, they could make inferences about which people might have asthma and lymphoma. They said this would make the activity more challenging. Correspondingly, the researcher reduced the information about the relatives who has such diseases. The other suggestion for Dr. Ahmet's Will activity is that they ask for a brochure to be prepared for their relatives at the end of the activity. Hence, the researcher asked the students to prepare a brochure using the Canva program about the most suitable place for relatives in the actual study.

3.7. Data Analysis

Analyzing qualitative data requires analyzing a large number of transcripts in search of similarities and differences, and then discovering themes and constructing categories (Creswell, 2012). In this process, the aim is to reduce the amount of data into meaningful components so that the research question can be answered (Merriam, 2009). In this regard, multiple sources provided the data to draw conclusions while preserving a clear chain of evidence in the current study (Yin, 2003). Creswell (2012) specifies the six steps that should be followed in the approach for analyzing qualitative data: prepare the data for analysis by arranging and organizing it; read all of the obtained data; code the data; produce the themes and/or the descriptions from the data; interrelate the themes and/or the descriptions; and interpret the meaning of the themes and/or the descriptions. In this study, data analysis was conducted using the mentioned steps.

In particular, the current study aims to investigate the cognitive modeling competencies of elementary gifted students when they are engaged in engineering-based MEAs and to determine to what extent they display cognitive modeling competencies and related sub-modeling competencies as a group. Correspondingly, the obtained data was analyzed based on the adapted version of classification of cognitive competency to investigate and assess proposed by de Villiers (2018). The seven cognitive modeling competencies include *internalizing*, *interpreting*, *structuring*, *symbolizing, adjusting*, *organizing,* and *generalizing*. In addition, the extent to which elementary gifted students in groups displayed such cognitive modeling competencies when they are engaged in engineering-based MEAs, namely, *Bridge Construction*, *Mars Lunarcrete,* and *Dr. Ahmet's Will* were determined based on the adapted version of the group modeling competency observation guide proposed by de Villiers (2018). To sum up, the cognitive modeling competency framework which describe the related mathematical modeling and engineering sub-competencies was used to explain the details of each cognitive modeling competency and its related sub-modeling competency. In addition, the cognitive modeling competency observation guide which identify the detail of each level with the score from zero to three used to determine the level of groups' cognitive modeling competency.

First of all, the video recordings of each group's work and their interviews were transcribed. For uncertain points, audio recordings and video recordings of the classroom section were also used to make the data more understandable and applicable. To gain a general sense of the data (Creswell, 2012), the researcher read all transcribed data sentence-by-sentence and identified words and phrases that described the participants' responses.

In the process of coding, there are two common analysis methods: (a) open coding approach, in which researchers construct codes based on their conceptual knowledge; (b) employing predetermined codes, in which researchers use codes developed based on the phenomena or literature findings (Creswell, 2012). After a review of the relevant literature, it was determined to conduct the initial analysis of gifted students' modeling competencies when they are engaged in engineering-based MEAs using the provided framework by de Villiers (2018). De Villiers (2018) developed the component of this framework pertaining to mathematical modeling competencies and sub-competencies by associating the seven steps of the modeling cycle proposed by Blum and LeiB (2005) with the taxonomy categories developed by Knott (2014). By matching the mathematical modeling competencies defined in the literature with the engineering technician competencies recommended by national and international professional accreditation engineering organizations, the final form of the mentioned framework was developed by de Villiers (2018) to identify competencies that support mathematical reasoning and understanding among first-year engineering students. Hence, the researcher mapped engineering competencies with mathematical modelling competencies in order to generate assessment guidelines that can monitor the evolution of all relevant competencies through mathematical modelling (de Villiers, 2018). Since engineering-based MEAs were used in the current study, initial data analysis was conducted using this framework that matches mathematical modeling competencies with engineering competencies.

In addition, the other aim of the current study was to determine to what extent elementary gifted students' display modeling competencies when they are engaged in engineering-based MEAs. Based on the integration of studies by Arter and McTighe (2001), Jensen (2007), and Knott (2014), de Villiers (2018) developed an observation guide to assign scores corresponding to the extent to which students demonstrate the relevant competencies when performing MEAs. The mentioned guide was intended to assign levels of competence of all the investigated competencies, not only their presence or absence. Through coding, the researcher indicated the level of competence of the groups in the present study during the engineering-based MEAs using this guide.

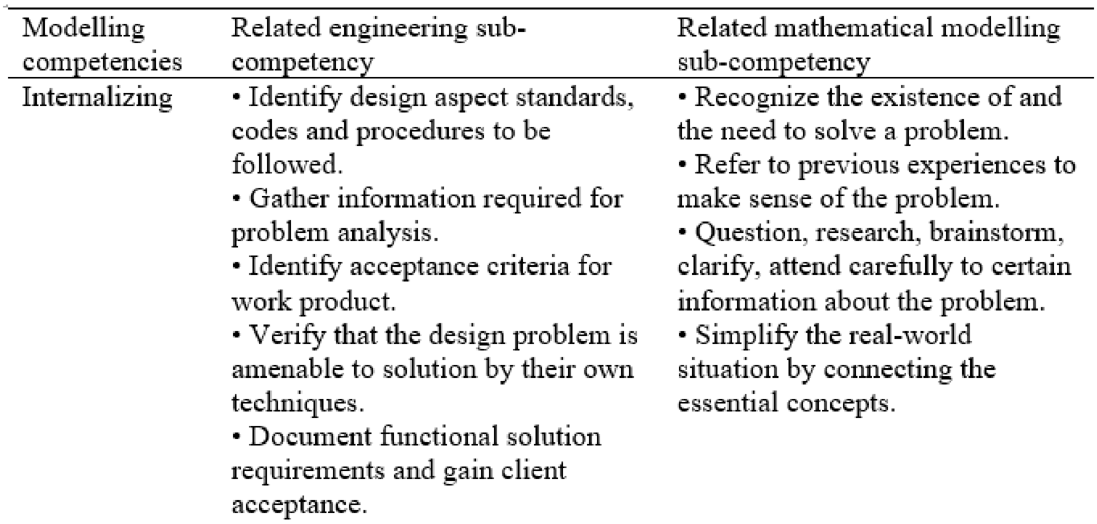
In the data analysis process, each of the transcribed videotape and interview sessions, in addition to the solution papers, were coded in order to assist in defining modeling competencies that were involved in each stage of the design process. Throughout each activity, this procedure was carried out on a one-by-one basis for each group. The following stage of the data analysis consisted of identifying recurrent patterns within the coded data by making comparisons between the patterns that had emerged for each category of the activities. The researcher read through the coded data multiple times, focusing on similarities and differences each time. This allowed her to generate the names of the sub-modeling competencies. After repeatedly using certain codes and phrases relating to students' modeling competencies, the names of the students' sub-modeling competencies began to emerge, and as a result, initial codes were produced. The researcher then went back through the transcribed data to make any necessary changes to the categories (competencies) and subcategories (sub-modeling competencies) before forming the final categories and subcategories. When the initial coding was finished, the researcher got together with the second coder to discuss the findings. The second coder is a mathematics teacher with 9 years of experience and a master's degree with studies on the application of MEAs. Hence, it is asserted that she has an adequate knowledge related to analysis of qualitative data and students' experiences with MEAs. After the framework and group modeling observation guide proposed by de Villiers (2018) presented to her along with comprehensive explanation. After providing her with a thorough description of the framework and group modeling observation guide proposed by de Villiers (2018), the generated codes were discussed. When there was a disagreement between the researcher and the second coder regarding the codes, the meanings of the codes were updated by looking for their instances in the literature once more and making their meaning explicit. The process continued until there was agreement on final codes both the researcher and the second coder. For example, the researcher highlighted the codes related to ethical issues with red color. Then, the researcher and second coder came together to discuss the code since the observation guide proposed by de Villiers does not include such a code. Following that, both coders discussed about how the code of ethical consideration relates to each of the sub-modeling competencies for internalizing. This was done as part of the first modeling competency since both of them agreed that the ethical consideration should be carried out to internalize the problem situation. By examining the horizontal relationship that the code has with other codes, it was decided that rather than introducing a new level of sub-modeling competency, a new sub-modeling competency was added.

Through the discussion, it was determined that some of the data obtained from the findings of this study did not fully comply with the proposed framework. Therefore, there was a need to expand the framework in line with the data of this study. The above-mentioned procedure was followed for all new codes between coders in this study. Accordingly, the new indicators that emerged from the data of the present study related to engineering sub-modeling competency and mathematical modeling sub-competency were classified under each modeling competency. Then, within the scope of this study, there was a need to match the modeling competencies and sub-competencies with the mentioned related engineering and mathematical modeling competencies.

In the following, the saturated codes and categories for each modeling competency and its related sub-modeling competency are presented using a consistent presentation. That is to say, at first table, the competency is formed by merging the indicators related to mathematical modeling and engineering sub-competencies. In the following tables, new codes and indicators that emerged within the scope of the current research were highlighted with "*". Subsequently, the revised framework based on the data of the current study was provided. In this table, the indicators related to mathematical modeling and engineering sub-competencies were categorized under the sub-modeling competency according to findings of this study. This new categorization method is another aspect that distinguishes this study from de Villiers (2018). The last table for each modeling sub-competency indicates the level of display of cognitive modeling sub-competency. All the details and changes are provided in more detail as follows.

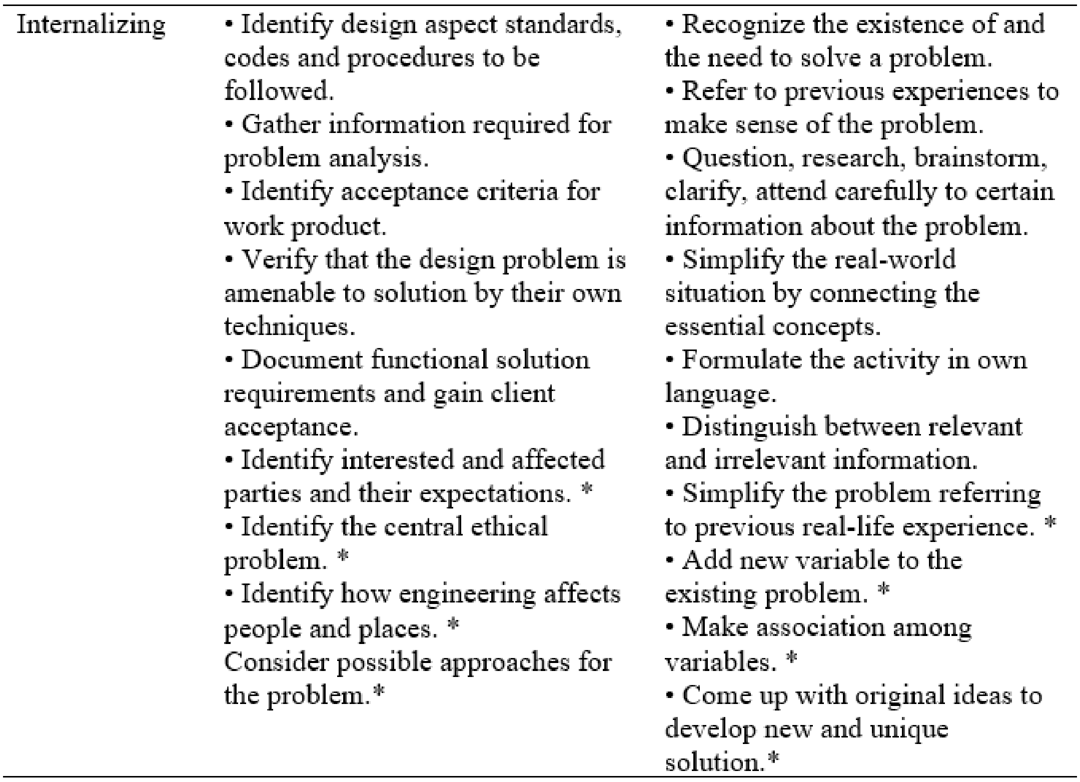
The procedures performed for each modeling competency are described in detail below. To prevent confusion, the competencies derived from the merging of engineering competencies and mathematical modeling competencies within the scope of this study are referred to as modeling competencies. The part of the framework that corresponds to each internalizing modeling competency is provided in the following section.

Table 3.5. Classification of internalizing competency to investigate and assess (Villiers, 2018, pp. 136-138)



Modelling competencies	Related engineering sub-competency	Related mathematical modelling sub-competency
Internalizing	• Identify design aspect standards, codes and procedures to be followed. • Gather information required for problem analysis. • Identify acceptance criteria for work product. • Verify that the design problem is amenable to solution by their own techniques. • Document functional solution requirements and gain client acceptance.	• Recognize the existence of and the need to solve a problem. • Refer to previous experiences to make sense of the problem. • Question, research, brainstorm, clarify, attend carefully to certain information about the problem. • Simplify the real-world situation by connecting the essential concepts.

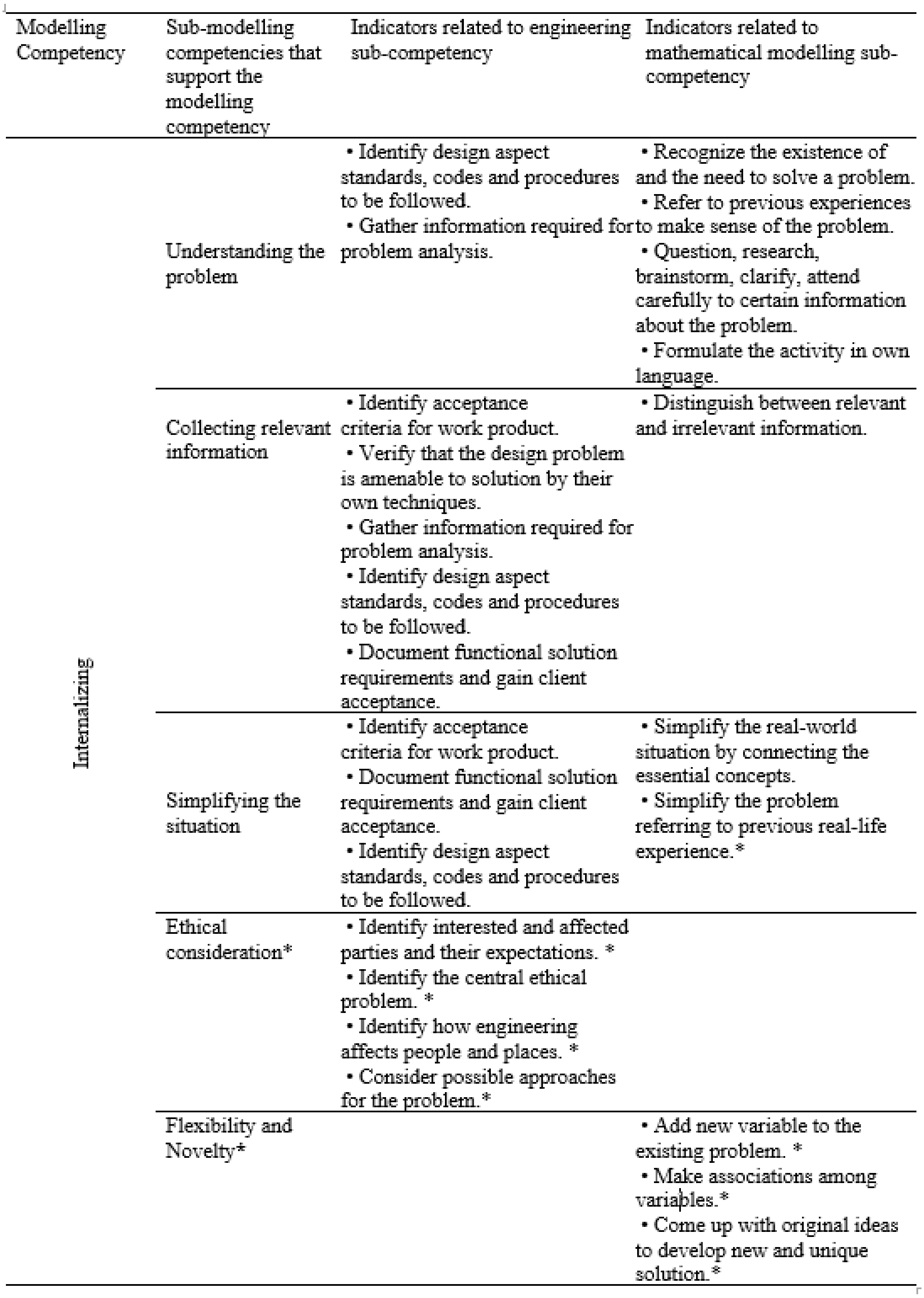
Table 3.5. (continued)



Internalizing	• Identify design aspect standards, codes and procedures to be followed. • Gather information required for problem analysis. • Identify acceptance criteria for work product. • Verify that the design problem is amenable to solution by their own techniques. • Document functional solution requirements and gain client acceptance. • Identify interested and affected parties and their expectations. * • Identify the central ethical problem. * • Identify how engineering affects people and places. * Consider possible approaches for the problem.*	• Recognize the existence of and the need to solve a problem. • Refer to previous experiences to make sense of the problem. • Question, research, brainstorm, clarify, attend carefully to certain information about the problem. • Simplify the real-world situation by connecting the essential concepts. • Formulate the activity in own language. • Distinguish between relevant and irrelevant information. • Simplify the problem referring to previous real-life experience. * • Add new variable to the existing problem. * • Make association among variables. * • Come up with original ideas to develop new and unique solution.*

As shown in the table above, the first modeling competency were defined with its related engineering sub-competency and mathematical modeling sub-competency. In other words, the indicators (identifying interested and affected parties and expectations, identifying the central ethical problem and considering possible approaches for the problem) were added to related engineering sub-competency of internalizing competency since these are the descriptions of new codes emerged from the data. In addition, the indicators (simplifying the problem referring to previous real-life experience, adding new variable to the existing problem, making association among variables and coming up with original ideas to develop new and unique solution) were added to related mathematical modeling sub-competency of internalizing. Based on the data of the current study, the part of the framework related to internalizing competency was extended as shown in Table 3.6.

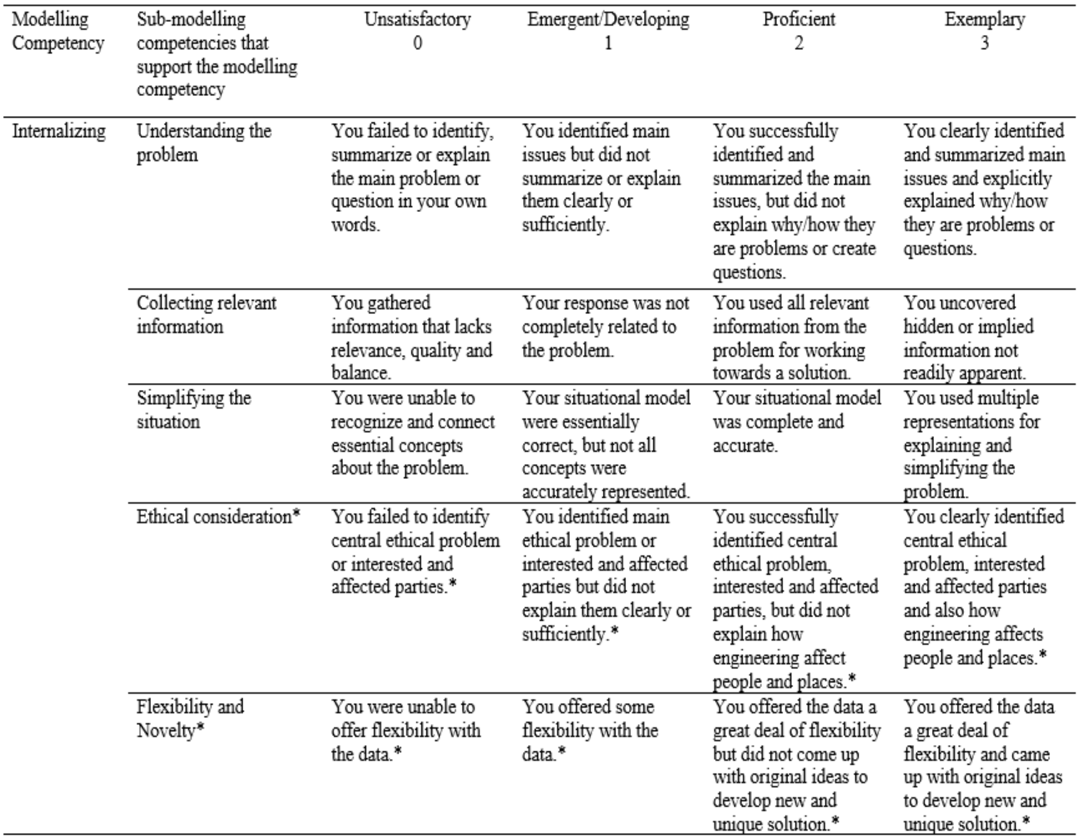
Table 3.6. Classification of internalizing competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Internalizing	Understanding the problem	• Identify design aspect standards, codes and procedures to be followed. • Gather information required for problem analysis.	• Recognize the existence of and the need to solve a problem. • Refer to previous experiences to make sense of the problem. • Question, research, brainstorm, clarify, attend carefully to certain information about the problem. • Formulate the activity in own language.
	Collecting relevant information	• Identify acceptance criteria for work product. • Verify that the design problem is amenable to solution by their own techniques. • Gather information required for problem analysis. • Identify design aspect standards, codes and procedures to be followed. • Document functional solution requirements and gain client acceptance.	• Distinguish between relevant and irrelevant information.
	Simplifying the situation	• Identify acceptance criteria for work product. • Document functional solution requirements and gain client acceptance. • Identify design aspect standards, codes and procedures to be followed.	• Simplify the real-world situation by connecting the essential concepts. • Simplify the problem referring to previous real-life experience.*
	Ethical consideration*	• Identify interested and affected parties and their expectations. * • Identify the central ethical problem. * • Identify how engineering affects people and places. * • Consider possible approaches for the problem.*	
	Flexibility and Novelty*		• Add new variable to the existing problem. * • Make associations among variables.* • Come up with original ideas to develop new and unique solution.*

As presented in Table 3.6., new sub-modeling competencies, ethical consideration, and flexibility and novelty were emerged based on the data of this study. When labeling these sub-modeling competencies, the descriptions in the relevant literature were considered. For example, adding new variable to the existing problem and making associations among variables are the descriptors of flexibility in the literature related to creativity (Siswono, 2010; Taşkın, 2016). Since flexibility and novelty concepts were interrelated in this study, they were categorized under the same modeling sub-competency. The same procedures were followed throughout all newly emerging categories. As another example, the indicators of related engineering sub-modeling competency were considered the descriptors of ethical considerations in the literature related on engineering design (Moore et al., 2014; Wheeler, 2019). After internalizing modeling competency and its sub-modeling competencies were established as in the table above, the observation guide to assess the extent of each sub-modeling competency was reviewed in line with the findings of this study.

Table 3.7. Group modeling competency observation guide for internalizing



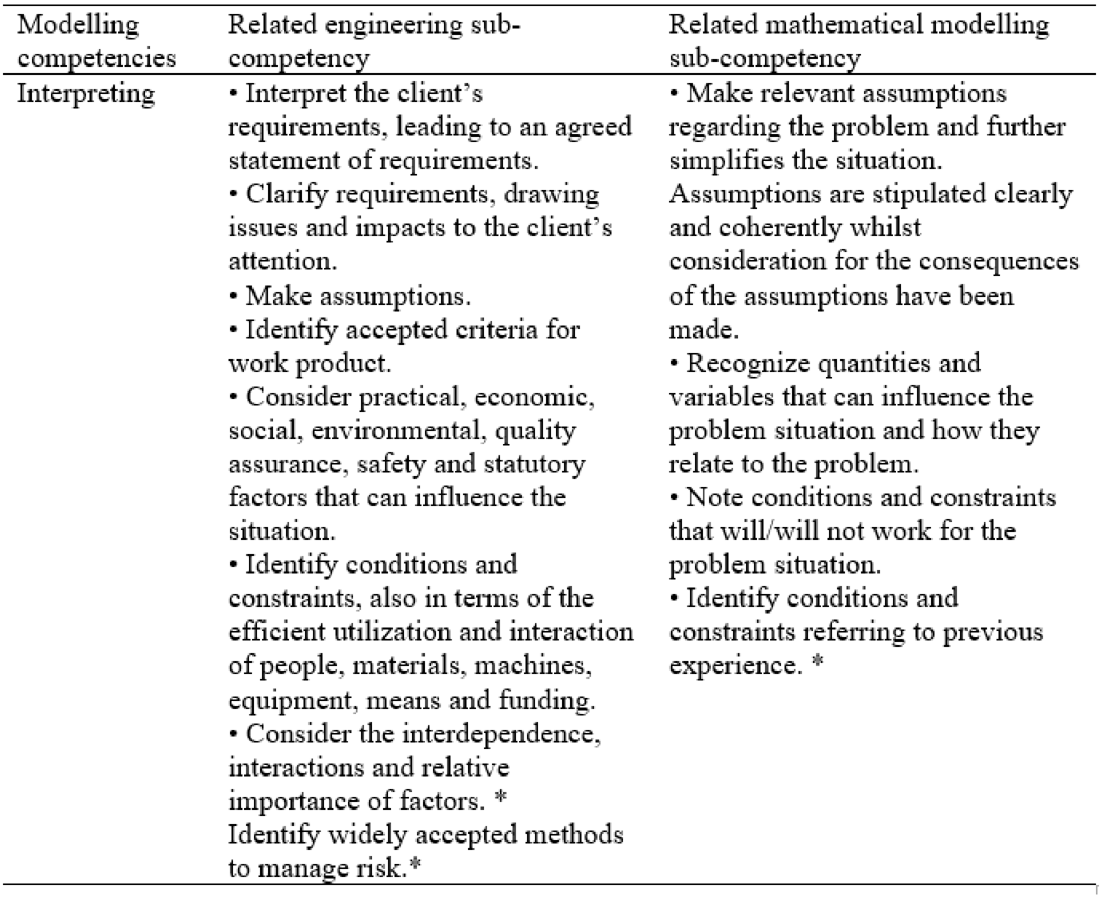
Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent/Developing 1	Proficient 2	Exemplary 3
Internalizing	Understanding the problem	You failed to identify, summarize or explain the main problem or question in your own words.	You identified main issues but did not summarize or explain them clearly or sufficiently.	You successfully identified and summarized the main issues, but did not explain why/how they are problems or create questions.	You clearly identified and summarized main issues and explicitly explained why/how they are problems or questions.
	Collecting relevant information	You gathered information that lacks relevance, quality and balance.	Your response was not completely related to the problem.	You used all relevant information from the problem for working towards a solution.	You uncovered hidden or implied information not readily apparent.
	Simplifying the situation	You were unable to recognize and connect essential concepts about the problem.	Your situational model were essentially correct, but not all concepts were accurately represented.	Your situational model was complete and accurate.	You used multiple representations for explaining and simplifying the problem.
	Ethical consideration*	You failed to identify central ethical problem or interested and affected parties.*	You identified main ethical problem or interested and affected parties but did not explain them clearly or sufficiently.*	You successfully identified central ethical problem, interested and affected parties, but did not explain how engineering affect people and places.*	You clearly identified central ethical problem, interested and affected parties and also how engineering affects people and places.*
	Flexibility and Novelty*	You were unable to offer flexibility with the data.*	You offered some flexibility with the data.*	You offered the data a great deal of flexibility but did not come up with original ideas to develop new and unique solution.*	You offered the data a great deal of flexibility and came up with original ideas to develop new and unique solution.*

The above table represents the updated version of group modeling observation guide related for internalizing competency and its sub-competencies. In this table, “*” denotes the emergence of novel codes and their descriptions within the scope of the present study.

As shown, the group modeling observation guide consists of two parts: scores and their descriptive expressions. Each of these criteria focuses on a different aspect of performance. Descriptive criteria were employed to categorize the modeling competencies of elementary gifted students that emerged during the process. It was determined that evaluating each modeling activity based on the four criteria proposed by de Villiers (2018) was sufficient. In this regard, the level of sub-modelling competencies for the groups in this study was classified as 0 *(unsatisfactory),* 1 *(emergent/developing),* 2 *(proficient)* and 3 *(exemplary).* To be able to assess performance with group modeling observation guide, the best performance indicators are defined for each emerging sub-modeling competence. To put it more precisely, different performances were evaluated and compared, and the best as well as other performance indicators were described for each sub-modeling competency. In other words, depending on the best group performance for each sub-modeling competency, other performance criteria were determined.

Moreover, the second modeling competency, interpreting was revised with its related engineering sub-competency and mathematical modeling sub-competency as shown in Table 3.8.

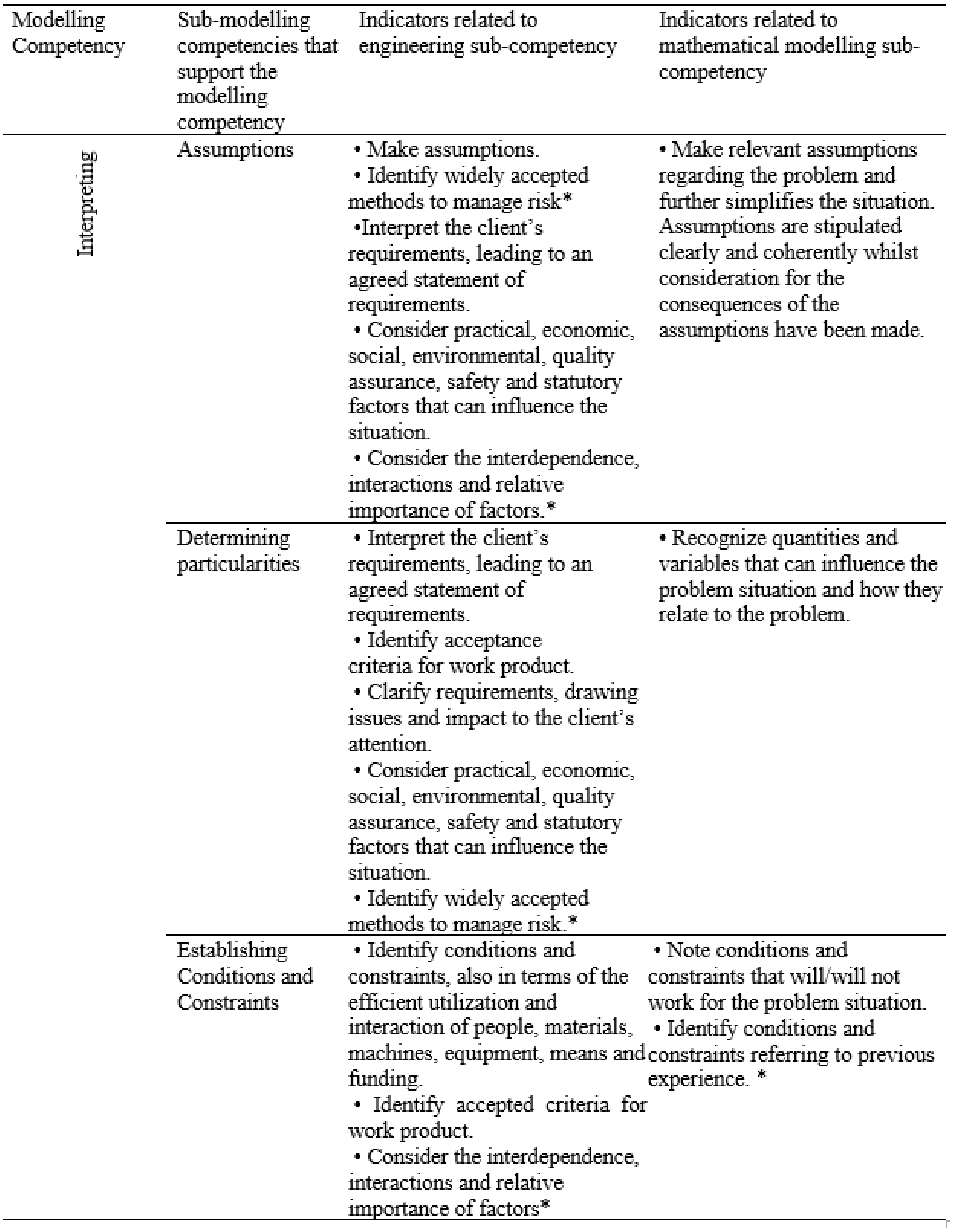
Table 3.8. Classification of interpreting competency to investigate and assess (De Villiers, 2018, pp. 136-138)



Modelling competencies	Related engineering sub-competency	Related mathematical modelling sub-competency
Interpreting	• Interpret the client's requirements, leading to an agreed statement of requirements. • Clarify requirements, drawing issues and impacts to the client's attention. • Make assumptions. • Identify accepted criteria for work product. • Consider practical, economic, social, environmental, quality assurance, safety and statutory factors that can influence the situation. • Identify conditions and constraints, also in terms of the efficient utilization and interaction of people, materials, machines, equipment, means and funding. • Consider the interdependence, interactions and relative importance of factors. * Identify widely accepted methods to manage risk.*	• Make relevant assumptions regarding the problem and further simplifies the situation. Assumptions are stipulated clearly and coherently whilst consideration for the consequences of the assumptions have been made. • Recognize quantities and variables that can influence the problem situation and how they relate to the problem. • Note conditions and constraints that will/will not work for the problem situation. • Identify conditions and constraints referring to previous experience. *

As shown in the Table 3.8., the indicators shown with '*', (considering the interdependence, interactions and relative importance of factors and identifying widely accepted methods to manage risk) were added to related engineering sub-competency of interpreting competency. Moreover, the indicator (identifying conditions and constraints referring to previous experience) was added to related mathematical modeling competency of interpreting. In light of the findings of the present study, the section of modeling competency pertaining to the interpreting was extended, as can be seen in Table 3.9.

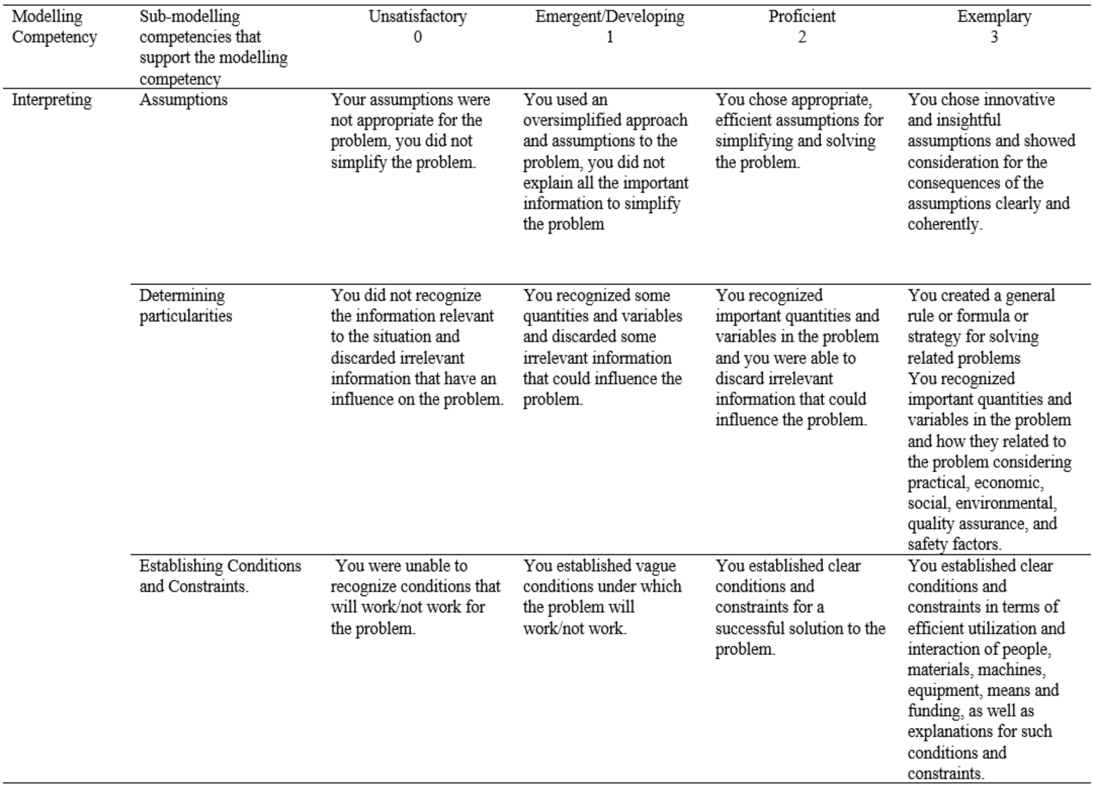
Table 3.9. Classification of interpreting competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Interpreting	Assumptions	• Make assumptions. • Identify widely accepted methods to manage risk* •Interpret the client's requirements, leading to an agreed statement of requirements. • Consider practical, economic, social, environmental, quality assurance, safety and statutory factors that can influence the situation. • Consider the interdependence, interactions and relative importance of factors.*	• Make relevant assumptions regarding the problem and further simplifies the situation. Assumptions are stipulated clearly and coherently whilst consideration for the consequences of the assumptions have been made.
	Determining particularities	• Interpret the client's requirements, leading to an agreed statement of requirements. • Identify acceptance criteria for work product. • Clarify requirements, drawing issues and impact to the client's attention. • Consider practical, economic, social, environmental, quality assurance, safety and statutory factors that can influence the situation. • Identify widely accepted methods to manage risk.*	• Recognize quantities and variables that can influence the problem situation and how they relate to the problem.
	Establishing Conditions and Constraints	• Identify conditions and constraints, also in terms of the efficient utilization and interaction of people, materials, machines, equipment, means and funding. • Identify accepted criteria for work product. • Consider the interdependence, interactions and relative importance of factors*	• Note conditions and constraints that will/will not work for the problem situation. • Identify conditions and constraints referring to previous experience. *

As shown in Table 3.9, some indicators are present in more than one sub-modeling competencies. That is to say, elementary gifted students in this study showed these indicators while demonstrating more than one sub-modeling competency. For example, they consider the interdependence, interactions and relative importance of factors when both making assumptions and establishing conditions and constraints. Correspondingly, group modeling observation guide as shown in Table 3.10 was extended based the analysis of the data from the current study.

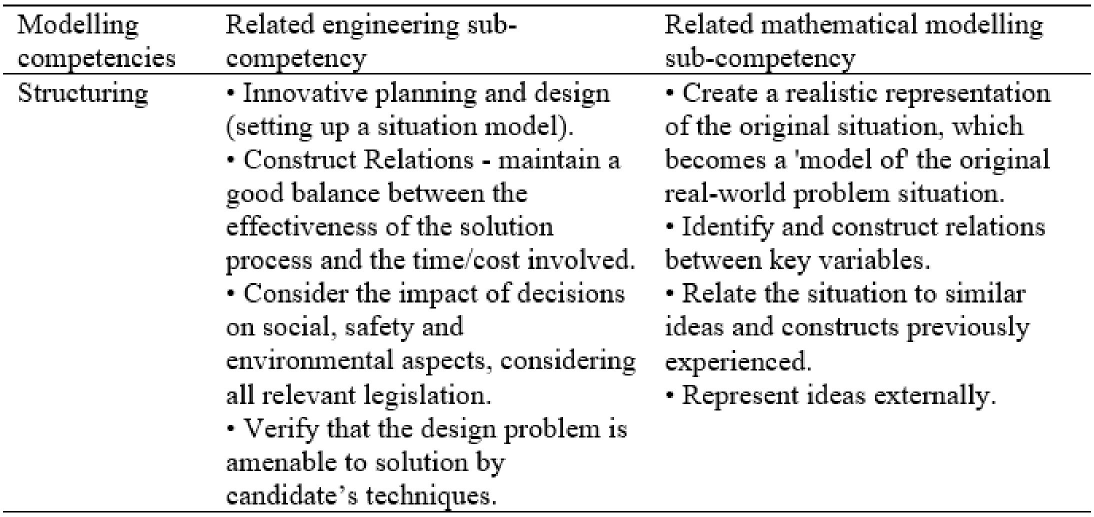
Table 3.10. Group modeling competency observation guide for interpreting



Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent/Developing 1	Proficient 2	Exemplary 3
Interpreting	Assumptions	Your assumptions were not appropriate for the problem, you did not simplify the problem.	You used an oversimplified approach and assumptions to the problem, you did not explain all the important information to simplify the problem	You chose appropriate, efficient assumptions for simplifying and solving the problem.	You chose innovative and insightful assumptions and showed consideration for the consequences of the assumptions clearly and coherently.
	Determining particularities	You did not recognize the information relevant to the situation and discarded irrelevant information that have an influence on the problem.	You recognized some quantities and variables and discarded some irrelevant information that could influence the problem.	You recognized important quantities and variables in the problem and you were able to discard irrelevant information that could influence the problem.	You created a general rule or formula or strategy for solving related problems You recognized important quantities and variables in the problem and how they related to the problem considering practical, economic, social, environmental, quality assurance, and safety factors.
	Establishing Conditions and Constraints.	You were unable to recognize conditions that will work/not work for the problem.	You established vague conditions under which the problem will work/not work.	You established clear conditions and constraints for a successful solution to the problem.	You established clear conditions and constraints in terms of efficient utilization and interaction of people, materials, machines, equipment, means and funding, as well as explanations for such conditions and constraints.

In the above observation guide, the explanation of the criteria of exemplary level related to determining particulates and establishing conditions and constraints were revised and extended. First of all, the extra explanation, recognizing factors that can influence the situation for determining the particularities sub-modeling competency was removed since it was preferred to express the sub-modeling competencies in a clear form. In this regard, the description (you recognized important quantities and variables in the problem and how they related to the problem considering practical, economic, social, environmental, quality assurance, and safety factors) was added the criteria of exemplary level. In addition to rules and formulas, the appropriate explanation has been updated to include the concept of strategy. Regarding establishing conditions and constraints sub-modeling competency, the phase (in terms of efficient utilization and interaction of people, materials, machines, equipment, means and funding) was included. When third modeling competency is considered, the following table indicates the related engineering sub-competency and mathematical modeling sub-competency proposed by de Villiers (2018).

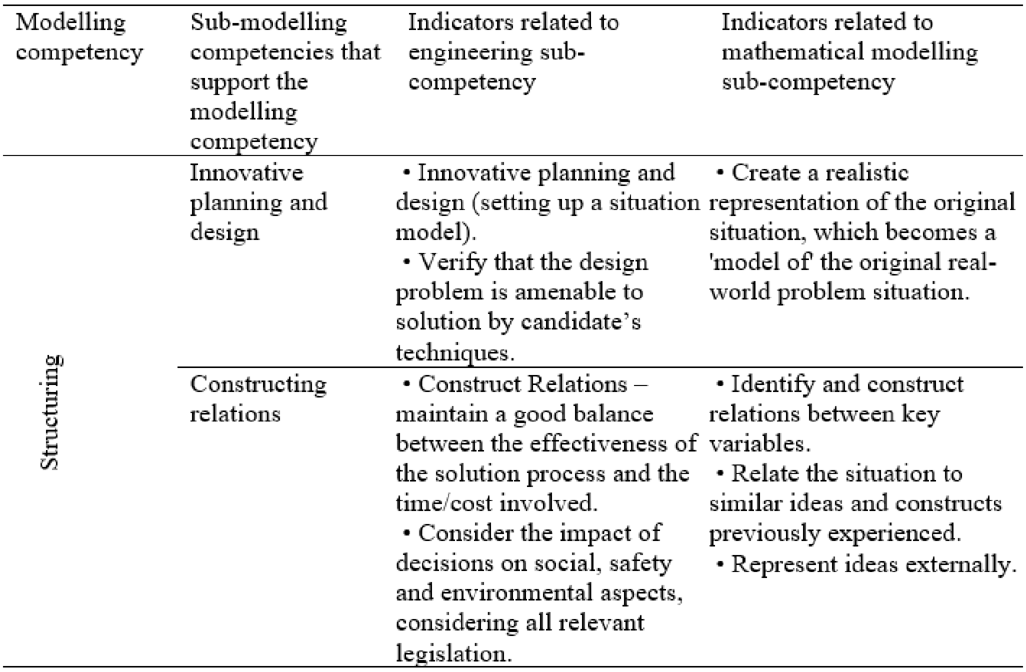
Table 3.11. Classification of structuring competency to investigate and assess (De Villiers, 2018, pp. 136-138)



Modelling competencies	Related engineering sub-competency	Related mathematical modelling sub-competency
Structuring	• Innovative planning and design (setting up a situation model). • Construct Relations - maintain a good balance between the effectiveness of the solution process and the time/cost involved. • Consider the impact of decisions on social, safety and environmental aspects, considering all relevant legislation. • Verify that the design problem is amenable to solution by candidate's techniques.	• Create a realistic representation of the original situation, which becomes a 'model of' the original real-world problem situation. • Identify and construct relations between key variables. • Relate the situation to similar ideas and constructs previously experienced. • Represent ideas externally.

As indicated in Table 3.11, no changes were essential to the indicators of structuring modeling sub-competency according to the data obtained from this study. As with other modeling competencies, sub-modeling competencies of structuring are matched with appropriate modeling competency indicators, as indicated in Table 3.12.

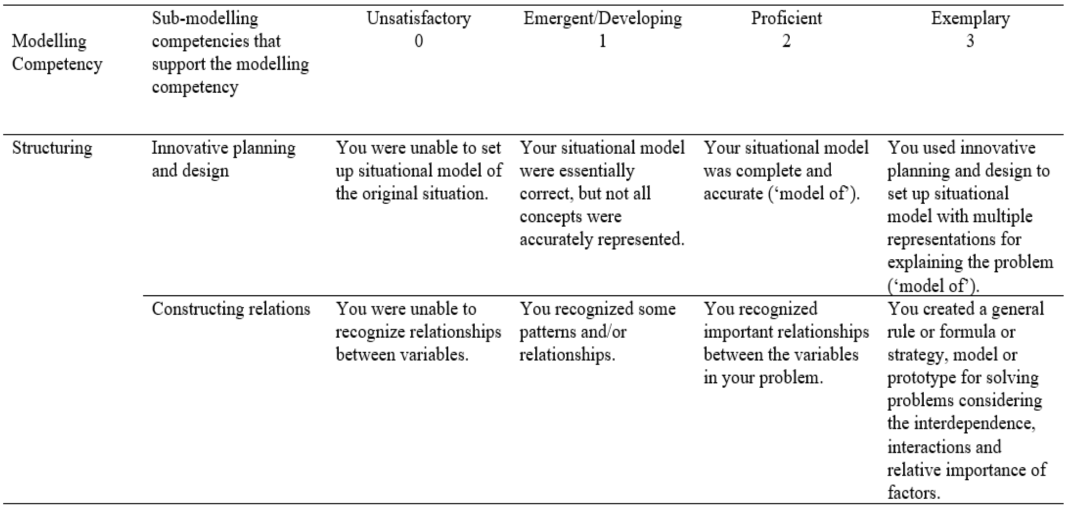
Table 3.12. Classification of structuring competency framework



Modelling competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Structuring	Innovative planning and design	• Innovative planning and design (setting up a situation model). • Verify that the design problem is amenable to solution by candidate's techniques.	• Create a realistic representation of the original situation, which becomes a 'model of' the original real-world problem situation.
	Constructing relations	• Construct Relations – maintain a good balance between the effectiveness of the solution process and the time/cost involved. • Consider the impact of decisions on social, safety and environmental aspects, considering all relevant legislation.	• Identify and construct relations between key variables. • Relate the situation to similar ideas and constructs previously experienced. • Represent ideas externally.

Above table indicated that the indicators related to engineering sub-competency and mathematical modeling sub-competency proposed by de Villiers (2018) overlapped with the data from this study. However, the criteria to assess sub-modeling competencies were modified as shown in Table 3.13.

Table 3.13. Group modeling competency observation guide for structuring



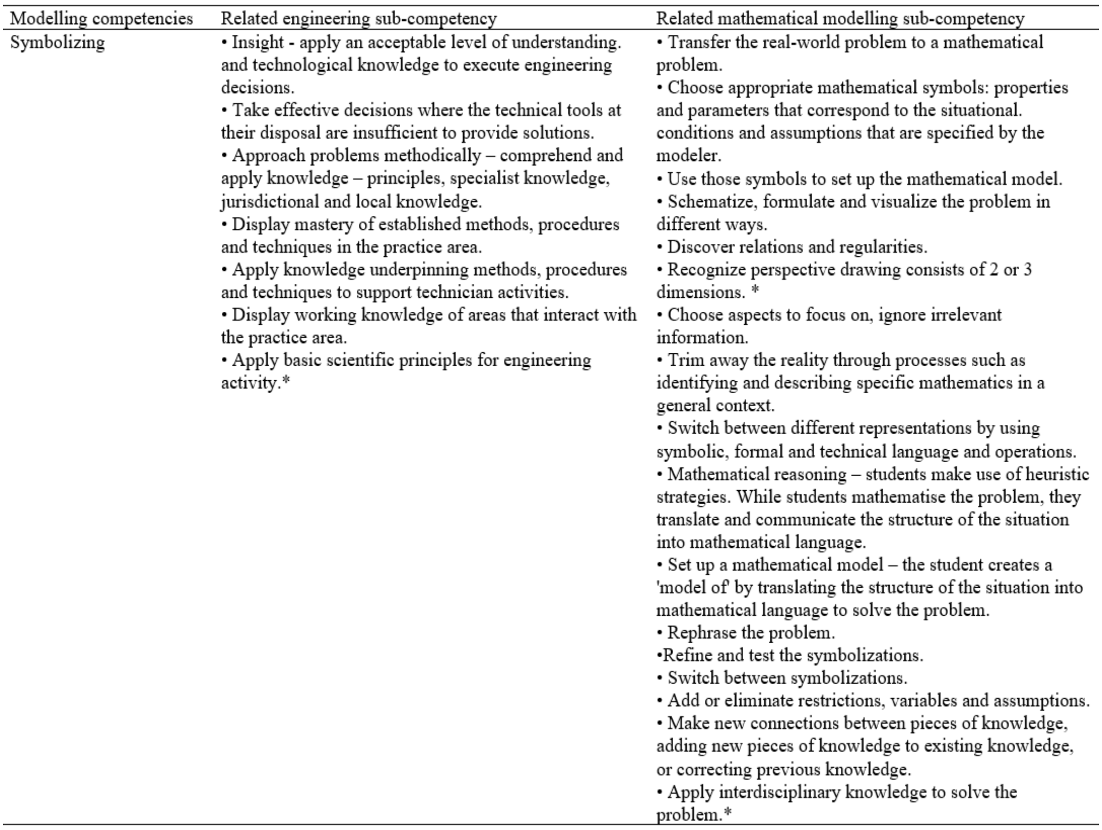
Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent/Developing 1	Proficient 2	Exemplary 3
Structuring	Innovative planning and design	You were unable to set up situational model of the original situation.	Your situational model were essentially correct, but not all concepts were accurately represented.	Your situational model was complete and accurate ('model of').	You used innovative planning and design to set up situational model with multiple representations for explaining the problem ('model of').
	Constructing relations	You were unable to recognize relationships between variables.	You recognized some patterns and/or relationships.	You recognized important relationships between the variables in your problem.	You created a general rule or formula or strategy, model or prototype for solving problems considering the interdependence, interactions and relative importance of factors.

As can be seen from the table above, the extra explanations for sub-modeling competencies, setting up a situational model for innovative planning and design and considering the interdependence, interactions, and relative importance of various factors for constructing relations were excluded. For the innovative planning and design sub-modeling competency, the phrase (used innovative planning and design to set up situational model with) was included in criteria of the exemplary level. In order to preserve the parallelism among the criteria, the explanation of unsatisfactory level (you were unable to recognize and connect essential concepts about the problem) was removed and replaced with another expression (you were unable to set up situational model of the original situation).

In addition, the exemplary level criteria of constructing relations were revised by adding the concepts (strategy, model or prototype) and phrase (considering the interdependence, interactions and relative importance of factors) after the pilot study. Since the nature of engineering-based MEAs for the current study requires to develop strategy, model or prototype, the revision was made.

Furthermore, the third model competency, symbolizing and its related engineering sub-modeling competency and related sub-modeling competency were presented in Table 3.14.

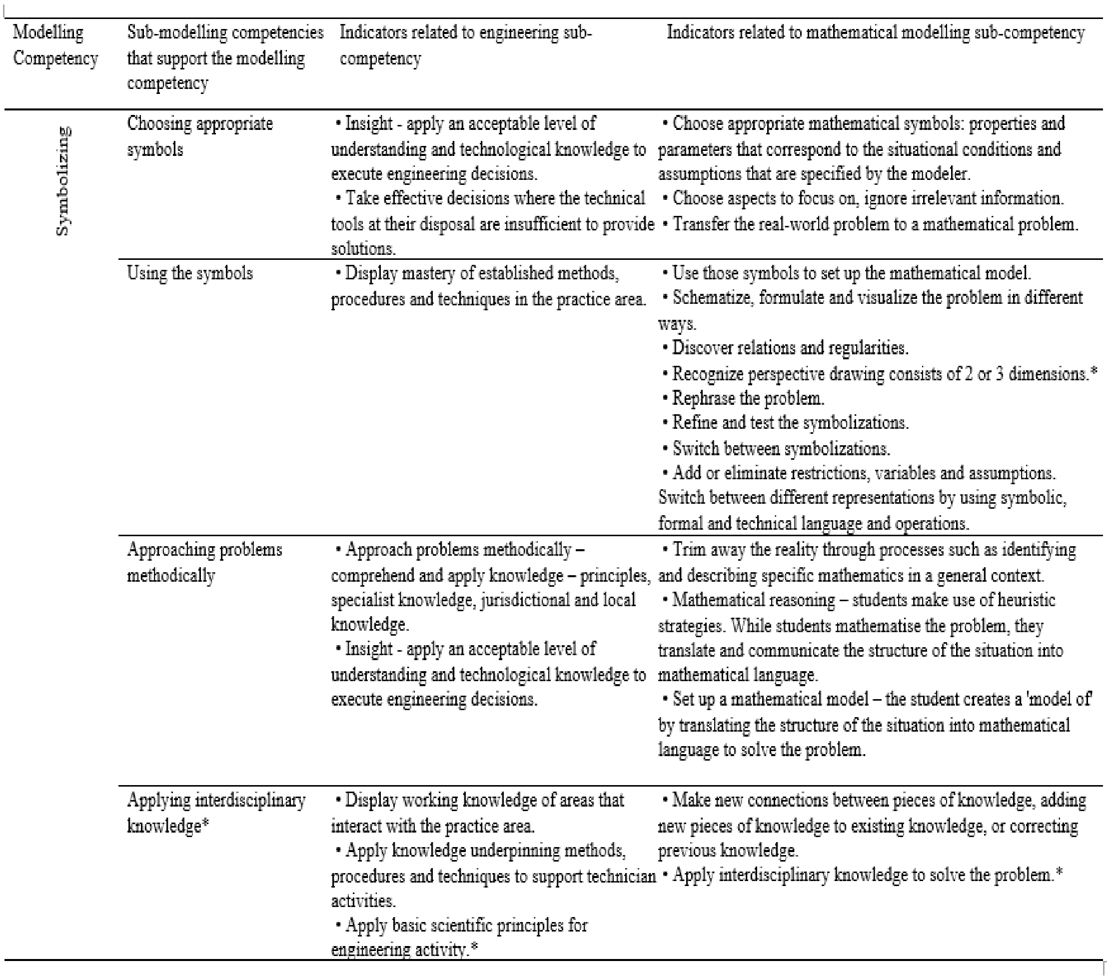
Table 3.14. Classification of symbolizing competency to investigate and assess (Villiers, 2018, pp. 136-138)



Modelling competencies	Related engineering sub-competency	Related mathematical modelling sub-competency
Symbolizing	• Insight - apply an acceptable level of understanding. and technological knowledge to execute engineering decisions. • Take effective decisions where the technical tools at their disposal are insufficient to provide solutions. • Approach problems methodically – comprehend and apply knowledge – principles, specialist knowledge, jurisdictional and local knowledge. • Display mastery of established methods, procedures and techniques in the practice area. • Apply knowledge underpinning methods, procedures and techniques to support technician activities. • Display working knowledge of areas that interact with the practice area. • Apply basic scientific principles for engineering activity.*	• Transfer the real-world problem to a mathematical problem. • Choose appropriate mathematical symbols: properties and parameters that correspond to the situational. conditions and assumptions that are specified by the modeler. • Use those symbols to set up the mathematical model. • Schematize, formulate and visualize the problem in different ways. • Discover relations and regularities. • Recognize perspective drawing consists of 2 or 3 dimensions. * • Choose aspects to focus on, ignore irrelevant information. • Trim away the reality through processes such as identifying and describing specific mathematics in a general context. • Switch between different representations by using symbolic, formal and technical language and operations. • Mathematical reasoning – students make use of heuristic strategies. While students mathematise the problem, they translate and communicate the structure of the situation into mathematical language. • Set up a mathematical model – the student creates a 'model of' by translating the structure of the situation into mathematical language to solve the problem. • Rephrase the problem. •Refine and test the symbolizations. • Switch between symbolizations. • Add or eliminate restrictions, variables and assumptions. • Make new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge. • Apply interdisciplinary knowledge to solve the problem.*

As shown in Table 3.14., the framework components related to symbolizing competency was revised based on the data of this study. The description (applying basic scientific principles for engineering activity) was inserted to related engineering sub-competency of symbolizing. On the other hand, the indicator related to mathematical modeling competency (recognizing isomorphic aspects in different problems) was not observed in the current study and removed. Based on the data of this study, two new explanations (recognizing perspective drawing 2 or 3 dimensions and applying interdisciplinary knowledge to solve the problem) were provided to this component. Additionally, the above-mentioned indicators were matched with the sub-modeling competencies of symbolizing shown below.

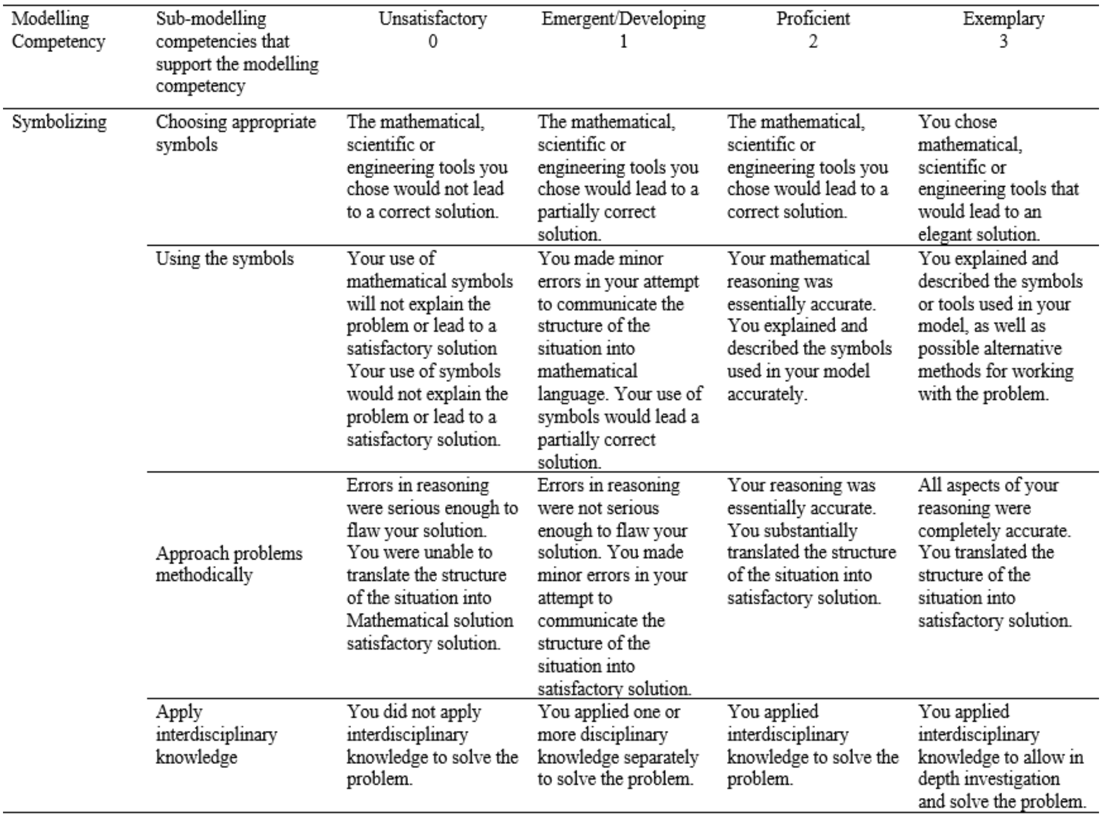
Table 3.15. Classification of symbolizing competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Symbolizing	Choosing appropriate symbols	• Insight - apply an acceptable level of understanding and technological knowledge to execute engineering decisions. • Take effective decisions where the technical tools at their disposal are insufficient to provide solutions.	• Choose appropriate mathematical symbols: properties and parameters that correspond to the situational conditions and assumptions that are specified by the modeler. • Choose aspects to focus on, ignore irrelevant information. • Transfer the real-world problem to a mathematical problem.
	Using the symbols	• Display mastery of established methods, procedures and techniques in the practice area.	• Use those symbols to set up the mathematical model. • Schematize, formulate and visualize the problem in different ways. • Discover relations and regularities. • Recognize perspective drawing consists of 2 or 3 dimensions.* • Rephrase the problem. • Refine and test the symbolizations. • Switch between symbolizations. • Add or eliminate restrictions, variables and assumptions. Switch between different representations by using symbolic, formal and technical language and operations.
	Approaching problems methodically	• Approach problems methodically – comprehend and apply knowledge – principles, specialist knowledge, jurisdictional and local knowledge. • Insight - apply an acceptable level of understanding and technological knowledge to execute engineering decisions.	• Trim away the reality through processes such as identifying and describing specific mathematics in a general context. • Mathematical reasoning – students make use of heuristic strategies. While students mathematise the problem, they translate and communicate the structure of the situation into mathematical language. • Set up a mathematical model – the student creates a 'model of' by translating the structure of the situation into mathematical language to solve the problem.
	Applying interdisciplinary knowledge*	• Display working knowledge of areas that interact with the practice area. • Apply knowledge underpinning methods, procedures and techniques to support technician activities. • Apply basic scientific principles for engineering activity.*	• Make new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge. • Apply interdisciplinary knowledge to solve the problem.*

As shown in the above table, along with the new indicators to both mathematical modeling and engineering sub-competency, a new sub-modeling competency emerged as a sub-category. Consequently, a revision was required in the group modeling observation guide for determining the degree of modeling sub-competencies as in the following.

Table 3.16. Group modeling competency observation guide for symbolizing

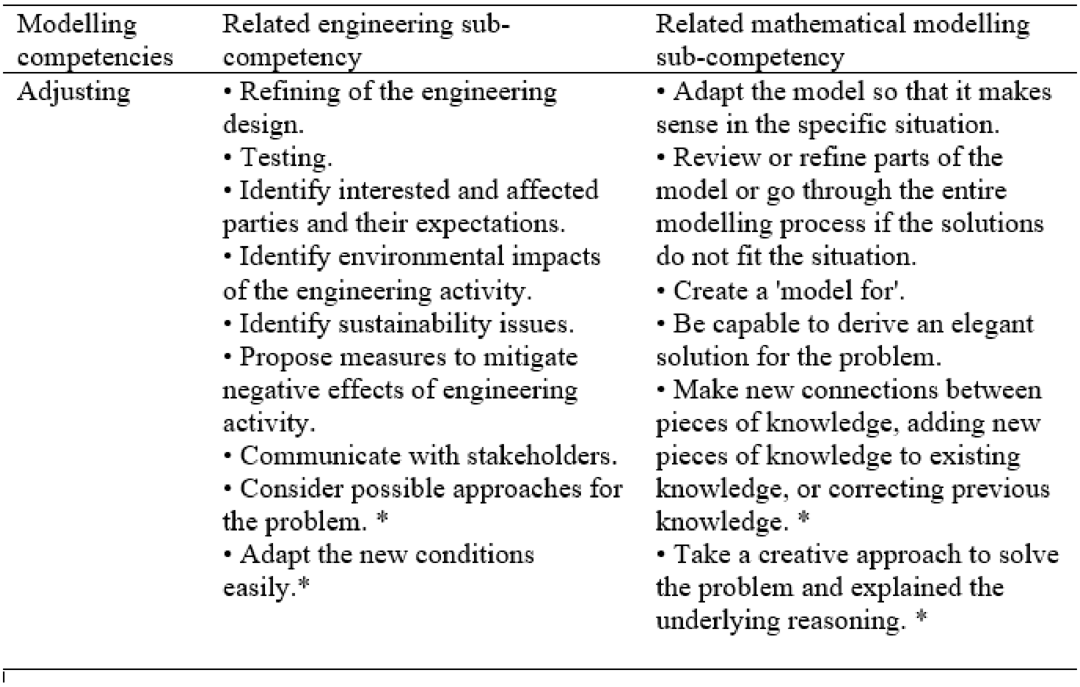


Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent/Developing 1	Proficient 2	Exemplary 3
Symbolizing	Choosing appropriate symbols	The mathematical, scientific or engineering tools you chose would not lead to a correct solution.	The mathematical, scientific or engineering tools you chose would lead to a partially correct solution.	The mathematical, scientific or engineering tools you chose would lead to a correct solution.	You chose mathematical, scientific or engineering tools that would lead to an elegant solution.
	Using the symbols	Your use of mathematical symbols will not explain the problem or lead to a satisfactory solution Your use of symbols would not explain the problem or lead to a satisfactory solution.	You made minor errors in your attempt to communicate the structure of the situation into mathematical language. Your use of symbols would lead a partially correct solution.	Your mathematical reasoning was essentially accurate. You explained and described the symbols used in your model accurately.	You explained and described the symbols or tools used in your model, as well as possible alternative methods for working with the problem.
	Approach problems methodically	Errors in reasoning were serious enough to flaw your solution. You were unable to translate the structure of the situation into Mathematical solution satisfactory solution.	Errors in reasoning were not serious enough to flaw your solution. You made minor errors in your attempt to communicate the structure of the situation into satisfactory solution.	Your reasoning was essentially accurate. You substantially translated the structure of the situation into satisfactory solution.	All aspects of your reasoning were completely accurate. You translated the structure of the situation into satisfactory solution.
	Apply interdisciplinary knowledge	You did not apply interdisciplinary knowledge to solve the problem.	You applied one or more disciplinary knowledge separately to solve the problem.	You applied interdisciplinary knowledge to solve the problem.	You applied interdisciplinary knowledge to allow in depth investigation and solve the problem.

In this table, the criteria of each sub-modeling competency were revised in some extent. For choosing appropriate symbols sub-modeling competency, scientific or engineering phrase were added after pilot study in addition to mathematics to all dimensions. In addition, the phase (or tools) were included in the exemplary level description of using the symbols sub-modeling competency. In order to provide consistency among the criteria, the explanations of unsatisfactory level (your use of mathematical symbols will not explain the problem or lead to a satisfactory solution), emergent /developing (you made minor errors in your attempt to communicate the structure of the situation into mathematical language) and proficient (your mathematical reasoning were essentially accurate) were removed. Instead of them, the explanations of unsatisfactory (your use of symbols would not explain the problem or lead to a satisfactory solution), emergent /developing (you explained and described the symbols used in your model accurately) and proficient (your use of symbols would lead to a partially correct solution) were added based on the criteria of exemplary level. In the proposed framework by de Villiers (2018), there were two dimensions related to approaching problem methodically. As one of the dimension serves only the mathematical model, the four criteria related to it was completely removed. Just mathematical models were not developed based on all of the data collected for this study. Accordingly, the phase (mathematical solution) change with other phase (satisfactory solution) for above shown criteria of approaching problem methodically in table 3.14. In order to maintain consistency among criteria, other changes included the addition of descriptions of emergent/developing (errors in reasoning were not serious enough to flaw your solution) and proficient (you substantially translated the structure of the situation into satisfactory solution). While determining the performance criteria of the applying interdisciplinary knowledge sub-competency that emerged from the data analysis of this study, the highest level was determined first, as in other newly emerged sub-modeling competencies. Then, the description of unsatisfactory level of this sub-modeling competency were determined. Lastly, the descriptions of intermediate-level related to applying interdisciplinary knowledge were determined.

Similar procedures were applied to the fourth modeling competency, adjusting. Nevertheless, there were some indicators of this modeling competency proposed by de Villiers (2018) that the students in this study did not display. Hence, the related engineering sub-modeling competency (engineers must keep themselves informed of new technological developments in their various fields) and mathematical modeling competency (rephrasing the problem and question his/her own model) were removed from the table 3.17.

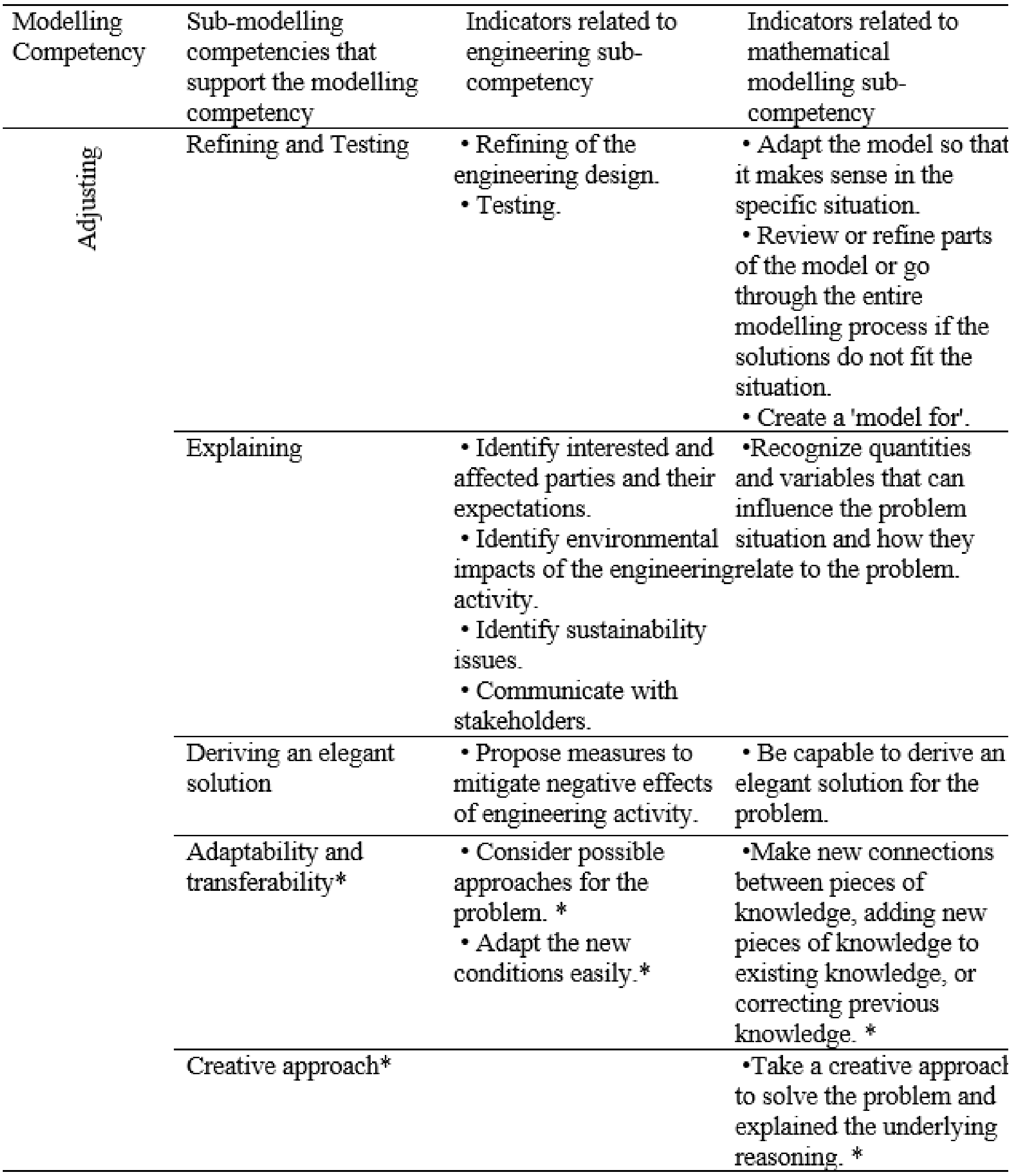
Table 3.17. Classification of adjusting competency to investigate and assess (Villiers, 2018, pp. 136-138)



Modelling competencies	Related engineering sub-competency	Related mathematical modelling sub-competency
Adjusting	• Refining of the engineering design. • Testing. • Identify interested and affected parties and their expectations. • Identify environmental impacts of the engineering activity. • Identify sustainability issues. • Propose measures to mitigate negative effects of engineering activity. • Communicate with stakeholders. • Consider possible approaches for the problem. * • Adapt the new conditions easily.*	• Adapt the model so that it makes sense in the specific situation. • Review or refine parts of the model or go through the entire modelling process if the solutions do not fit the situation. • Create a 'model for'. • Be capable to derive an elegant solution for the problem. • Make new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge. * • Take a creative approach to solve the problem and explained the underlying reasoning. *

As seen above, some new indicators shown with ‘*’ were added to both dimension of adjusting. In this regard, the descriptions (considering possible approaches for the problem and adapting the new conditions easily) were added to related engineering sub-competency and the descriptions (making new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge and taking creative approach to solve the problem and explained the underlying reasoning) were added to related mathematical modeling-competency. Then, all the above mentioned indicators were categorized under the sub-modeling competencies of adjusting as shown in Table 3.18.

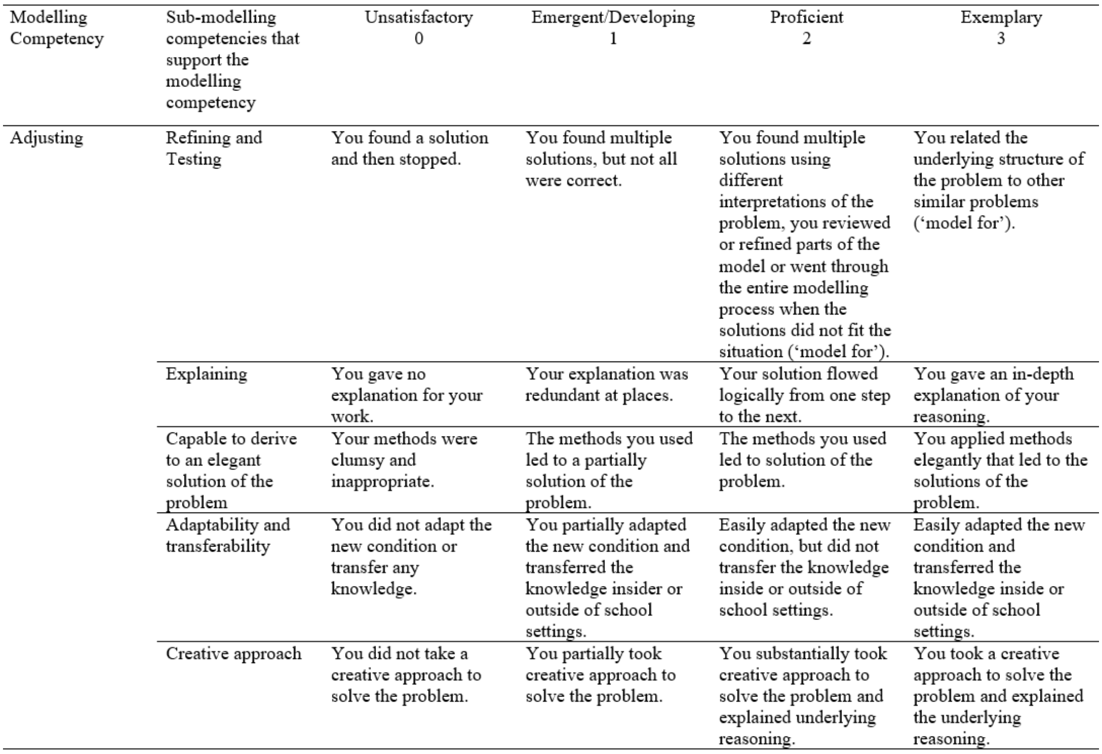
Table 3.18. Classification of symbolizing framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Adjusting	Refining and Testing	• Refining of the engineering design. • Testing.	• Adapt the model so that it makes sense in the specific situation. • Review or refine parts of the model or go through the entire modelling process if the solutions do not fit the situation. • Create a 'model for'.
	Explaining	• Identify interested and affected parties and their expectations. • Identify environmental impacts of the engineering activity. • Identify sustainability issues. • Communicate with stakeholders.	•Recognize quantities and variables that can influence the problem situation and how they relate to the problem.
	Deriving an elegant solution	• Propose measures to mitigate negative effects of engineering activity.	• Be capable to derive an elegant solution for the problem.
	Adaptability and transferability*	• Consider possible approaches for the problem. * • Adapt the new conditions easily.*	•Make new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge. *
	Creative approach*		•Take a creative approac to solve the problem and explained the underlying reasoning. *

As seen above, two new emerging sub-modeling competencies were categorized under the adjusting competency. The indicators identified in the relevant literature were employed in the procedure of labeling these sub-modeling competencies. For instance, creative approach is the concept for both gifted education literature (Mann et al., 2011) and modeling literature (Şengil-Akar, 2017). Similar to the work of Renzulli (2020), adaptability and transferability appeared as relevant concepts for gifted students in this study. Besides, the assessment criteria of adjusting competency were presented below.

Table 3.19. Group modeling competency observation guide for adjusting

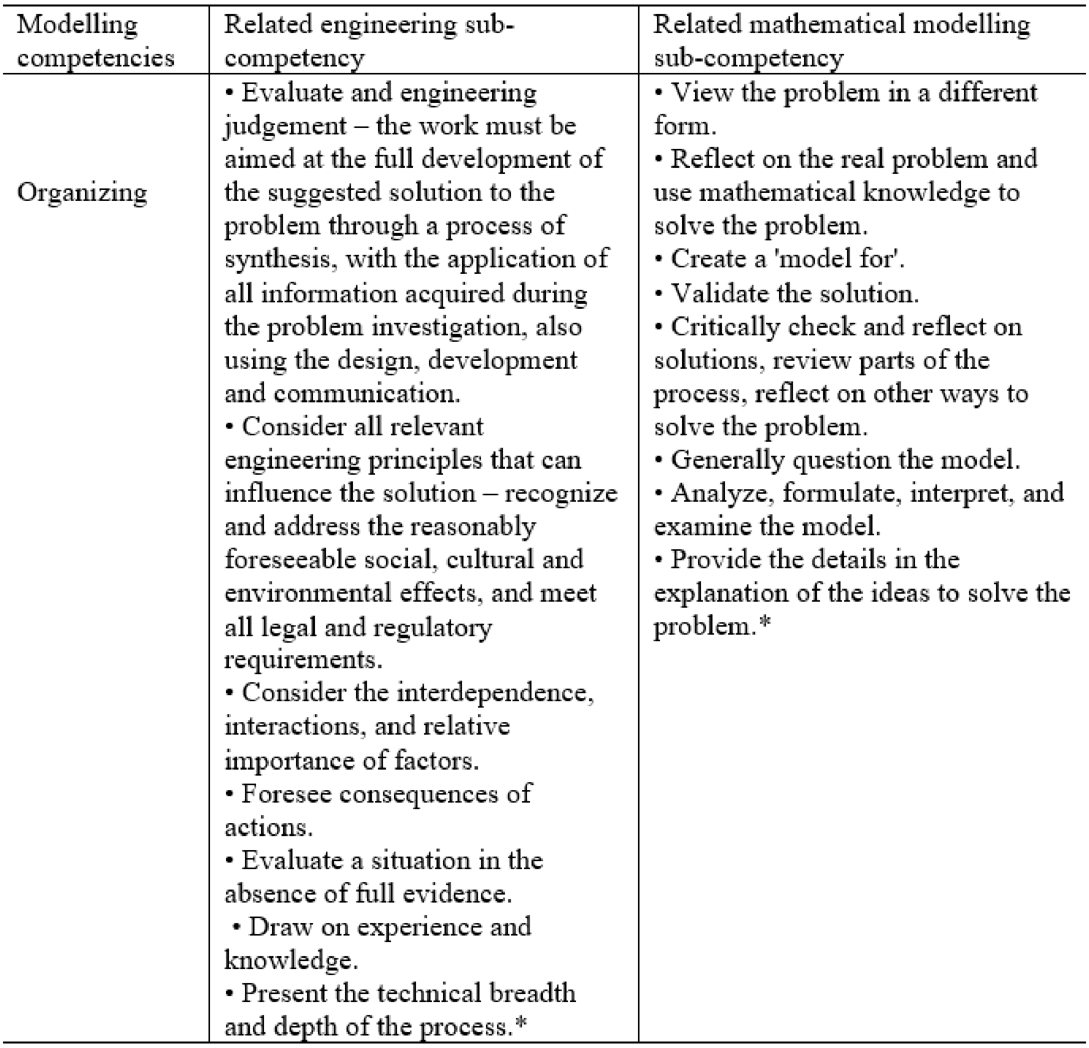


Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent/Developing 1	Proficient 2	Exemplary 3
Adjusting	Refining and Testing	You found a solution and then stopped.	You found multiple solutions, but not all were correct.	You found multiple solutions using different interpretations of the problem, you reviewed or refined parts of the model or went through the entire modelling process when the solutions did not fit the situation ('model for').	You related the underlying structure of the problem to other similar problems ('model for').
	Explaining	You gave no explanation for your work.	Your explanation was redundant at places.	Your solution flowed logically from one step to the next.	You gave an in-depth explanation of your reasoning.
	Capable to derive to an elegant solution of the problem	Your methods were clumsy and inappropriate.	The methods you used led to a partially solution of the problem.	The methods you used led to solution of the problem.	You applied methods elegantly that led to the solutions of the problem.
	Adaptability and transferability	You did not adapt the new condition or transfer any knowledge.	You partially adapted the new condition and transferred the knowledge insider or outside of school settings.	Easily adapted the new condition, but did not transfer the knowledge inside or outside of school settings.	Easily adapted the new condition and transferred the knowledge inside or outside of school settings.
	Creative approach	You did not take a creative approach to solve the problem.	You partially took creative approach to solve the problem.	You substantially took creative approach to solve the problem and explained underlying reasoning.	You took a creative approach to solve the problem and explained the underlying reasoning.

In the above table, the descriptions of four levels for the new emerging codes related to adjusting, adaptability and transferability, and creative approach were determined based on the data of the present study. After determining the highest and lowest levels display of sub-modeling competency, the definitions for intermediate levels were established, respectively.

Moreover, the related engineering and mathematical modeling sub-competency indicators of organizing sub-modeling competency were revised according to data of this study as shown in Table 3.20.

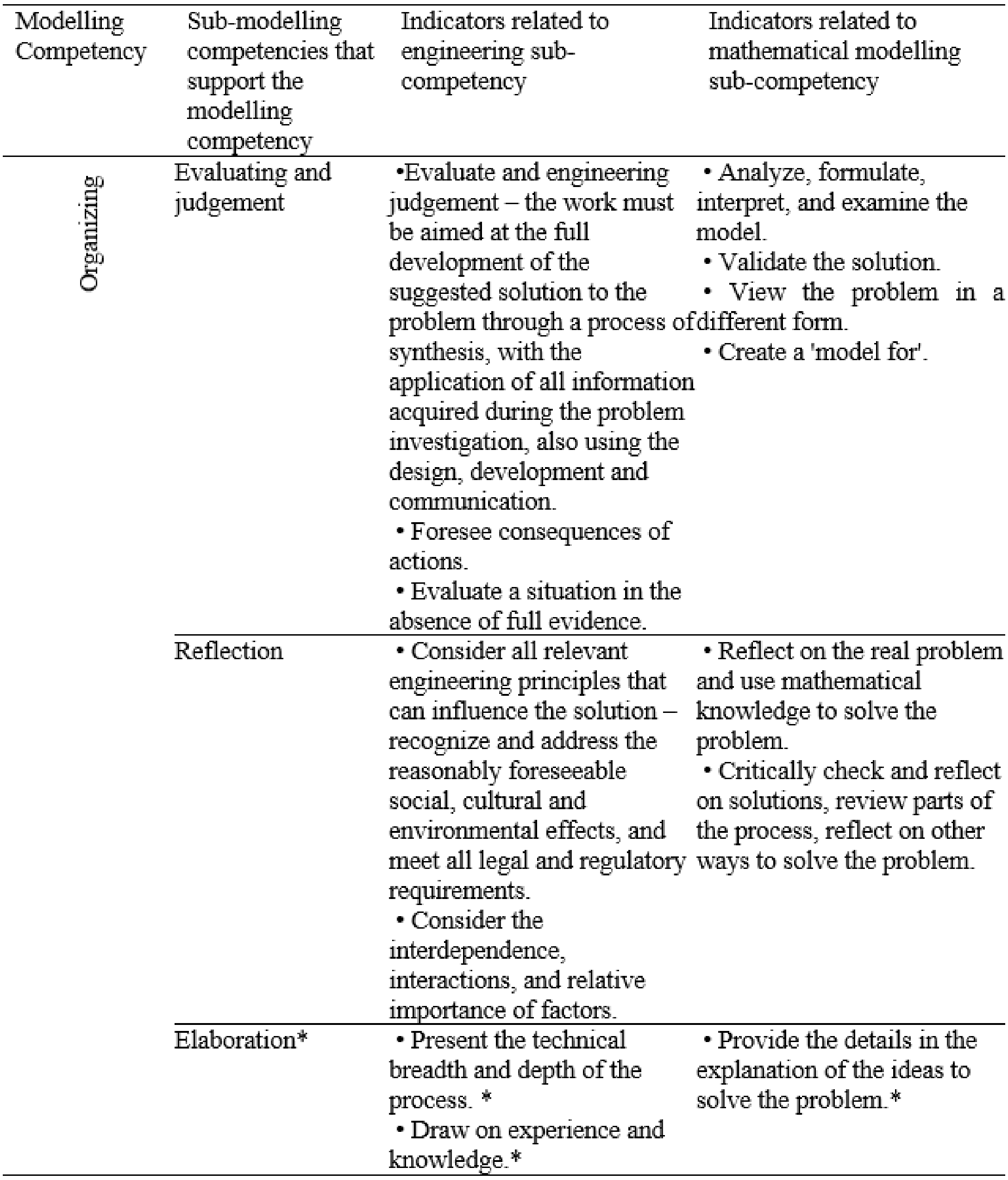
Table 3.20. Classification of organizing competency to investigate and assess (De Villiers, 2018, pp. 136-138)



Modelling competencies	Related engineering sub-competency	Related mathematical modelling sub-competency
Organizing	• Evaluate and engineering judgement – the work must be aimed at the full development of the suggested solution to the problem through a process of synthesis, with the application of all information acquired during the problem investigation, also using the design, development and communication. • Consider all relevant engineering principles that can influence the solution – recognize and address the reasonably foreseeable social, cultural and environmental effects, and meet all legal and regulatory requirements. • Consider the interdependence, interactions, and relative importance of factors. • Foresee consequences of actions. • Evaluate a situation in the absence of full evidence. • Draw on experience and knowledge. • Present the technical breadth and depth of the process.*	• View the problem in a different form. • Reflect on the real problem and use mathematical knowledge to solve the problem. • Create a 'model for'. • Validate the solution. • Critically check and reflect on solutions, review parts of the process, reflect on other ways to solve the problem. • Generally question the model. • Analyze, formulate, interpret, and examine the model. • Provide the details in the explanation of the ideas to solve the problem.*

Above table indicated that the description (presenting the technical breadth and depth of the process) was included in related engineering sub-competency and the description (providing the details in the explanation of ideas to solve the problem). Accordingly, the new emerging sub-modeling competency and its related indicators were presented in Table 3.21.

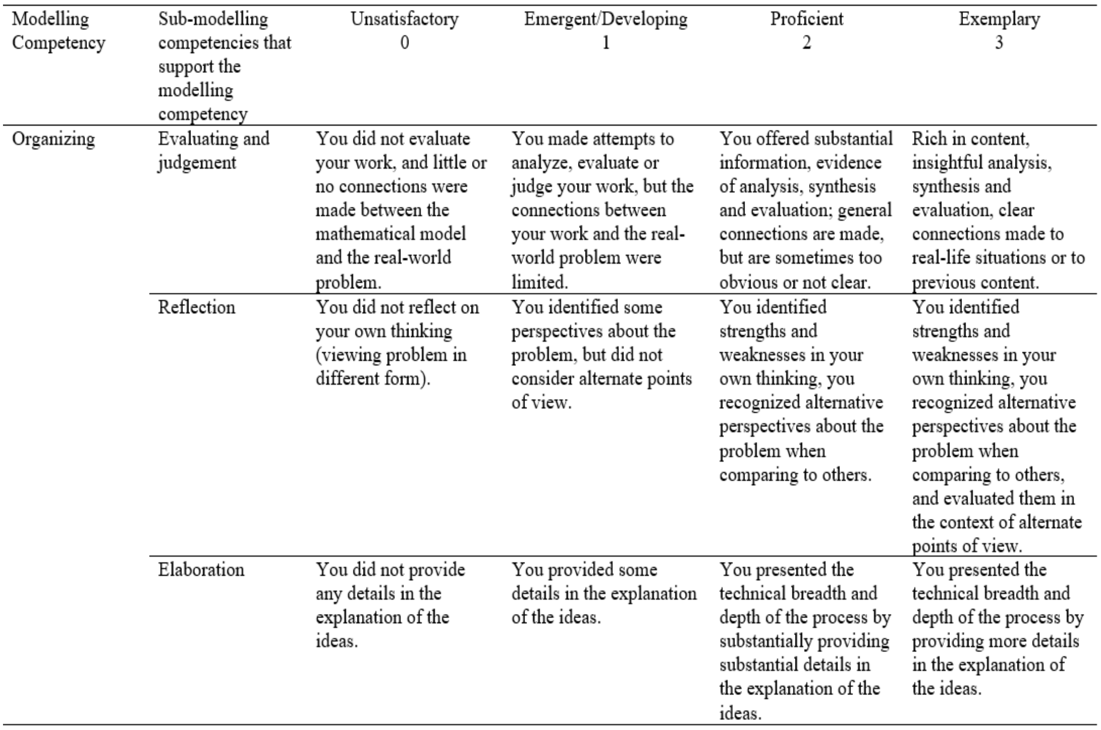
Table 3.21. Classification of organizing competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Organizing	Evaluating and judgement	•Evaluate and engineering judgement – the work must be aimed at the full development of the suggested solution to the problem through a process of synthesis, with the application of all information acquired during the problem investigation, also using the design, development and communication. • Foresee consequences of actions. • Evaluate a situation in the absence of full evidence.	• Analyze, formulate, interpret, and examine the model. • Validate the solution. • View the problem in a different form. • Create a 'model for'.
	Reflection	• Consider all relevant engineering principles that can influence the solution – recognize and address the reasonably foreseeable social, cultural and environmental effects, and meet all legal and regulatory requirements. • Consider the interdependence, interactions, and relative importance of factors.	• Reflect on the real problem and use mathematical knowledge to solve the problem. • Critically check and reflect on solutions, review parts of the process, reflect on other ways to solve the problem.
	Elaboration*	• Present the technical breadth and depth of the process. * • Draw on experience and knowledge.*	• Provide the details in the explanation of the ideas to solve the problem.*

As it can be seen in Table 3.21, consistent with the literature on creativity (Guilford, 1967), the analysis of the data from the present study indicated elaboration was emerged as sub-modeling competency. Consequently, group modeling observation guide propped by de Villliers (2018) was revised with the new emerging sub-modeling competency as indicated in Table 3.22.

Table 3.22. Group modeling competency observation guide for organizing

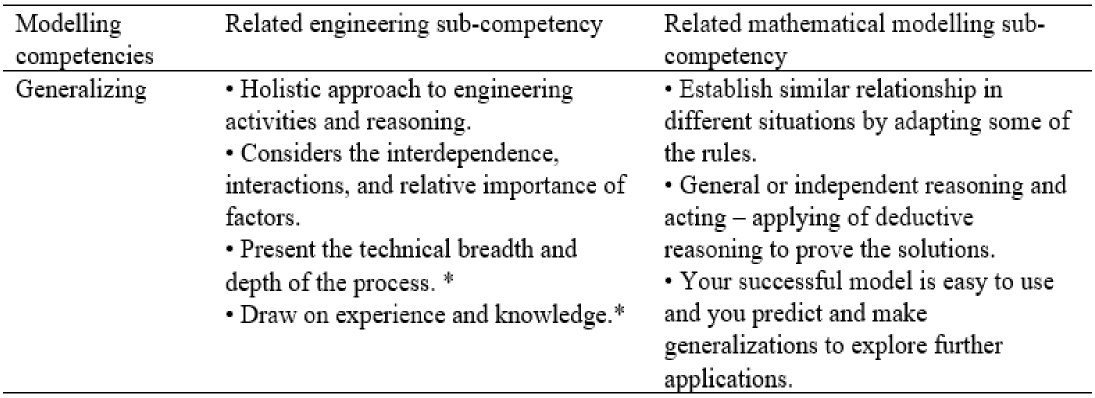


Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent/Developing 1	Proficient 2	Exemplary 3
Organizing	Evaluating and judgement	You did not evaluate your work, and little or no connections were made between the mathematical model and the real-world problem.	You made attempts to analyze, evaluate or judge your work, but the connections between your work and the real-world problem were limited.	You offered substantial information, evidence of analysis, synthesis and evaluation; general connections are made, but are sometimes too obvious or not clear.	Rich in content, insightful analysis, synthesis and evaluation, clear connections made to real-life situations or to previous content.
	Reflection	You did not reflect on your own thinking (viewing problem in different form).	You identified some perspectives about the problem, but did not consider alternate points of view.	You identified strengths and weaknesses in your own thinking, you recognized alternative perspectives about the problem when comparing to others.	You identified strengths and weaknesses in your own thinking, you recognized alternative perspectives about the problem when comparing to others, and evaluated them in the context of alternate points of view.
	Elaboration	You did not provide any details in the explanation of the ideas.	You provided some details in the explanation of the ideas.	You presented the technical breadth and depth of the process by substantially providing substantial details in the explanation of the ideas.	You presented the technical breadth and depth of the process by providing more details in the explanation of the ideas.

As seen in the table 3.22, the exemplary level description of elaboration was determined based on the high display of group work as presenting the technical breadth and depth of the process by providing more details in the explanation of the ideas. Then, the unsatisfactory, emergent/ developing and proficient level were determined respectively.

As the last sub-modeling competency, generalizing and its related engineering and mathematical modeling sub-competency were presented below.

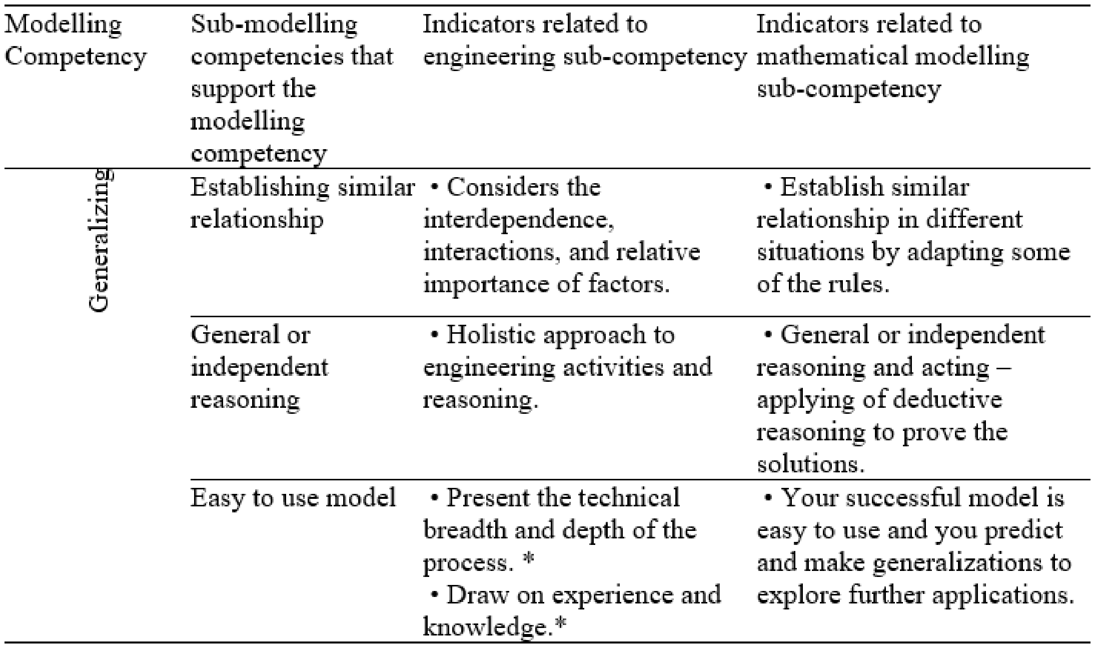
Table 3.23. Classification of generalizing competency to investigate and assess (Villiers, 2018, pp. 136-138)



Modelling competencies	Related engineering sub-competency	Related mathematical modelling sub-competency
Generalizing	• Holistic approach to engineering activities and reasoning. • Considers the interdependence, interactions, and relative importance of factors. • Present the technical breadth and depth of the process. * • Draw on experience and knowledge.*	• Establish similar relationship in different situations by adapting some of the rules. • General or independent reasoning and acting – applying of deductive reasoning to prove the solutions. • Your successful model is easy to use and you predict and make generalizations to explore further applications.

As seen in the Table 3.23., the descriptions (presenting the technical breadth and depth of the process and drawing on experience and knowledge) were included in the related engineering sub-competency of generalizing. According to data of the current study, sub-modeling competencies of generalizing and their related indicators were shown in Table 3.24.

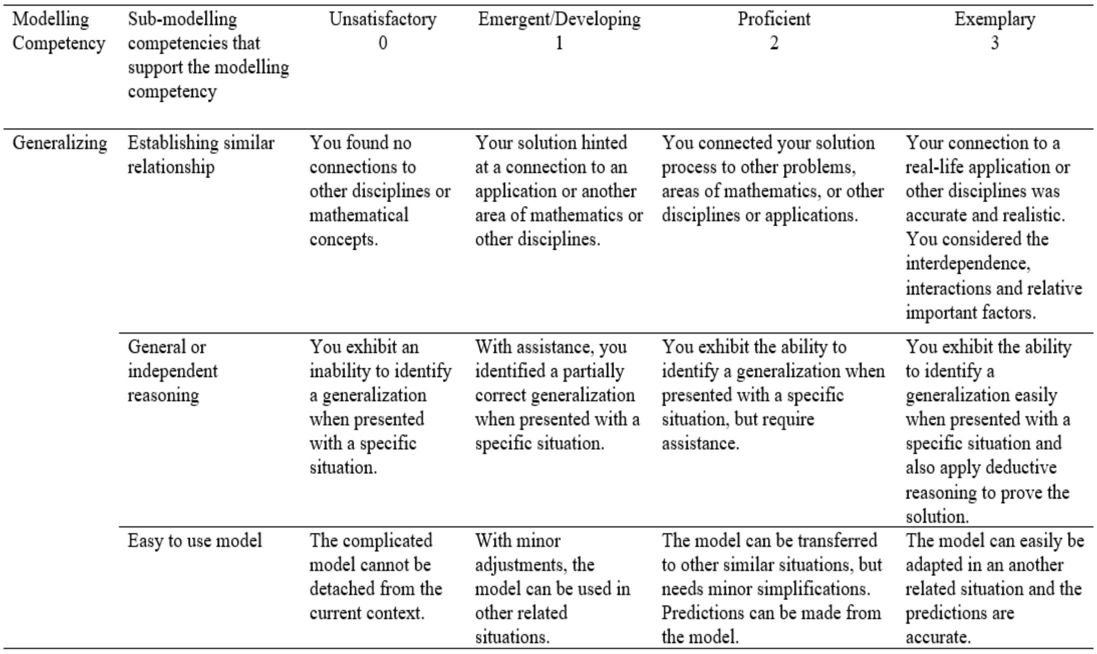
Table 3.24. Classification of generalizing competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Generalizing	Establishing similar relationship	• Considers the interdependence, interactions, and relative importance of factors.	• Establish similar relationship in different situations by adapting some of the rules.
	General or independent reasoning	• Holistic approach to engineering activities and reasoning.	• General or independent reasoning and acting – applying of deductive reasoning to prove the solutions.
	Easy to use model	• Present the technical breadth and depth of the process. * • Draw on experience and knowledge.*	• Your successful model is easy to use and you predict and make generalizations to explore further applications.

The above table indicated that new emerging codes were categorized under the sub-modeling competency of easy to use model. Accordingly, the observation guide to determine generalizing sub-modeling competency of generalizing was revised.

Table 3.25. Group modeling competency observation guide for generalizing



Modelling Competency	Sub-modelling competencies that support the modelling competency	Unsatisfactory 0	Emergent/Developing 1	Proficient 2	Exemplary 3
Generalizing	Establishing similar relationship	You found no connections to other disciplines or mathematical concepts.	Your solution hinted at a connection to an application or another area of mathematics or other disciplines.	You connected your solution process to other problems, areas of mathematics, or other disciplines or applications.	Your connection to a real-life application or other disciplines was accurate and realistic. You considered the interdependence, interactions and relative important factors.
	General or independent reasoning	You exhibit an inability to identify a generalization when presented with a specific situation.	With assistance, you identified a partially correct generalization when presented with a specific situation.	You exhibit the ability to identify a generalization when presented with a specific situation, but require assistance.	You exhibit the ability to identify a generalization easily when presented with a specific situation and also apply deductive reasoning to prove the solution.
	Easy to use model	The complicated model cannot be detached from the current context.	With minor adjustments, the model can be used in other related situations.	The model can be transferred to other similar situations, but needs minor simplifications. Predictions can be made from the model.	The model can easily be adapted in an another related situation and the predictions are accurate.

As it can be seen above, the labels of all sub-competencies of the generalization modeling competency were simplified within the scope of this study for clarity. That is to say, the details of establishing similar relationships (in different situations by adapting some of the rule), general or independent reasoning (applying deductive reasoning to prove solutions) and the easy to use model (the successful model allows for prediction). These descriptions were integrated into criteria of the related sub-modeling competencies. Firstly, the phrase (other disciplines) were added to the dimensions of unsatisfactory, emergent/developing and proficient levels of establishing similar relationships sub-modeling competency. In addition to this, the description (you considered the interdependence, interactions and relative important factors) was added to the exemplary level criteria of this sub-modeling competency. Secondly, the description (also apply deductive reasoning to prove the solution) was added to the exemplary level indicator of general or independent reasoning sub-modeling competency. Lastly, the description (predictions can be made from the model) were added proficient level and description (and the predictions are accurate) were added to exemplary level of easy to use model sub-modeling competency.

3.8. Trustworthiness of the Study

In qualitative research, the trustworthiness of a study is the degree of confidence in the data, interpretation, and methods employed to assure the quality of a study (Lincoln & Guba, 1985). The researcher should establish the procedures followed to ensure reliability and validity of the study (Shenton, 2004). In this regard, Lincoln and Guba (1985) proposed four criteria that should be considered by qualitative researchers in pursuit of a trustworthy study. They suggested using the terms credibility, transferability, dependability, and confirmability instead of internal validity, external validity, reliability, and objectivity, respectively. In the following, each of these criteria and how these concerns are addresses in this study are provided in detail.

Firstly, credibility, which represents internal validity, is one of the most essential aspects in qualitative research to ensure reliability (Lincoln & Guba, 1985). According to the definition provided by Merriam (2009), “Internal validity deals with the topic of how study findings mirror reality. How congruent are the findings with reality?” (p.213). There are some strategies, “prolong engagement, persistent observation, triangulation, peer debriefing, member checking and the reflexive journal” (Lincoln & Guba, 1985, p.328) to ensure the credibility of the researcher’s conclusions. Among these strategies, most of them were employed in the current study. As a participant observer, the researcher in this study spent extended periods of time observing and taking field notes in the context of the research setting (Lincoln & Guba, 1985; Merriam, 2009). In order to establish a long-term interaction with the participants prior to the pilot study, the researcher in this thesis study observed the students in BILSEM for one month, two lesson hours per week, and ensured the development of a trust-based environment with participants. Hence, the researcher spent a significant amount of time (four months) in the research area, and as a result was exposed to different aspects of the subject under investigation. Triangulation through multiple data sources was also used to enhance the accuracy of the acquired data (Creswell, 2012; Merriam, 2009). By collecting data from a variety of sources, including video recordings of the different engineering-based MEAs, documents, field notes from observations, and group interviews allowed the researcher in this study to cross-check and to form themes among data sources so that it enhanced credibility of the study (Creswell, 2009;

Shenton, 2004). In addition, a second coder who is a mathematics teacher with nine years of experience and an expert in mathematical modeling contributed to triangulation. After reviewing and debating the information that had been generated, the coders used a color-coding scheme consisting of green, yellow, and red to evaluate the degree to which each data point converged across method and data source. The level of agreement between the coders was continuously high for the green and red codes. However, the yellow codes required a more in-depth analysis in order to determine the final triangulation code. By applying the literature about the yellow and red codes, the final green codes were accepted by both coders.

Secondly, transferability, referred to external validity, is another criterion for ensuring the trustworthiness of qualitative research (Lincoln & Guba, 1985). Due to the selection of the participants and nature of the qualitative research, it is not appropriate to generalize the results to the entire population (Merriam, 2009). Instead of generalizability, there is another concept for qualitative research, transferability (Shenton, 2004). It defines the responsibility of a qualitative researcher as discussing the possibility of transferring findings of the research to similar settings (Merriam, 2009; Shenton, 2004). Therefore, the researcher should describe whole process in detail in order readers to transfer the descriptions to similar settings (Lincoln & Guba, 1985). In this regard, the context and participants of the study, data collection methods and data analysis procedures are provided in detail to ensure transferability of the current study.

Thirdly, dependability, which represents reliability, refers to the consistency the research procedure employed over time and is concerned with the ability to replicate research findings (Merriam, 2009). To address the issue of dependability more directly, the processes within the study should be described in sufficient detail a future researcher to repeat the work, although not necessarily obtain exactly the same results (Lincoln & Guba, 1985). Thus, the researcher in the current study described the research procedure in detail. The code-recode strategy was another technique that was used to enhance the dependability of the study. This strategy suggested that whether or not another rater with the same theoretical framework could have interpreted the same phenomena in the same way (Anney, 2014). Accordingly, the researcher waited three weeks after coding a section of data before returning to recode the same data and examine the findings. In addition, as mentioned in the data analysis section of this chapter, the researcher discussed her research findings with colleague who has experience on mathematical modeling research and teaching. Hence, this peer debriefing provides a qualified peer researcher to review and evaluate transcripts, emerging and final categories derived from those transcripts, and the final themes or findings of the current study (Yıldırım& Şimşek, 2011).

Fourthly, confirmability which refers to objectivity, is concerned with demonstrating that facts and interpretations of the findings are not figments of the investigator's imagination, but rather are clearly drawn from the data (Yıldırım & Şimşek, 2011). The confirmability of this study was established through audit trail and reflective journals (Lincoln & Guba, 1985). Using an audit trail at critical decision points throughout a qualitative study can strengthen the research's rigor and confirmability, while also revealing any biases (Creswell, 2012). In qualitative research, an audit trail is a record of how a qualitative study was conducted and how researchers reached their conclusions. In this regard, science teacher who is a PhD candidate examined the data collection process and documented critical decisions throughout the study. A reflective journal that further describes the reasoning for decisions made within this study is another tool that was used to consistently establish and maintain confirmability. In order to minimize the influence of investigator bias (Shenton, 2004), it is essential that the importance of the role that triangulation plays in ensuring such confirmability is once again emphasized. In addition to all of these, another crucial consideration is the researcher's role to establish trustworthiness in the qualitative research presented in the following section in detail.

3.9. Researcher Background and Role in the Study

In qualitative research, one of the crucial issues to consider is the researcher's background. In the current study, researcher is a PhD student at Department of Mathematics and Science Education in Faculty of Education in a public university. Before gathering the data, she took a course in qualitative research, and learned the qualitative research paradigms and qualitative research procedures. The researcher attended the thematic working group of the conference on mathematical modeling. She also enrolled in the Scientific and Research Technological Council of Turkey (In Turkish, TUBITAK) 2237 project, *Mathematical Modeling and Applications in Scientific Process.* She conducted research on differentiation strategies for gifted students by using qualitative research. In addition to research experience, she had five-year teaching experience with varying grade levels in elementary education. More specifically, she had a teaching experience with gifted students in both homogeneous and heterogeneous classrooms. Moreover, she participated one-year in-service teacher training program for gifted students. Taking into account all of these aspects, it is possible to conclude that the research and teaching expertise of the researcher matches with the subject of interest of the current study, making her an appropriate candidate for carrying it out.

Furthermore, the researchers in most of the qualitative studies are the primary instrument of data collection; therefore, it is crucial for the credibility of qualitative research that researchers are recognized for their roles in data collection (Creswell, 2012). For this reason, it is essential for the researcher in the current study to play an active role in the data collection process, prepare the engineering-based MEAs, organize the research environment, interview and observe the participants, analyze the data, and be personally involved in the process when the students are engaged in engineering-based MEAs. Since the researcher took an active role in the process of the research as both a teacher and a researcher while the data was being collected, she was considered a “participant as an observer” in this study (Merriam, 2009). In other words, the role of the researcher was to be actively involved in this study, interacting directly with the study group, meeting directly with the group, and, when necessary, intensely experiencing the experiences of the study group, and then using these experiences in the analysis of the data that was collected.

The researcher designed the activities and learning environment, implemented the activities herself, and answered the questions posed by the students during the process. During the application process, she did not provide direct information and interventions to the students. Instead, she prompted them to think by asking pertinent questions and assisted them in finding their own answers. During the presentation phase of the process, the researcher adopted the position of moderator and guide, ensuring that the process run smoothly, that students communicated their ideas properly and defended their models, and opposing viewpoints were discussed in the research setting. All of these allowed the researcher to participate in the process and to objectively describe and reflect the process. In addition, in qualitative research, the researcher spends a significant amount of time in the natural setting of the study and maintains direct contact with the participants (Merriam, 2009). Accordingly, the researcher observed the students in their natural settings at BILSEM once a week for a month before starting the pilot study. After three or four observations, the students were accustomed to the researcher, and she was able to begin the actual progress of data collection.

3.10. Ethical Considerations

In conducting qualitative research, the researchers have a responsibility to address a number of ethical considerations, including the protection of subjects from harm, the right to privacy, the concept of informed consent, and the issue of deception (Creswell, 2012, p.230). In this regard, the ethical issues that were followed throughout the current study were stated as follows.

Before conducting this research, the appropriate authorizations were obtained from the Applied Ethics Research Center at Middle East Technical University (METU). Appendix A provides evidence that the Human Subjects Ethics Committee given their consent to the research. In addition, the Ministry of National Education provided their approval, which was required in order to carry out the research. Then, the necessary permission was obtained from the principal of each of the BILSEM by presenting a legal document issued by MoNE. Upon the approval of the principal, the researcher informed all teachers and students attending individual talent recognition program about the purpose and scope of the study. After informing students, the researcher requested their voluntary participation in the study. Subsequently, parents of students who volunteered to participate in the study were given an informed consent form to sign (See Appendix D).

After receiving the consent from parents, the researcher explained the details of the study to students. Before the study, the researcher conducted informal introduction meetings with the participants to provide them with the necessary information about the nature of the study. The participants were informed about the research topic and research questions, the structure of the study and interviews and also the time schedule of the study. In addition, they were informed about the usage of voice recorders and cameras during the experiment. The researcher ensured that the participants understood their rights as participants, that their involvement in the research study was not related to any program evaluation within the school district or BILSEM, and that they could withdraw themselves from study participation at any time. Hence, the conformity of the participants for the study understood both by the researcher and the participants. Moreover, the anonymity of the participants by assigning numbers and the groups by assigning letters was preserved. Taking into account the confidentiality of student identity, careful consideration was given to the selection of images to illustrate the findings section of this study.

CHAPTER 4

FINDINGS

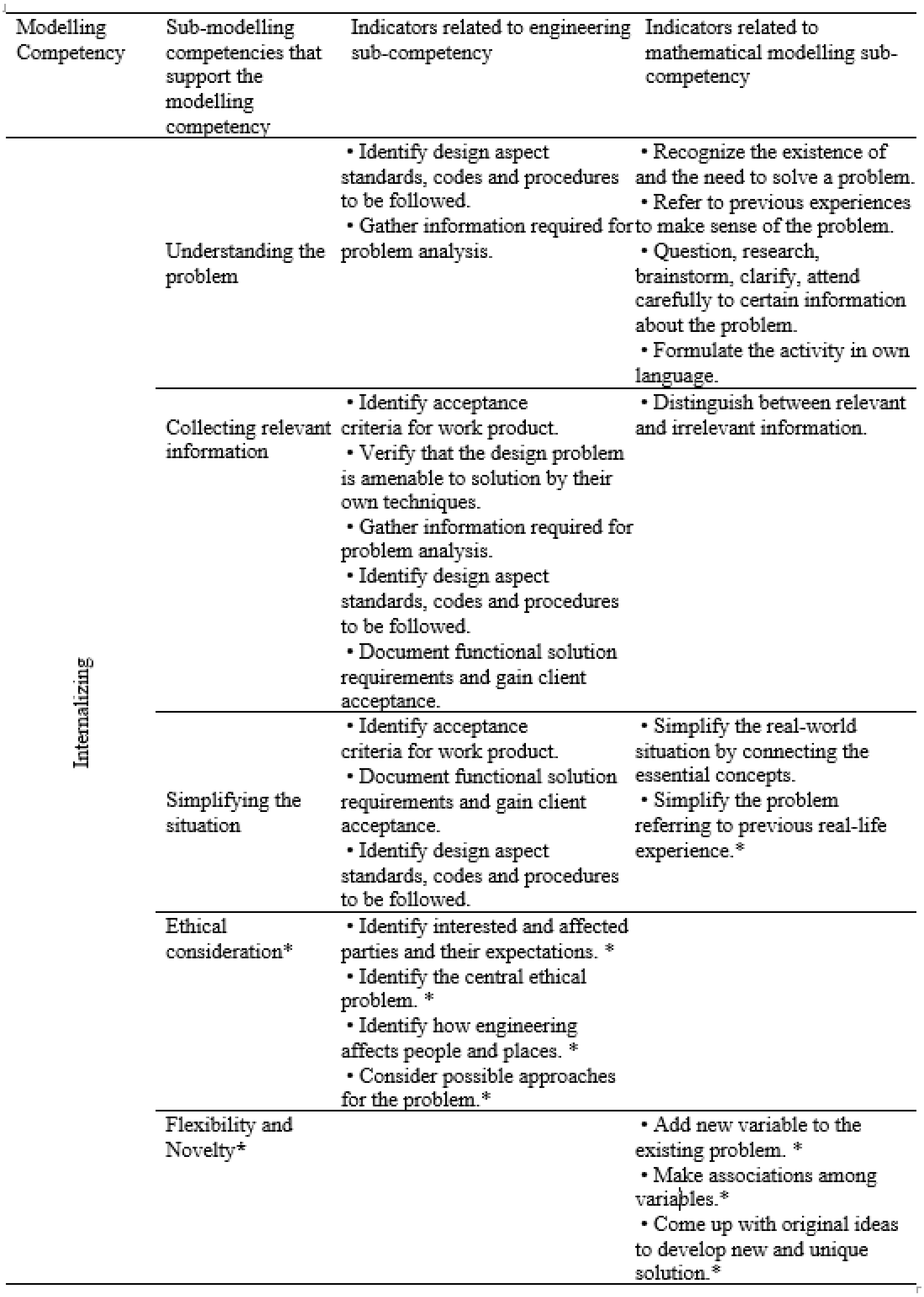
The aim of this research study was to investigate the cognitive modeling competencies of elementary gifted students when they are engaged in engineering-based model eliciting activities (MEAs). In line with this aim, this chapter categorizes the findings of this study in seven main sections and their related subsections, each of which focuses on different cognitive modeling competencies. In this regard, the findings of three engineering-based MEAs are presented based on the adapted version of the group modeling competency observation guide proposed by de Villiers (2018) as stated in the methodology chapter. The seven main sections are: internalizing, interpreting, structuring, symbolizing, adjusting, organizing, and generalizing. Each section examines the extent to which elementary gifted students in groups displayed such cognitive modeling competencies when they are engaged in engineering-based MEAs, namely, *Bridge Construction*, *Mars Lunarcrete*, and *Dr. Ahmet's Will*.

A consistent reporting method is used to present the findings in each section. Firstly, a brief description of the cognitive modeling competency and its related sub-competencies is provided. Then, the saturated codes and categories for the relevant competency are stated in the table along with their accompanying indicators. In the table, the competency is formed by merging the indicators related to mathematical modeling and engineering sub-competencies. Subsequently, the cognitive modeling competency and related sub-competencies of the two different groups are provided in detail through the presentation of examples for each activity. Lastly, the findings obtained from these three engineering-based MEAs are summarized in terms of the extent to which elementary gifted students in groups displayed the cognitive modeling competency and its related sub-competencies. The following seven sections report the findings in more detail concerning the emergent categories mentioned previously.

4.1. Modeling Competency 1: Internalizing

The data-driven findings related to cognitive modeling competencies were classified into seven main sections based on the classification of cognitive modeling competencies suggested by de Villiers (2008). This section presents the findings of the first modeling competency, internalizing. The internalizing competency was investigated under the sub-modeling competencies: *understanding the problem*, *collecting relevant information*, *simplifying the situation*, *ethical consideration*, and *flexibility and novelty*. In addition, the extent to which elementary gifted students in groups displayed the internalizing modeling competency and its sub-modeling competencies when they engaged in Bridge Construction, Mars Lunarcrete, and Dr. Ahmet's Will activities are presented in this section. In each activity, examples of two different groups were provided to demonstrate sub-modeling competencies of elementary gifted students. Specifically, the internalizing competency of elementary gifted students was analyzed through its sub-modeling competencies and their indicators related to the engineering and mathematical modeling sub-competencies as shown in Table 4.1. In this table, new codes that emerged within the scope of the current research are highlighted with '*'.

Table 4.1. Classification of internalizing competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Internalizing	Understanding the problem	• Identify design aspect standards, codes and procedures to be followed. • Gather information required for problem analysis.	• Recognize the existence of and the need to solve a problem. • Refer to previous experiences to make sense of the problem. • Question, research, brainstorm, clarify, attend carefully to certain information about the problem. • Formulate the activity in own language.
	Collecting relevant information	• Identify acceptance criteria for work product. • Verify that the design problem is amenable to solution by their own techniques. • Gather information required for problem analysis. • Identify design aspect standards, codes and procedures to be followed. • Document functional solution requirements and gain client acceptance.	• Distinguish between relevant and irrelevant information.
	Simplifying the situation	• Identify acceptance criteria for work product. • Document functional solution requirements and gain client acceptance. • Identify design aspect standards, codes and procedures to be followed.	• Simplify the real-world situation by connecting the essential concepts. • Simplify the problem referring to previous real-life experience.*
	Ethical consideration*	• Identify interested and affected parties and their expectations. * • Identify the central ethical problem. * • Identify how engineering affects people and places. * • Consider possible approaches for the problem.*	
	Flexibility and Novelty*		• Add new variable to the existing problem. * • Make associations among variables.* • Come up with original ideas to develop new and unique solution.*

As seen in Table 4.1, the analysis of this study revealed that in addition to the sub-codes suggested by de Villiers (2018), internalizing sub-competencies, understanding the problem, collecting relevant information, and simplifying the situation, the following sub-codes emerged based on the findings of the current study, ethical consideration and flexibility and novelty. Accordingly, the indicators of internalizing sub-competencies in terms of engineering and mathematical modeling sub-competencies when elementary gifted students are engaged in engineering-based MEAs are presented in Table 4.1.

First of all, the findings of group A and group B regarding internalizing competency and related sub-competencies while they are engaged in the *Bridge Construction* activity are presented in this part of the section. The internalizing competency of elementary gifted students is examined under the sub-modeling competencies: *understanding the problem*, *collecting relevant information*, *simplifying the situation*, *ethical consideration*, and *flexibility and novelty*. In addition, the extent to which elementary gifted students in groups displayed the internalizing modeling competency and its sub-modeling competencies is shown in Figure 4.1. Based on the group modeling competency observation guide, the level of sub-modeling competencies for the groups was classified as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)* and 3 *(exemplary)*, proposed by de Villiers (2018).

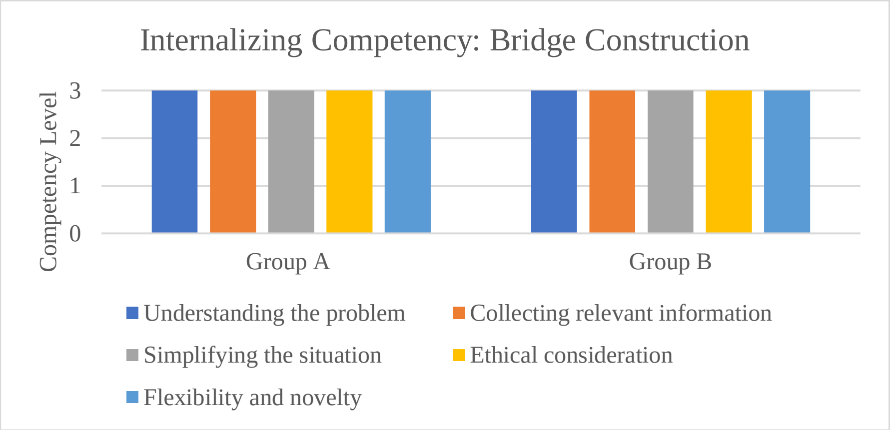


Figure 4.1. Internalizing Competency: Bridge Construction

As shown in Figure 4.1, the findings of the current study indicated that elementary gifted students in groups A and B in the Bridge Construction activity exhibited *exemplary* modeling sub-competencies of internalizing. To clarify and expand on the aforementioned categories, sample excerpts representative of each category are provided below.

Understanding the problem. Following the presentation of the video related to the Bridge Construction problem situation in Harşit Stream, both groups of elementary gifted students asked to watch the video two more times in order to observe the conditions in the village. The members of group A identified the problem as "*now, our problem is that there is a 60-year-old wooden suspension bridge and it is about to collapse. So, a girl named Ayşe asked us for help, and we designed a bridge*". The excerpt showed that the group members summarized the problem in their own language. Differently, the members of group B read the problem one more time to internalize it. Then, they negotiated what they knew about the problem.

> *Student 6*: In the video, the suspension bridge, which is 10 meters above the stream, is requested to be renewed.
> *Student 5*: Its location is important for three villages. It is also located close to residential areas.
> *Student 4*: In the video, someone said that vehicles and school buses passed over this wooden bridge. It is too old and dangerous.
> *Student 5*: I agree. It is too dangerous for students.

The above indicates that they clarify and attend carefully to problem details. The students in group B tried to perceive what information from the video and the problem text was different. Both groups explicitly determined and summarized the major concerns and also clarified why and how Bridge Construction is a problem for the peasantry. This can be accepted as evidence that both groups displayed the *exemplary* modeling sub-competency in *understanding the problem*. In addition, the members of group B identified the central ethical problem and considered the impact of engineering on people and places in the above extract. These were indicators of the *ethical consideration* of the sub-modeling competency.

Collecting relevant information and flexibility and novelty. The findings of this study revealed that elementary gifted students internalize the task by distinguishing between relevant and irrelevant information *(collecting relevant information).* For instance, none of the groups preferred to construct truss, arch, or girder bridges separately, as given in Bridge Construction Data Set 1. Instead, they stated that they combined the properties of the three to construct their bridges. Hence, this showed evidence that elementary gifted students identified acceptance criteria for work product as an indicator of the engineering sub-competency under *collecting relevant information*. In a similar vein, neither group found the materials sufficient to construct a bridge. These groups expressed their opinion that they bring recyclable household materials for constructing their bridges in the next session. They shared the responsibilities among the group members investigating who can bring which materials from the house. The researcher asked the whole class how they would plan to determine the cost of their household materials. After the whole class discussion, they reached a consensus that it was best to search for the cost of the material via the internet and then they calculated the cost of the material required, keeping in mind the current internet price. Such preferences showed that elementary gifted students do not strictly depend on the given data. Both groups offered a great deal of flexibility with the given data related to the type of bridge, materials, and material cost. As a result, they added new variables to the existing problem and made associations among variables *(flexibility)*. The ideas stated above can be accepted as original ideas to develop new and unique solutions since it is an extraordinary situation for the scope of the problem *(novelty)*. Thus, the stated indicators can be accepted as evidence of the *exemplary flexibility and novelty* modeling sub-competency. Moreover, both groups uncovered hidden information not readily apparent and showed indicators of the *exemplary collecting relevant information* modeling sub-competency. Together these findings provided evidence regarding the engineering sub-competency that elementary gifted students gathered information required for problem analysis. Accordingly, they showed that it is possible to solve the Bridge Construction problem employing their own techniques. To construct their bridges, elementary gifted students specified their rules, methods, and materials as the engineering sub-competency of *collecting relevant information*. Furthermore, both groups approached the design process with a positive approach, namely that improvements can be made in design identifying possibilities and opportunities in situations. This indicated that they considered possible approaches for the problem as an indicator of the *ethical consideration* modeling sub-competency.

Simplifying the situation and ethical consideration. When they engaged in the Bridge Construction activity, both groups simplified the real-world situation by integrating the core ideas. In order to create the prototype, the groups clarify the real-world situation to determine the factors affecting the context given in the activity. An excerpt from group A to simplify the situation in Bridge Construction activity is given below.

> *Student 1*: In the video, the bridge in the video lasts 60 years. So, the bridge prototype should be durable when we compare it with the existing one.
> *Student 2*: The bridge is swaying now. It must be strong enough to weigh vehicles and people as well.
> *Student 3*: I agree with both of them. Its use should be suitable for peasants and children. According to the conditions of the village, there should be a road for cars and pedestrians. We also keep in mind that we have a restricted budget because these peasants have difficulty paying.

As seen above the excerpt, group A members considered various factors such as durability, strength, and economic feasibility in order to simplify the problem situation and reached a consensus for the prototype. On the other hand, Group B members discussed the problem concerning Student 6's observation in real life:

> *Student 6*: There is a residential area where my grandmother lives similar to this one, and there is little water there. There are two bridges across the water. Why did they construct two bridges there?
> *Student 4*: I think they should be for different purposes. Are they not?
> *Student 6*: One of them is for people, and the other is for cars. Their appearance is the same, but they carry different weights. The materials they used are also different. While the abutments of one of them are iron, the other's is steel. The main body of the bridge is wood, and the other is iron.
> *Student 5*: We think of the people and cars for constructing our bridge, too. Constructing two bridges may increase the cost for peasants. It should be available for cars and people. The strength is also important for our prototype.

The above indicates that Student 6 referred to past experience to internalize the problem. The group discussed a similar situation considering the situation in the Bridge Construction activity. They *simplified the situation* referring to previous real-life experience. The dialogues from both groups also showed *exemplary* evidence related to the *ethical consideration* modeling sub-competency, namely that elementary gifted students identified interested and affected parties and their expectations, acceptance criteria for work product, and also how engineering affects people and places. As an engineering sub-competency of *simplifying the situation*, they documented functional solution requirements to be accepted by peasants. In addition to this, elementary gifted students determined the design aspect standards of the bridges to ensure safety, durability and serviceability. When all of the above instances were considered, elementary gifted students displayed *exemplary simplifying the situation* competency since they used multiple representations to explain the problem.

Secondly, this part of the section presents the findings from group C and D on internalizing competency and related sub-competencies when engaging in the Mars Lunarcrete activity. Through the sub-modeling competencies: *understanding the problem*, *collecting relevant information*, *simplifying the situation*, *ethical considerations*, and *flexibility and novelty*, the internalizing competency of elementary gifted students in groups were examined when they engaged in this activity. Additionally, Figure 4.2 indicated the extent to which elementary gifted students displayed the internalizing modeling competency and related sub-modeling competencies when they are engaged in the *Mars Lunarcrete* activity. The level of sub-modeling competencies for the groups was categorized as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)*, and 3 *(exemplary)*, based on the group modeling competence observation guide proposed by de Villiers (2018).

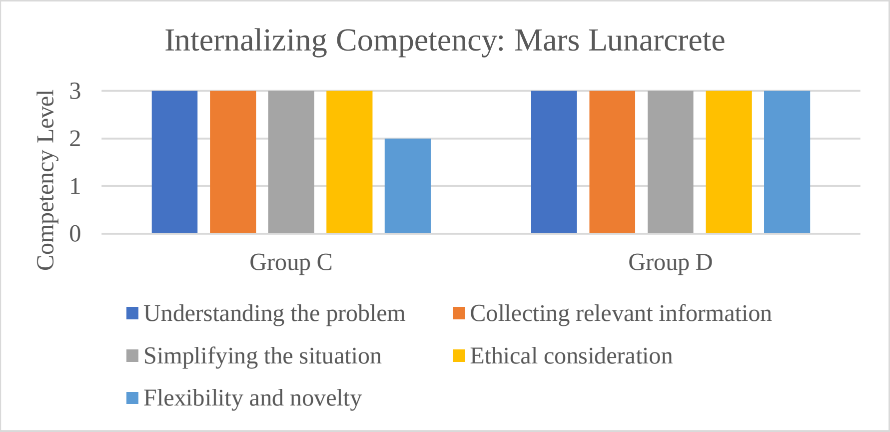


Figure 4.2. Internalizing Competency: Mars Lunarcrete

As illustrated in Figure 4.2, the findings of this study indicate that elementary gifted students in groups C and D generally exhibited *exemplary* modeling sub-competencies of internalizing when they are engaged in the *Mars Lunarcrete* activity except for flexibility and novelty. The above figure also showed that group C displayed *proficient* flexibility and novelty sub-modeling competency engaging in the activity.

Accordingly, the sample instances of this competency related to the aforementioned sub-categories are provided below.

Understanding the problem and ethical consideration. When the *Mars Lunarcrete* problem situation, data set 1, and a recipe for this activity were given to the groups, each group approached the problem in a different way. For example, group C negotiated what is given and what is asked in the problem:

> *Student 8*: Do we understand what the corporation requested from us? What was given to us?
> *Student 7*: Density, wind resistance, friability. The ratio of regolith and binders were also given.
> *Student 9*: The recipe was given to us. The ratios in the recipe are not exactly the same in data set 1, but close to what we would choose.
> *Student 8*: I see you both forgot the price. You continue the discussion; I will read the texts again in order not to miss anything.
> *Student 7*: We determine the best regolith for settlement on Mars and this is what the corporation requested.
> *Student 9*: In the letter, a company is looking to build efficient, affordable housing, townhomes and single family homes.
> *Student 8*: If there is a trip to Mars and settlement there in the future, we can also use our building material.

The above indicates that elementary gifted students examined the given data in detail. Then, they compared the data in both data set 1 and the recipe to understand the problem. Hence, they identified the main issues and gathered information required for problem analysis, reflecting the engineering sub-competency of *understanding the problem.* As seen in the dialogue above, the students recognized the existence of and the need to solve the problem, referring to the request of the company *(understanding the problem).* In addition, they identified design aspect standards, codes, and procedures to be followed and showed evidence of the engineering sub-competency of *understanding the problem*. Also, Student 9 identified affected parties and their interests, and so group C displayed evidence of the *ethical consideration* modeling sub-competency. On the other hand, group D carefully read each variable in the data set and discussed what they had to solve in the problem:

> *Student 11:* Everyone, take a look into the data. Let me know if you have another idea. I think we will take into account those who say let us do it this way.

> *Student 12:* We should discuss our variables for the condition of Mars. We should make our choices based on temperature differences.
> *Student 10:* Normal concrete cannot be used there because it cannot withstand the heat and cold of Mars. We should use a building material called lunarcrete there because it can withstand the weather conditions there.
> *Student 12:* We should combine regolith and binder solutions to create the building model the corporation desires.

As it is presented above, elementary gifted students identified design aspect standards and procedures to be followed by recognizing the presence of and the necessity to address the problem, indicating the engineering sub-competency of *understanding the problem*. They tried to understand the real-life situation beyond the data in order to make the problem more meaningful. Thus, they referred to previous experiences about the condition of Mars to make sense of the problem *(understanding the problem).* Further, they made associations among variables and came up with original ideas to create the model since they generated numerous original and different types of ideas. This can be accepted as *exemplary* evidence of the *flexibility and novelty* modeling sub-competency. The above dialogue also indicates that they collaborate with their group members and value their viewpoints as they attempt to understand the problem. As seen in the dialogues above, both groups clearly defined and summarized the main issues and explicitly explained why this is a problem. This can be accepted as *exemplary* evidence of the *understanding the problem* modeling sub-competency. In addition, both groups recognized interested and affected parties as well as their expectations and also how engineering affects people and places *exemplary* indicators of the *ethical consideration* modeling sub-competency.

Collecting relevant information. The findings from the *Mars Lunarcerete* activity indicated that elementary gifted students scrutinized carefully relevant information about the problem by questioning, brainstorming, and clarifying. For example, group D criticized all the information in the data set and recipe:

> *Student 11*: What is the effect of density on the structure?
> *Student 10*: Density keeps the structure more solid. For instance, it really stays strong in an earthquake.
> *Student 12*: So, why is wind resistance important?
> *Student 11*: Wind resistance is also important. Let’s see.

> *Student 10*: Density and wind resistance, both of them can create a stronger structure.
> *Student 12*: Cost is an important factor for corporation.
> *Student 11:* What is the effect of salt?
> *Student 12:* I think it holds tighter.
> *Student 10*: I also think salt is important. On the other hand, I think diluting the glue before adding it to the solids adds strength, but it increases the stickiness of the solution.
> *Student 11*: For example, if we add spices, it always smells.
> *Student 12*: Does flour hold better or corn flour?
> *Student 10*: I can try it at home without trying the recipe next week. I don't have a guess for now.

As it was deduced from the dialogue above, this group uncovered implied information not readily apparent and so exhibited *exemplary* indicators of *collecting relevant information*. In addition to this, they made associations among variables and explored solutions to engineering problems utilizing original ideas (*flexibility and novelty)*. In a similar vein, group C discussed the relevant information required for the problem situation:

> *Student 9:* Density and wind resistance help us to build strong and durable structures. Both of them are like the same thing. The higher the density, the higher the wind resistance. Of course, we also consider the cost.
> *Student 7:* I think the cost is not important.
> *Student 8:* How is it not important? Cost is also important. The company could go bankrupt.
> *Student 9:* Should we add salt?
> *Student 8:* I think it works. Will it be flourless? Not at all. I think it would be unstable.
> *Student 7:* But there are options below, I think we can use sand.
> *Student 8:* We will vote on the ideas. I agree with the idea that cost is important. However, we can use such options if our most suitable recipe does not work.
> *Student 9:* Nice idea.
> *Student 7:* It is ok for me.

This dialogue shows that there were some disagreements among the group members. However, they reached consensus by voting. This indicated evidence that the members of group C collaborate with their group members and value their viewpoints. In the above excerpt, the group members made associations among variables to generate different types of ideas and so showed evidence of flexibility. Accordingly, they considered possible approaches if their recipe did not work *(ethical consideration)*. Besides, they documented functional solution requirements to gain client acceptance, reflecting the engineering sub-competency of *collecting relevant information*. On the other hand, group C also exhibited an *exemplary* indicator of collecting *relevant information* since they uncovered information not readily apparent such as the relationship between the density and wind resistance. As a concluding point, both groups distinguished between relevant and irrelevant information based on their arguments as presented in the above dialogues *(collecting relevant information).*

Simplifying the situation, and flexibility and novelty. The findings from the *Mars Lunarcrete* activity showed that elementary gifted students simplified the situation by connecting the essential concepts. In this activity, the members of group D discussed the real-world situation from different perspectives. For instance,

> *Student 11*: Wind resistance should be considered because there can be sandstorms there. Although sand is such a small thing, it can swing homes.
> *Student 12*: Cost is also important information in order to transfer the material into Mars.
> *Student 10:* But there are too many factors we should consider. We have difficulty deciding. Let us start with the most important. I think strength.
> *Student 12:* You said the temperature differences in Mars. The building material we will make should be heat resistant.
> *Student 11:* Now, we eliminate any substance that can be affected by temperature.

The above excerpt shows that elementary gifted students simplified the situation by considering the given data and actual context. This simplified the situation to create a real model by referring to previous real-life experience *(simplifying the situation)*. Correspondingly, group D made associations among variables both in the given data and previous real-life experience *(flexibility)*. Moreover, they came up with original ideas to develop new and unique solutions by explaining why wind resistance is an important variable to simplify the situation *(novelty)*. Hence, elementary gifted students in group D displayed an *exemplary flexibility and novelty* modeling sub-competency since they offered a great deal of flexibility and generated original ideas to support their claim. As an engineering sub-competency of *simplifying the situation*, elementary gifted students determined the design aspect standards of the building material to ensure strength, heat resistance, and economy. They also documented the functional solution requirements to be accepted by corporation as an indicator of the engineering sub-competency of *simplifying the situation*. As *exemplary* indicators of the *ethical consideration* sub-competency in the above dialogue, they identified interested and affected parties and their expectations and also how engineering affects people and places.

On the other hand, group C simplified the situation using mathematical data in the recipe. For example:

> *Student 8:* We said we would add salt. First, let us eliminate the 2:1 regolith recipe because there is no salt in this recipe. The rest is all the same amount of salt.
> *Student 9*: Let's eliminate that too. The amount of water is too much.
> *Student 7*: This is also a little close, but the amount of water is too much. It also spreads like dough. We do not want to add too much water either. Now, the density should be high. So, we should add enough flour.
> *Student 9:* Wind resistance is very important. Let us try something less costly first.

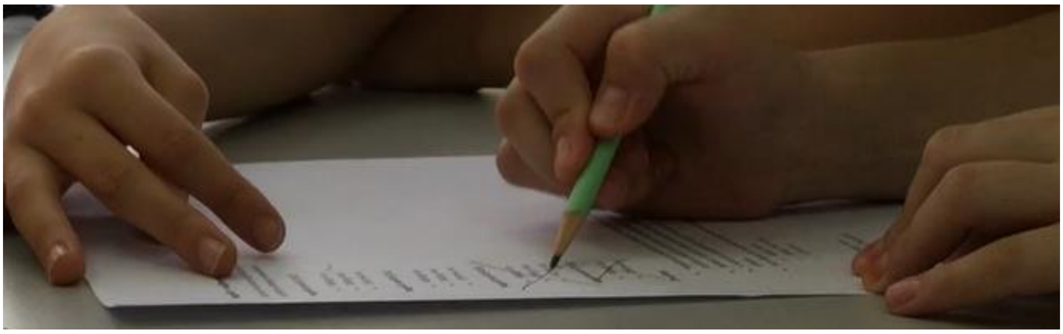


Figure 4.3. An example of group C to simplify the problem

As can be deduced from the dialogue and Figure 4.3, group C simplified the problem by reducing the regolith recipe. They compared mathematical information in the given data and eliminated them considering the important factors in creating a real model. This means that they made associations among variables *(flexibility)* but they did not come up with original ideas to develop new and unique solutions. Hence, group C displayed *proficient* evidence of the *flexibility and novelty* sub-modeling competency. In addition, they showed evidence of *ethical consideration* by considering possible approaches for the problem. The dialogues above also show evidence related to the engineering sub-competency that elementary gifted students in both groups identified acceptance criteria for work product and documented functional solution requirements to be accepted by corporation *(simplifying the situation)*. When all of the above-mentioned instances of both groups were taken into consideration, elementary gifted students exhibited *exemplary* indicators of the *simplifying the situation* modeling sub-competency by using multiple representations to explain and simplify the problem.

Finally, this part of the section presented the findings from groups E and F about the internalization of competency and related sub-competencies when they are engaged in the *Dr. Ahmet's Will* activity. The internalizing competency of elementary gifted students engaged in this activity was investigated in terms of the sub-modeling competencies: *understanding the problem*, *collecting relevant information*, *simplifying the situation*, *ethical consideration*, and *flexibility and novelty*. Moreover, the extent to which elementary gifted students in groups displayed the internalizing modeling competency and its sub-modeling competencies when they are engaged in the *Dr. Ahmet's Will* activity is shown in Figure 4.4. According to the group modeling competence observation guide, the sub-modeling competencies of the groups were classified as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)*, and 3 *(exemplary)*, proposed by de Villiers (2018).

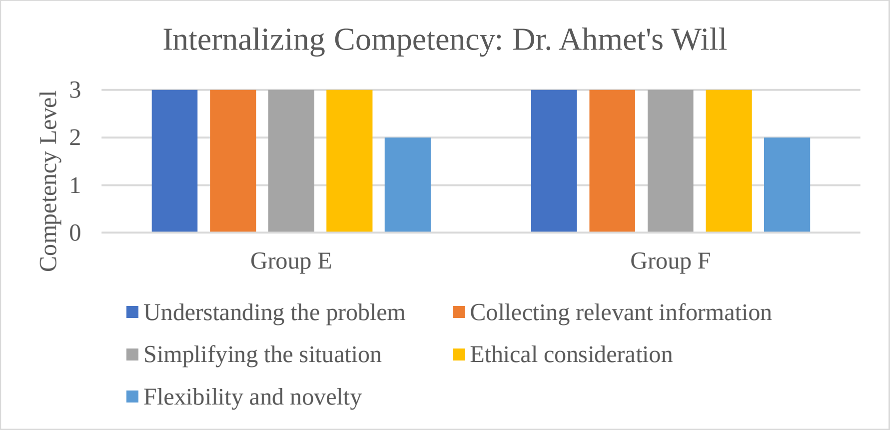


Figure 4.4. Internalizing Competency: Dr. Ahmet's Will

As seen in Figure 4.4, when elementary gifted students in groups E and F engaged in the *Dr. Ahmet's Will* activity, they generally exhibited *exemplary* modeling sub-competencies of internalizing, except for flexibility and novelty. From the graph above, it can be asserted that both groups displayed *proficient* evidence of the flexibility and novelty sub-modeling competency. In order to explain and expand on the components of internalizing competency, sample extracts, which are representative of each sub-competency, are given below.

Understanding the problem. The findings of the current study revealed that elementary gifted students made a concerted attempt to understand the problem. The members of group E attempted to make sense of what they had to do at the beginning of the problem investigation, but they did not address the essence of the problem situation. It seemed apparent to them that they should match the location of the relatives with one of the most appropriate cities in the data set. They did not internalize the problem in their attempt to select a common meeting point for all relatives. For instance:

> *Student 13:* I am not sure what the table is all about. There are travel times and flight costs.
> *Student 14:* There are six relatives here. But, there are five options to go.
> *Student 13:* Will two people go to the same city?
> *Student 15:* Let us try to choose the best option for each of our relatives.

This conversation indicates that the students initially had difficulty in understanding the main problem. When they probed further into the problem, the members of group E tried to clarify different information in the data set and problem situation *(understanding the problem)*. After a long discussion, all the group members agreed that six relatives should meet in a common place to read Dr. Ahmet's Will. They commented "*if someone did not come, Dr. Ahmet would donate his entire will to the foundation*". They thus explained and summarized the main issues in the problem situation. In a similar vein, the members of group F spent a significant amount of time reading the problem aloud, focusing on particular information for each city and attempting to understand what the numbers in the table meant. Hence, they gathered information required for problem analysis, a reflection of the engineering sub-competency of *understanding the problem*. Also, they made comparisons among cities to gain a better understanding of the problem by showing the position on a world globe sphere 3D model in Figure 4.5.

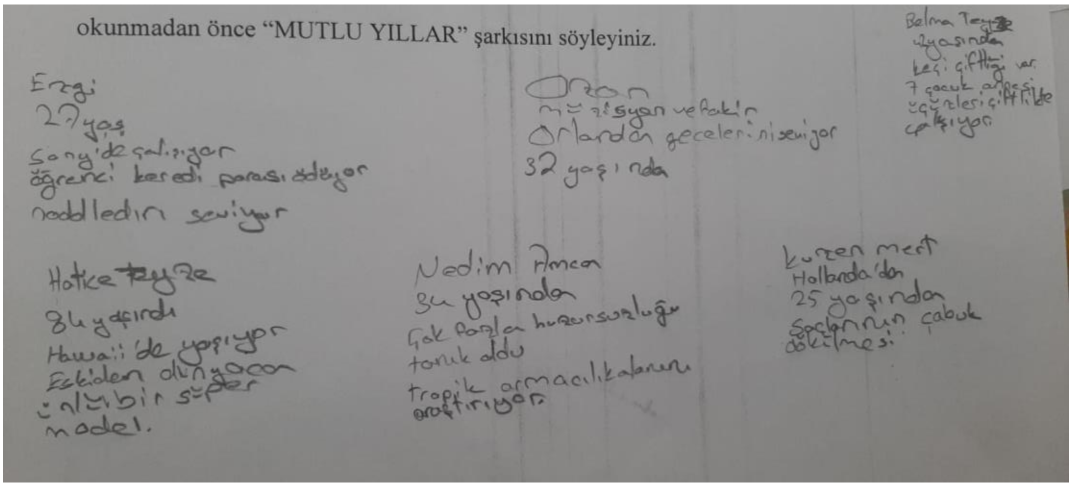
Figure 4.5. An example of group F trying to understand the problem

Then, they marked on the map the cities where the lawyer’s office is located and the cities where the relatives live as shown in Figure 4.5. Group F summarized the main problem as “*Dr. Ahmet died and left a will for us. On his birthday, April 1, his will can be read if one person from each family attends. On a rainy day, he also desires to have his request granted”.* This showed that they explained the main problem situation in their own words *(understanding the problem).* The above excerpts provide evidence that having the relatives meet at a common point and satisfying the conditions determined by Dr. Ahmet were the main issues in the problems for both groups. Thus, both groups exhibited *exemplary* evidence of the *understanding the problem* modeling sub-competency since they clearly identify the main issues and explicitly establish why they are problems.

Collecting relevant information and ethical consideration. Another sub-competency of internalizing obtained from the findings of elementary gifted students in the *Dr. Ahmet’s Will* activity is related to the collection of relevant information and ethical consideration. When examining the relevant information, both groups preferred to prioritize the given data. Elementary gifted students initially examined travel time and flight cost together. Other relevant information for working towards a solution was determined as the lowest temperature and average rainfall. As students tried to better understand the relevant information, they made comparisons among the importance of the information to select the best option for relatives. Both groups documented the functional solution requirements for the problem situation in terms of flight costs, travel time, temperature, altitude, snowfall, and pollution levels, indicative of the engineering sub-competency *(collecting relevant information)*. The following is a discussion example from among the members of group F:

> *Student 18:* First we have to look at travel time and temperature.
> *Student 17:* I think that travel time is more important.
> *Student 16:* I also think that the temperature is more important.
> *Student 18:* The degree of pollution is more important than rainfall.
> *Student 19:* Lastly, the amount of snowfall is important.
> *Student 17:* Altitude is also important. When places are higher, it rains more.
> *Student 18:* The snowfall was not that important anyway.

As can be seen in the above extract, elementary gifted students tried to reach a consensus on relevant information in this problem situation. Student 17 presented the relevance of the information for the problem situation with an explanation and the reason was associated with the desired situation. Hence, group F distinguished between relevant and irrelevant information to internalize the problem *(collecting relevant information)*. Besides, both groups considered the characteristics of each relative such as age, job, and where they live. Differently, group E took some notes about certain information about the problem as in Figure 4.6.


okunmadan önce "MUTLU YILLAR" şarkısını söyleyiniz.

Ezgi
27 yaş
Sony'de çalışıyor
öğrenci keredi parası ödüyor
naddledin seviyor

Orhan
müzisyen ve fakir
Orlando gecelerini seviyor
32 yaşında

Belma Teyze
44 yaşında
keçi çiftliği var.
7 çocuk annesi
üçüzlesi çiftlikte
çalışıyor

Hatice Teyze
84 yaşında
Hawaii'de yaşıyor
Eskiden dünyaca
ünlü bir süper
model.

Nedim Amca
54 yaşında
Çok fazla huzursuzluğu
tanık oldu
tropik ormancılık alanını
araştırıyor

kuzen mert
Hollanda'dan
25 yaşında
Saçlarının çabuk
dökülmesi

Figure 4.6. The notes of group E related to Dr. Ahmet's relatives

As shown in Figure 4.6, they recorded the name of the relatives, the age, professions, economic situation, where the relatives live, and certain characteristics such as raising seven children and premature hair loss. Hence, the elementary gifted students considered not only externally given data but also implied information related to the characteristics of each relative to internalize the problem. This could be accepted as exemplary evidence of the collecting relevant information modeling sub-competency for both groups. In light of all of this, these students clarified relevant information about the problem by identifying interested and affected parties and their expectations, and by considering the impact of decisions on people and places and so showed evidence of the exemplary ethical consideration modeling sub-competency.

Simplifying the situation and flexibility and novelty. The findings from groups E and F engaging in Dr. Ahmet's Will activity suggest that elementary gifted students progressed in simplifying the problem from its initial conditions to the specific problem based on the characteristics of the relatives (*simplifying the situation*). They also tried to eliminate some variables to reduce the options. The following dialogue among members of Group F could be given as an example:

> *Student 18:* Since there is no flight to Kanpur, it certainly will not be an option.
> *Student 19:* Why not? Do they have to fly; they can go by car or another way?
> *Student 16:* Maybe one of the relatives will come and get him and they will go together.
> *Student 17*: How should they go? Then it will be costlier.
> *Student 18*: Let's take a look at the map, shall we? Is another way possible?
> *Student 16*: Let this be our last option for now. We can reconsider other situations.

The above excerpts show that the members of group F offered the data a great deal of flexibility but did not come up with original ideas to develop a new and unique solution *(flexibility).* Although they proposed some ideas in their attempt to find a new way, they did not critically discuss their reasons. Hence, these ideas could be categorized as *proficient* evidence of the *flexibility and novelty* modeling sub-competency since they exhibited flexibility but not novelty to the problem situation.

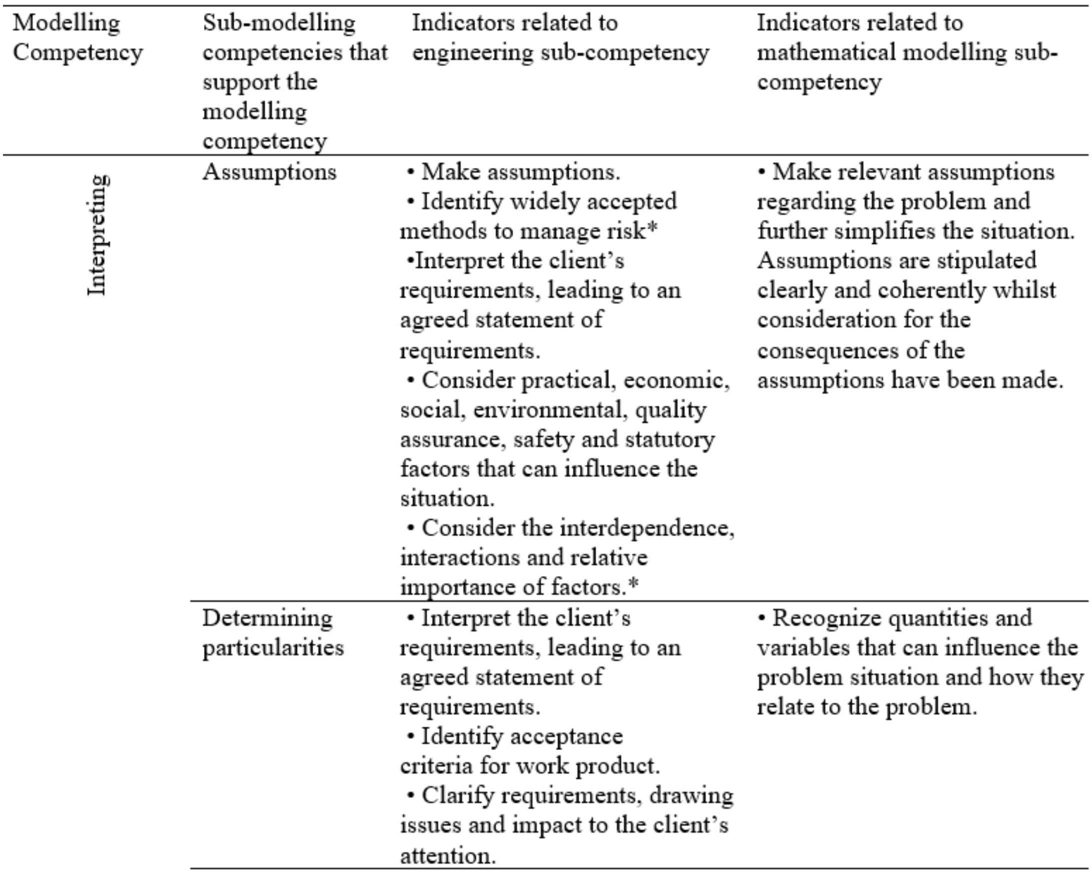
In addition to the excerpts above, group E eliminated Kanpur not only with no flight due to but also its high degree of pollution. Thus, elementary gifted students specified that certain pieces of information were irrelevant to provide the necessary condition. That is to say, they *simplify the situation* by connecting essential concepts within the problem. On the other hand, the members of group E found the total travel time of the relatives for each option in order to simplify the travel time of each relative to the location of the office. Hence, they applied mathematical operations to simplify the problem situation by connecting the essential concepts. This provided evidence regarding the *flexibility* modeling competency that group E made an association between the multiple representations of the data and the purpose of the problem. However, they did not generate original ideas in order to develop new and unique solutions. Thus, they displayed evidence of *proficient* demonstration of the *flexibility and novelty* modeling sub-competency. On the other hand, when all the above-mentioned instances for both groups are taken into consideration, they demonstrated *exemplary* evidence of the *simplifying the situation* modeling sub-competency by utilizing multiple representations to explain and simplify the problem.

To summarize, the preceding parts of this section presented the findings regarding the emerging categories of internalizing competency when elementary gifted students engaged in the engineering-based MEAs mentioned above in more detail. These sub-modeling competencies of internalizing covered *understanding the problem, collecting relevant information*, *simplifying the situation*, *ethical consideration,* and *flexibility and novelty*. Although the findings in the sub-modeling competencies of *understanding the problem*, *collecting relevant information,* and *simplifying the situation* were mostly classified according to the group modeling competency observation guide proposed by Villiers (2018), a new indicator related to a mathematical modeling sub-competency of simplifying the situation was added based on the emerging codes from the current study. In addition to the group modeling competency observation guide proposed by Villiers (2018), *ethical consideration*, and *flexibility and novelty* sub-modeling competencies emerged from the data of the current study. The findings of this study demonstrated that elementary gifted students in groups exhibited instances of *exemplary* engagement in engineering-based MEAs by understanding the problem, collecting relevant information, and simplifying the situation in order to internalize the problem. Beyond that, they displayed *exemplary* modeling competencies in specific instances of *ethical consideration* as well as *flexibility and novelty*.

4.2. Modeling Competency 2: Interpreting

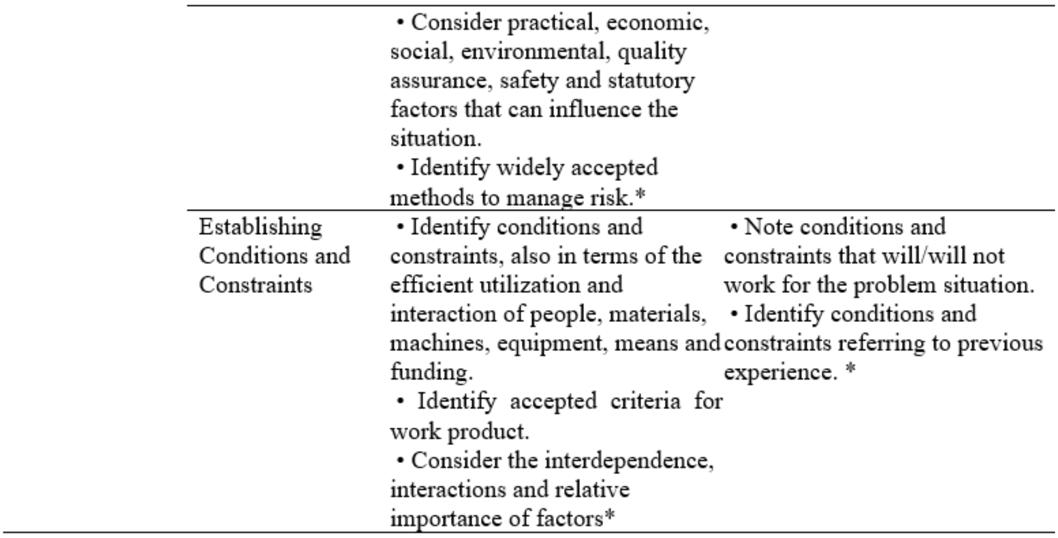
This section of the chapter investigates the findings for interpreting, the second modeling competency based on the classification of cognitive modeling competencies proposed by de Villiers (2018). The interpreting competency was analyzed with respect to the sub-modeling competencies: *assumptions*, *determining particularities,* and *establishing conditions and constraints*. Further, the extent to which elementary gifted students in groups displayed the interpreting modeling competency and its sub-modeling competencies when they are engaged in *Bridge Construction*, *Mars Lunarcrete*, and *Dr. Ahmet's Will* activities were investigated in this section. The findings for each activity included examples of two distinct groups exhibiting the sub-modeling competencies of elementary gifted students. In particular, the interpreting competency of the students was analyzed through these sub-modeling competencies and their related engineering and mathematical modeling sub-competency as shown in Table 4.2. In the table, new codes that emerged from the analysis of the current study are indicated with '*'.

Table 4.2. Classification of interpreting competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Interpreting	Assumptions	• Make assumptions. • Identify widely accepted methods to manage risk* •Interpret the client's requirements, leading to an agreed statement of requirements. • Consider practical, economic, social, environmental, quality assurance, safety and statutory factors that can influence the situation. • Consider the interdependence, interactions and relative importance of factors.*	• Make relevant assumptions regarding the problem and further simplifies the situation. Assumptions are stipulated clearly and coherently whilst consideration for the consequences of the assumptions have been made.
	Determining particularities	• Interpret the client's requirements, leading to an agreed statement of requirements. • Identify acceptance criteria for work product. • Clarify requirements, drawing issues and impact to the client's attention.	• Recognize quantities and variables that can influence the problem situation and how they relate to the problem.

Table 4.2. (continued)



	• Consider practical, economic, social, environmental, quality assurance, safety and statutory factors that can influence the situation. • Identify widely accepted methods to manage risk.*	
Establishing Conditions and Constraints	• Identify conditions and constraints, also in terms of the efficient utilization and interaction of people, materials, machines, equipment, means and funding. • Identify accepted criteria for work product. • Consider the interdependence, interactions and relative importance of factors*	• Note conditions and constraints that will/will not work for the problem situation. • Identify conditions and constraints referring to previous experience. *

As shown in Table 4.2, the analysis of this study revealed that the interpreting sub-competencies *assumptions*, *determining particularities*, and *establishing conditions and constraints*, proposed by de Villiers (2018), emerged as sub-codes. The indicators of interpreting sub-competencies in terms of engineering and mathematical modeling sub-competencies when elementary gifted students engaged in engineering-based MEAs are indicated in Table 4.2.

As the first example of engineering-based MEAs, the findings for the *Bridge Construction* activity related to the interpreting competency of groups A and B are presented regarding the sub-competencies of *assumptions*, *determining particularities,* and *establishing conditions and constraints*. Moreover, the extent to which elementary gifted students exhibited the interpreting modeling competency and related sub-modeling competencies when engaging in the *Bridge Construction* activity is illustrated in Figure 4.7. Based on the group modeling competency observation guide proposed by de Villiers (2018), the level of sub-modeling competencies for the groups was classified as 0 *(unsatisfactory),* 1 *(emergent/developing),* 2 *(proficient),* and 3 *(exemplary).*

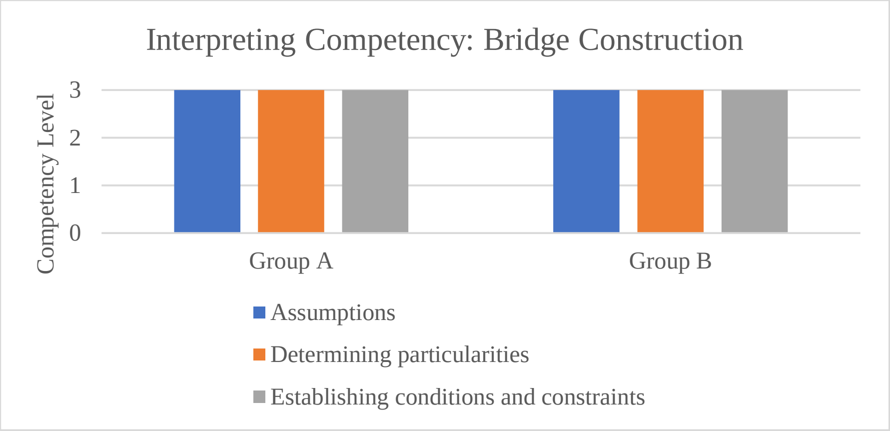


Figure 4.7. Interpreting Competency: Bridge Construction

As seen in Figure 4.7, when elementary gifted students in groups A and B engaged in the *Bridge Construction* activity, they exhibited exemplary sub-modeling competencies of interpreting. Examples of this competency in relation to the aforementioned sub-categories are presented below.

Assumptions. The elementary gifted students made *assumptions* drawn from real-life situations during the construction of their bridges. Their main focus was to make the bridge strong. This was revealed through their dialogue, as can be seen below:

> *Group A*: We thought about fixing abutments so that the road does not fall, how we will keep the road on abutments, then we thought about how to keep the road from above if the abutments fell down (Student 1).
> *Group B*: Abutments and arches hold the bridge (Student 5). If the abutments are broken, it would be held from the other side (Student 6).

This dialogue excerpt indicates that both groups made assumptions regarding the possibility of the bridge collapsing because of the failure of different parts and produced an alternative support mechanism for that condition, thus demonstrating evidence of the manage risk as engineering sub-competency. Excerpts for both groups also showed evidence of the engineering sub-competency in that they considered the interdependence, interactions, and relative importance of factors (assumptions). Not only did they clearly specify the assumptions but they also considered the consequences of the assumptions they made.

As stated above, both groups displayed *exemplary* evidence of the *assumptions* modeling sub-competency since they made innovative and insightful assumptions, and showed consideration for the consequences of the assumptions clearly and coherently.

Determining particularities. The analysis of the data revealed that elementary gifted students interpreted the client's requirements, leading to an agreed statement of requirements and reflecting the engineering sub-competency of *determining particularities*. For example, the students initially drew sketches of their bridges by defining predetermined requirements. As an engineering sub-competency of *determining particularities*, both groups identified accepted criteria in order to design their bridges in a way considerate of strength, durability, safety, and serviceability.

Specifically, the protocol proposed by Song and Agogino (2004) was used to characterize the design sketches of the elementary gifted students. In the analysis of the sketches in designing their bridges, the concept of annotation, "type of support notation metric, which includes labels, lists, narratives, dimensions and calculations" (Song & Agogino, 2004, p.2) guided the coding of sketches. As shown in the groups' sketches below (See Figure 4.8 and 4.9), both groups used basic and two-dimensional sketches. Their sketches included bridge measurements such as tall, width, and height coding. While group A labeled the materials used, the other group did not.

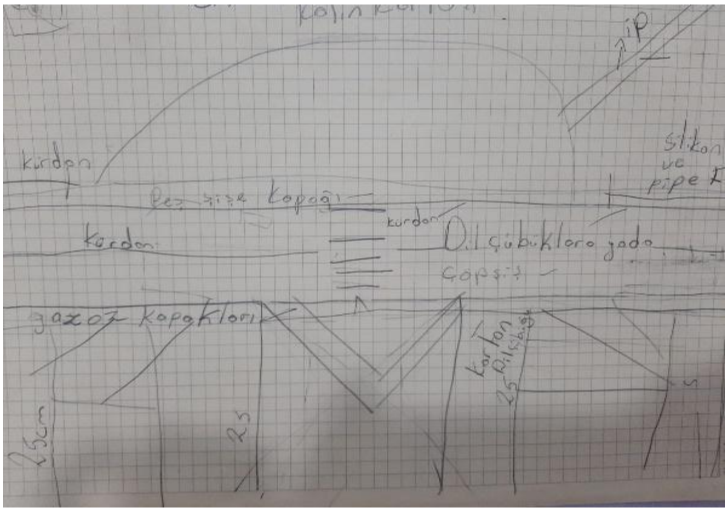


Figure 4.8. The bridge drawing of group A

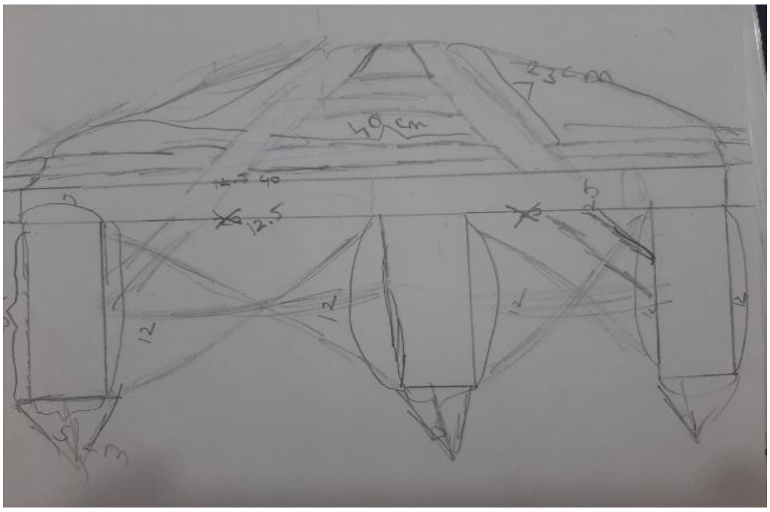


Figure 4.9. The bridge drawing of group B

As can be seen from Figures 4.8 and 4.9, elementary gifted students clarified requirements, drawing issues and impacting the client’s attention, demonstrating the engineering sub-competency of *determining particularities*. Moreover, both groups recognized the type of bridge, the materials, and the cost as important variables in the *Bridge Construction* problem. However, neither preferred to use only the given materials in the given data set. As stated before, they brought flexibility to the data related to materials and their cost without changing the scope of the problem. Thus, the students recognized quantities and variables that can influence the problem situation and how they relate to the problem (*determining particularities).*

In addition, these elementary gifted students reflected that their first priority was to make the bridge stable. Accordingly, they chose to use extra materials to reach this aim. Both groups considered economic factors that influenced the problem situation. In the following extract, the members of group B discussed the cost of the bridge they will construct:

> *Student 5*: It may be very expensive as it has to be strong. I estimated the cost at 1500 Turkish Liras (TL).
> *Student 6*: Is it 1500 TL? It might be 2500 TL.
> *Student 4*: 2500 TL makes sense to me.
> *Student 6*: How much is 1 kg of iron? We can calculate…

The above excerpt shows that they estimated the cost of the real-bridge rather than the prototype. They identified the accepted criteria related to the cost of the work product as engineering competency (*determining particularities*). Furthermore, group A found their bridge a little costly but they supported the idea that it will be worth what they do. Students' reflections on the cost of the bridge in their initial planning showed that constructing an economic bridge was not their first priority. On the other hand, both groups also considered practical, social, quality assurance, and safety factors that can influence the situation from different perspectives, an indicator of the engineering sub-competency (*determining particularities*). An instance of interpretation occurred as the members of group A specified the factors when they drew their bridge:

> *Student 2*: We draw a design sketch of our bridge like this. We will use pet bottle caps for those who go to one side and soda caps for those who go to the other side of the road. To avoid confusion and to prevent people from hitting each other.
> *Student 1*: We will also make a pedestrian crossing for people to cross to the other side if they take the wrong path, and we will use tongue sticks or toothpicks there.
> *Student 3*: We will also make the abutments with very thick cardboard. We will also support them with tongue sticks so that it does not fall out and open. These will be 25 cm.

The above could be accepted as evidence that members of group A gave detailed information about their drawing and care about the user-friendly bridge. As seen in the dialogue above, group A considered practical factors in terms of efficient utilization of the people, an indicator of the engineering sub-competency (*determining particularities*). Together these findings of both groups provide *exemplary* evidence regarding the sub-competency *determining particularities*, in that they recognized important quantities and variables in the problem and how they related to the problem considering practical, economic, social, safety, and quality assurance factors.

Establishing conditions and constraints. Elementary gifted students identified conditions and constraints, including the efficient use and interaction of people and materials, reflecting the engineering sub-competency *(establishing conditions and constraints)*. For example, the members of group A considered the maximum amount of material for each item and its cost as in the following excerpt:

> *Student 2*: The maximum amount of material we will use for tongue sticks is 300. The cost of it is 9.75 TL. Too expensive.
> *Student 1*: How can we construct our prototype?
> Student *3*: I can bring cardboard for the next session. I have one big enough at home. So, we can decrease the cost of bridges.

> *Student 1*: But, we can use the tongue stick for the road. Do you think it will work?
> *Student 2*: We think so, label the drawing. But, we will discuss when constructing the bridge.

This conversation above provided evidence of the engineering sub-competency that elementary gifted students identified cost as the acceptance criterion for work product *(establishing conditions and constraints)*. Different from group A, the members of group B discussed the cost of the material and decided to construct a small prototype for the peasants:

> *Student 5*: It is only a prototype, not a real bridge. We can make small prototypes.
> *Student 6*: Hmm... How can we demonstrate all the things we draw?
> *Student 4*: Its length is the same as a ruler. We do not need many materials to construct it.
> *Student 5*: Yes, in my opinion, putting two tongue sticks is enough according to our drawing.
> *Student 4*: So, we can construct it cheaper.

This snippet highlights the interpretation competency of elementary gifted students, how they investigate whether the conditions and constraints will work for the problem situation *(establishing conditions and constraints)*. Overall, the findings of this study revealed that both groups also considered the interdependence, interactions, and relative importance of factors to interpret the *Bridge Construction* activity, indicators of the engineering sub-competency *(establishing conditions and constraints)*. The dialogue excerpts from both groups also showed exemplary evidence related to the *establishing conditions and constraints* modeling sub-competency. They established clear conditions and constraints in terms of efficient utilization and interaction of people, material, equipment, means, and funding, as well as explanations for such conditions and constraints.

As a second example of engineering-based model eliciting activities, the findings of the *Mars Lunarcrete* activity related to interpreting competency of group C and D in the activity were investigated under the sub-competencies of *assumptions*, *determining particularities*, and *establishing conditions and constraints*. In addition, the extent to which the groups displayed the interpreting modeling competency and its sub-competencies when engaging in the *Mars Lunarcrete* activity are given in Figure 4.10. According to the group modeling competence observation guide proposed by de

Villiers (2018), the sub-modeling competencies of the groups for interpreting competency were classified as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)*, and 3 *(exemplary)* proposed by de Villiers (2018).

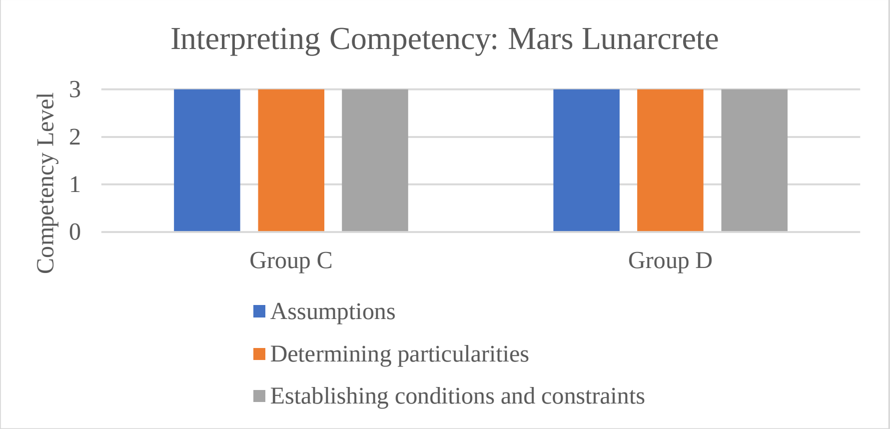


Figure 4.10. Interpreting Competency: Mars Lunarcrete

As illustrated in Figure 4.10, elementary gifted students in groups C and D generally displayed exemplary sub-modeling competencies of interpreting when they are engaged in the *Mars Lunarcrete* activity. In order to clarify and elaborate on the aforementioned categories, some specific instances of interpreting competency for each category are provided below.

Assumptions. Elementary gifted students made pertinent *assumptions* concerning the problem and simplified the situation even more. The following was an example of interpretation by members of group D while drawing assumptions on the *Mars Lunarcrete* activity:

> *Student 12*: First of all, the amount of building material we will make should be sufficient to establish a settlement on Mars. For example, we can build one house, but we do not have enough material to build the others. The spacecraft may need to be sent again, resulting in a higher cost.
> *Student 10*: The cost should be moderate. There must be more flour, otherwise, it will not be solid. It should turn into a solid.
> *Student 11:* Binder should be more. So it holds up better. Also, the regolith becomes like a brick. The structure becomes more durable and safer for people.

As shown in the dialogue above, elementary gifted students made their *assumptions* based on the variables that affect the desired situation in the problem. They not only stipulated *assumptions* clearly and coherently but also reflected the consequences of the assumptions they made. Correspondingly, the members of group D considered practical, economic, and safety factors that can influence the consequences of assumptions, indicators of the engineering sub-competency *(assumptions)*. On the other hand, group C interpreted the client's requirements. For instance:

> *Student 8:* We should develop a functional model to determine the ideal regolith and binder solution for Mars settlement.
> *Student 7:* The corporation has requested to create models of the colony. They want to use regolith to construct efficient, affordable condos, townhomes, and single-family houses on Mars.
> *Student 9:* The buildings must be durable and strong. It is also important whether they are airtight or not.
> *Student 8:* Comfort is also important because people will live there. Student 7 says we will build many types of houses for settlement. We need to create a model for people to live with their families.

The above excerpt showed that the students considered not only economic, social, and quality assurance factors that can influence the situation but also the usability of the model to live in the Mars. Hence, both groups made their *assumptions* considering the client's requirements to develop a real working model. Together these findings provided evidence regarding the engineering sub-competency of the *assumptions* modeling sub-competency that they considered the interdependence, interactions, and relative importance of factors. As seen in the dialogues above, the assumptions about airtightness for group C and the amount of building material for group D could be accepted as innovative and insightful instances. Thus, this can be asserted as *exemplary* evidence of the *assumptions* modeling sub-competency since they considered the consequences of their assumptions clearly and coherently beyond making innovative and insightful assumptions.

Determining particularities, and establishing conditions and constraints. The findings of this study showed that elementary gifted students determined particularities for the settlement on Mars by drawing. Both groups determined the accepted criteria in their attempts to create models of the colony, focusing on strength, durability and affordance, reflecting the engineering sub-competency *(determining particularities)*. Nevertheless, their drawings indicated that they determine the factors that can influence the settlement on Mars in a different way. Similar to the drawings in the Bridge Construction activity, Song and Agogino's (2004) concept of annotation, "type of support notation metric, which includes labels, lists, narratives, dimensions, and calculations" (p.2) guided the coding of sketches in the analysis of elementary gifted students' sketches in designing their settlement on Mars. Both groups used basic and two-dimensional sketches, as demonstrated in the sketches below.

In particular, the members of group C discussed different ideas when drawing their particularities. They firstly determined the building as shown in Figure 4.11. However, student 8 commended "*but there needs to be additions there, box chambers and so on*". This indicated that they need to clarify requirements, drawing issues and impacts to the client's attention, an indicator of the engineering sub-competency *(determining particularities).* Then, they agreed on the sketch of the building as shown in Figure 4.12.

> *Group C*: We are thinking of doing this. It is like that pyramid. It has solar panels on the sides and a generator inside. Power unit at the bottom. (Figure 4.12)

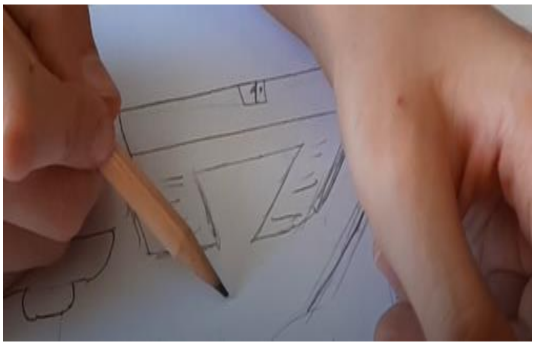


Figure 4.11. The first building sketch

Figure 4.12. The final building sketch

As shown in the excerpts from group C, they recognized the building, greenhouse, and power plant as important requirements for the settlement on Mars in the problem and specified the characteristics of these requirements by considering the condition of Mars. Thus, they exhibited *exemplary determining particularities* behavior since they stated how the variables relate to the problem situation considering practical, social, and environmental factors.

After they decided on the building, they *established the conditions and constraints* for a greenhouse and power plant. For example:

Student 7: What could be suitable as a greenhouse? Considering the conditions of Mars?
Student 9: In the movie The Martian, in his greenhouse, water was formed by steam in the greenhouse, and when they became water, the plants were watered. Thus, he was able to afford drinking water. For example, he was growing potatoes.
Student 8: There are also steps here. I think this drawing is suitable for a greenhouse. (See Figure 4.13)

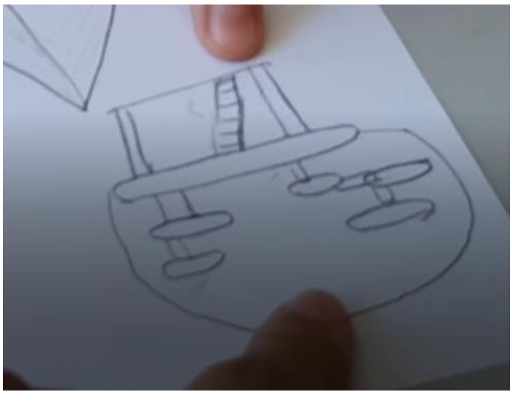


Figure 4.13. The sketch of greenhouse

As shown in the above excerpt, Student 9 referred to previous experience to *identify conditions and constraints*. The justifications of group C on drawings also showed that they considered the efficient utilization and interaction of people, material, equipment, and means, reflecting the engineering sub-competency *(establishing conditions and constraints)*. The following dialogue indicated that they established conditions and constraints with regards to efficient utilization. *"Student 9: That could also be a power plant in the shape of a pyramid* (See Figure 4.14). *Student 8: If it can produce energy, yes".*

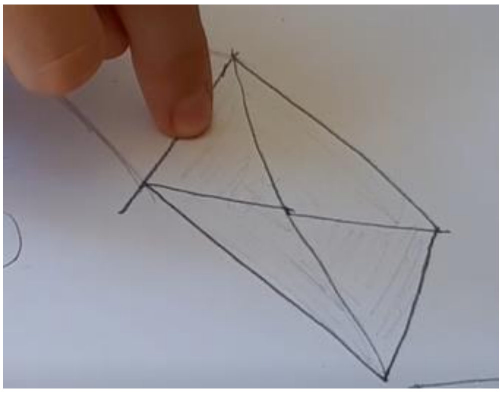


Figure 4.14. The sketch of power plant

As seen in the above dialogues, group C provided *exemplary* evidence of the *establishing conditions and constraints* modeling sub-competency since they established clear conditions and constraints in terms of efficient utilization and interaction of people, materials, equipment as well as explanations for such conditions and constraints. On the other hand, the justification of group D on their drawings revealed that they recognized different factors that can influence the settlement on Mars *(determining particularities)*:

> *Student 10:* This is the greenhouse (See Figure 4.15). There are such serums in the greenhouse. But the serum has more water capacity and the water goes faster, because all of them have to be distributed in the greenhouse. We put two for the least cost, and there are two fans to keep the air flowing inside.
> *Student 11:* The house we know is next to the greenhouse. We did not need to design it. It will be like the house we know, a regular house.
> *Student 12:* There is also a garage next to it. The garage has a forward sloping door. There is a folding door. We thought that the reason why it is inclined will be to be more resistant to the wind, since it will hit the direct winds in such a sloping way, not straight.
> *Student 11:* There will be spare materials in the garage, everything will be inside. For example, there will be vehicles. There will be backups of our fans here.
> *Student 10:* Because everything can go wrong at any time. We must be prepared for any situation. This is Mars.

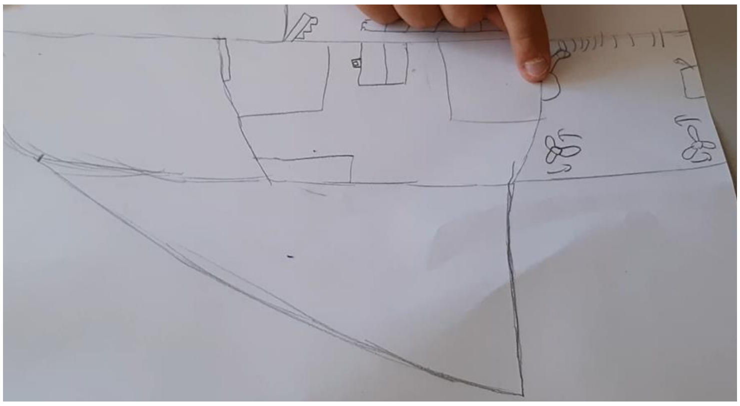


Figure 4.15. The design of the group D for the settlement on Mars

The conversations of group D demonstrate that they determined the particularities of greenhouse, building, garage, solar panel and basement by clarifying requirements, drawing issues and impacts to the client’s attention as engineering sub-competency *(determining particularities).* The above excerpt provided *exemplary* evidence of *determining particularities* that the members of group D considered practical, economic, social, environmental, quality assurance, and safety factors when recognizing important variables in the settlement on Mars and their relationship to the problem. The above dialogues also showed that the students identified widely accepted methods such as backup plans to manage risk, an indicator of the engineering sub-competency *(determining particularities)*.

Furthermore, the students in group D *established conditions and constraints* clearly as well as justifications for them. For instance:

> *Student 12:* Apart from that, there is a solar panel here (see Figure 4.16). The generator here collects the lights coming from here. The generator here collects the sun's rays as follows. That is how we use our vehicles from here. We will use a hair dryer to do it.
> *Student 10:* There is a small basement. There is food and more water in terms of storage if they are finished.
> *Student 12:* There is a second generator. Let us say we were using it in the evening, the light went out, this comes into play. It also gets its energy from the solar panel. It stores the energy here, and it sends it from here to here. When we run out of energy storage here, it goes to the one below.

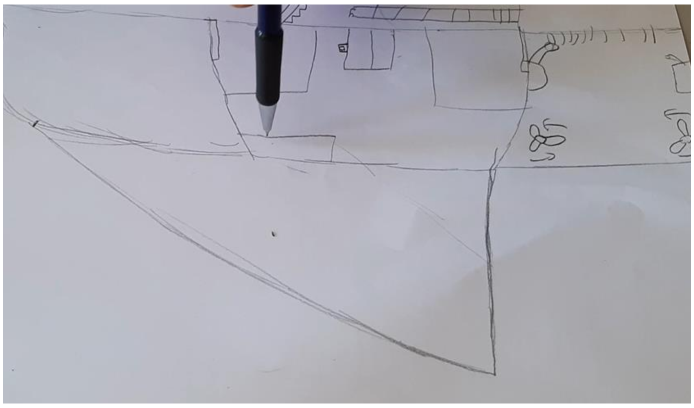


Figure 4.16. The solar panel design of the group D

As can be deduced from the dialogues above, the members of group D *established conditions and constraints* that will or will not work for the problem situation. Beyond that, they showed *exemplary* evidence of *establishing conditions and constraints* since they explained such conditions and constraints in terms of efficient utilization and interaction of people, material, machines, equipment, means, and funding. As a concluding point, it can be asserted that group D also provided evidence regarding the engineering sub-competency since they considered the interdependence, interactions, and relative importance of factors.

As the last example of engineering-based MEAs, the findings of the *Dr. Ahmet's Will* activity with respect to the interpreting competency of groups E and F were investigated. The sub-competencies of interpreting were categorized under the sub-title of *assumptions*, *determining particularities* and *establishing conditions and constraints.* In addition, the extent to which the elementary gifted students displayed the interpreting modeling competency and related sub-modeling competencies when engaging in the *Dr. Ahmet's Will* activity is presented as shown in Figure 4.17. According to the de Villiers's (2018) group modeling competence observation guide, the sub-modeling competencies of the groups for interpreting competency were classified as follows: 0 *(unsatisfactory),* 1 *(emerging/developing),* 2 *(proficient),* and *3 (exemplary).*

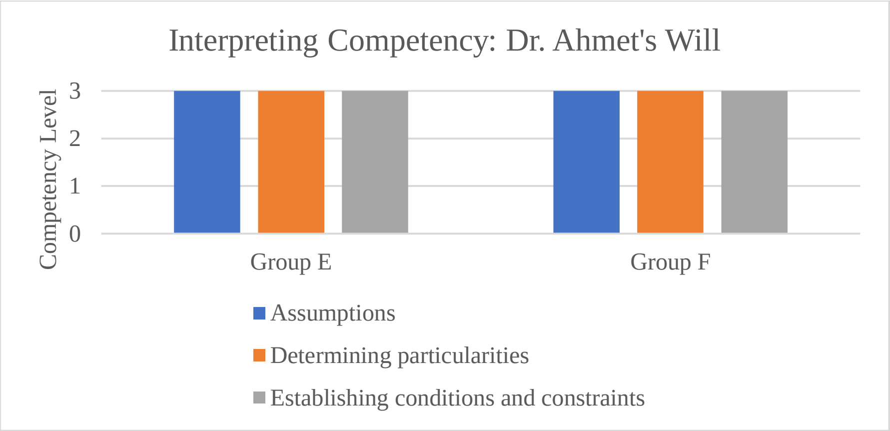


Figure 4.17. Interpreting Competency: Dr. Ahmet's Will.

As is shown in Figure 4.17, the findings of the current study revealed that the elementary gifted students in groups E and F displayed exemplary modeling sub-competencies of interpreting when engaged in the *Dr. Ahmet's Will* activity. As a result, the following part provides examples of this competency related to the sub-categories indicated above.

Assumptions. When interpreting the client's requirements, both groups made the *assumptions* that old persons could not withstand long journeys. In the following extract, the members of group F considered the consequences of the *assumptions* in terms of practical and social factors, an indication of the engineering sub-competency:

> *Student 18:* Actually, we expect it will rain more in Sydney, but Uncle Nedim's journey will take 49 hours.
> *Student 16:* I do not think Uncle Nedim can stand to go for that long.
> *Student 18:* Cousin Ozan arrives after 47 hours of travel.
> *Student 17:* It is a tremendous burden for Uncle Nedim.
> *Student 16:* Aunt Belma cannot stand to go to Kanpur with seven children, for example. We did not choose Kanpur from it either.

As can be deduced from the dialogue above, elementary gifted students further simplified the situation in terms of efficient utilization of people *(assumptions)*. In addition, both groups assumed that the financial situation of the relatives would afford them the ability to travel to the location they chose. Group E commented:

> *We considered the relatives' financial status as well. Cousin Ozan, for example, resided in the basement. Istanbul was a little budget-friendly for him. However, aunt Hatice's travel time to Istanbul was longer than her travel time to Geneva. We also investigated such factors.*

This showed evidence that elementary gifted students interpreted not only client requirements, leading to an agreed statement of requirements but also the particular characteristics of the relatives. The stated ideas above on assumptions can be accepted as indicators of innovative and insightful assumptions within the scope of the problem since they led the students to change the strategy of the solution. Thus, both groups indicated *exemplary* evidence of *assumptions* modeling sub-competency, in that they considered the consequences of the innovative and insightful assumptions in a clear and coherent way.

Determining particularities. The findings also showed that elementary gifted students recognized important quantities and variables that can influence the *Dr. Ahmet's Will* problem situation and how they relate to the problem *(determining particularities)*. The following was an example of interpretation used by members of group F to determine particularities:

> *Student 16:* Geneva seems the most suitable city for all relatives.
> *Student 18:* Most costly for whom*?*
> *Student 16:* It is costly for Cousin Ezgi, but I guess she works, too. I suppose Cousin Ezgi pays the loan as well.
> *Student 17:* Cousin Ezgi, after all, graduated from university and probably earns a certain amount of money.
> *Student 18:* She was employed. She still works at Sony.

> *Student 16:* She can pay this money because she works. She can save up and pay off this money for months.

As seen in the dialogue above, elementary gifted students identified economic factors influencing the problem and explained their relationship with the situation, a reflection of the engineering sub-competency. Similarly, the members of group E discussed the important variables in the problem as follows:

> *Student 13:* Geneva appears to be an appropriate location, as the travel time is short and the rainfall is not too unpleasant. It is already very high altitude. Due to the high altitude, there may be considerable rainfall.
> *Student 15:* If we had chosen Buenos Aires or Sydney, the costs and travel times would not have been suitable for the relatives.
> *Student 14:* In our opinion, Geneva is the cheapest and fits most of the factors.
> *Student 15:* Who will travel the longest?
> *Student 13:* Aunt Hatice, whose travel time is approximately 23 hours.
> *Student 14:* Can she withstand the journey that long?
> *Student 13:* I believe she can.
> *Student 15:* The flight cost of Geneva is available to all relatives.
> …
> *Student 13:* Also, Aunt Hatice lives in Hawaii, which is a warm place. If we had sent her to a cold place, she would probably get sick. There would also be rain, he might catch a cold and be sick, so she could not come to meet.

According to their conversations, elementary gifted students in both groups considered how practical, economic, social, and environmental factors would affect the situation beyond explaining the variables that could influence the situation and its relevance with the problem. This showed evidence that both groups displayed the *determining particularities* sub-competency in exemplary fashion.

Establishing conditions and constraints. Based on the findings of this study, both groups *established clear conditions and constraints* that will or will not work for the problem situation. An excerpt from group F reflecting their attempt to identify conditions and constraints in the *Dr. Ahmet's Will* activity is given below:

> *Student 17:* All the relatives should be together; this is our condition. The temperature degrees are also suitable for the rain.
> *Student 19:* The other condition is that only one person from each family will come.
> *Student 16:* I do not think any of the seven children have any expenses, except for water and toilet fees. Perhaps, they can stay at home if they are accompanied by an older person.

> *Student 19:* The triplets are under the age of twelve. As a result, she must also take them. That is why it is a little expensive for her.
> *Student 16:* For Aunt Hatice, the flight cost is $1340. Are the kids free?
> *Student 18:* Maybe.
> *Student 17:* She had goats, she could cover the cost by selling goat milk. Or if she sells the goats and buys donkeys, donkey milk is quite expensive.
> *Student 19:* Exactly, she can somehow cover the cost by selling donkey milk.
> *Student 18:* She has a farm and she can sell something from there.

Here, the students established clear conditions and constraints as well as explanations for them. An indicator of the engineering sub-competency, they also identified conditions and constraints in terms of the efficient utilization and interaction of people and funding. Considering the social factors that influence the situation, they determined the accepted criteria for the best location and suggested alternative solutions. Additionally, the members of group E considered the temperature and altitude conditions in which each relative was used to living and examined whether they would adapt to the conditions of their chosen location. For instance:

> *Student 15:* Let us compare cities by conditions. We said Geneva is our first option. Let us compare the temperatures.
> *Student 13:* Aunt Hatice can definitely endure. The highest temperature where she lives is 83 degrees. She can tolerate it.
> *Student 15:* Geneva appears to be in excellent condition at the moment, both at the highest and lowest temperatures.
> *Student 14:* But, the altitude is too high in Geneva.
> *Student 15:* Aunt Belma lives in Bogota at an altitude of 8612 ft.
> *Student 13:* Uncle Nedim, let us have a look at Nairobi. The highest temperature is 77 degrees. The lowest temperature is 56 degrees. While the highest temperature in Geneva is 56 degrees, it is the lowest there.
> *Student 14:* Both of them can handle it.
> *Student 13:* Now let us look at Cousin Mert in Amsterdam
> *Student 14:* But the highest temperature there is 55 degrees. Do you think you cannot tolerate it?
> *Student 13:* Of course it will. Then, that is ok. Let us look at Aunt Belma. The lowest temperature is 46 degrees and the highest temperature is 66 degrees in Bogota. She can come, too.
> *Student 14:* Cousin Ezgi also lived in Tokyo. The temperature is also ideal for her. But the altitude is too much for her. She will go from 59 ft. to 1210 ft.
> *Student 15:* So far, Geneva seems suitable for all conditions.
> *Student 14:* But the altitude is not suitable.
> *Student 13:* I think you’re right. However, the temperature is more important than the altitude.

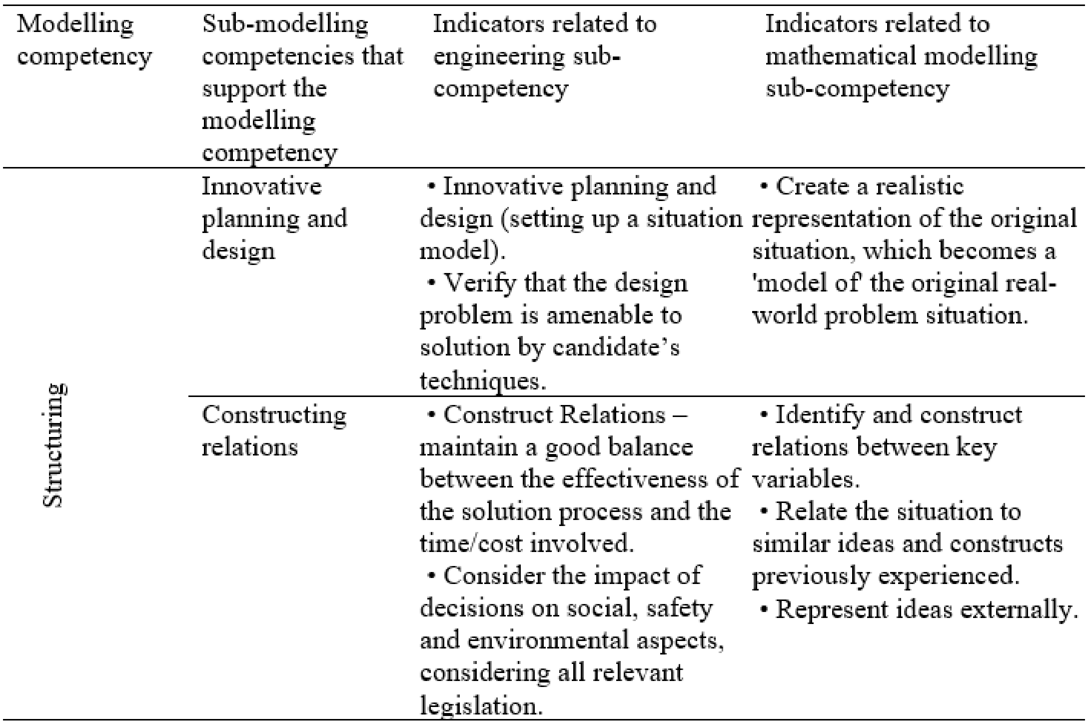
As can be understood from the extract presented above, the elementary gifted students identified conditions and constraints in terms of the efficient utilization and interaction of people. This could be accepted as indicators of *exemplary* demonstration of the *establishing conditions and constraints* sub-modeling competency. The dialogues from both groups presented above also showed evidence related to the engineering sub-competency, in that they considered the interdependence, interactions, and relative importance of factors for the problem situation *(establishing conditions and constraints).*

In summary, the second section of findings chapter investigated the elementary gifted students' sub-modeling competencies when they engaged in engineering-based MEAs, including *assumptions, determining particularities,* and *establishing conditions and constraints* under the interpreting competency. The group modeling competency observation guide proposed by de Villiers (2018) was revised during the analysis of the elementary gifted students' experiences in the engineering-based model eliciting activity and adapted according to the data analysis. In this scope, the findings of this study indicated that elementary gifted students in groups exhibited an *exemplary* modeling competency in determining particularities considering practical, economic, social, environmental, quality assurance, and safety factors. Besides these, they established conditions and constraints in terms of the efficient utilization and interaction of people, materials, machines, equipment, means, and funding. Such explanations were added to the relevant dimensions in line with the findings obtained from this study. On the other hand, elementary gifted students in groups displayed *exemplary* modeling competency in making innovative and insightful assumptions by consideration for the consequences of the assumptions in real life. In addition, the indicators related to the engineering sub-competency of the internalizing competency were included in the group modeling observation guide based on the analysis of the current study.

4.3. Modeling Competency 3: Structuring

The findings of the third modeling competency, structuring, are reported in this section based on the classification of cognitive modeling competencies proposed by de Villiers (2008). The structuring competency was examined as part of the sub-modeling competencies of *innovative planning and design*, as well as *constructing relations*. In addition, the extent to which elementary gifted students in groups displayed the structuring modeling competency and its sub-modeling competencies when they are engaged in Bridge Construction, Mars Lunarcrete, and Dr. Ahmet’s Will activities are provided in this section. In each activity, examples of two different groups are presented to reflect the sub-modeling competencies of elementary gifted students. In particular, the structuring competency of elementary gifted students is analyzed through its sub-modeling competencies and their indicators related to the engineering and mathematical modeling sub-competencies as indicated in Table 4.3.

Table 4.3. Classification of structuring competency framework



Modelling competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Structuring	Innovative planning and design	• Innovative planning and design (setting up a situation model). • Verify that the design problem is amenable to solution by candidate’s techniques.	• Create a realistic representation of the original situation, which becomes a 'model of' the original real-world problem situation.
	Constructing relations	• Construct Relations – maintain a good balance between the effectiveness of the solution process and the time/cost involved. • Consider the impact of decisions on social, safety and environmental aspects, considering all relevant legislation.	• Identify and construct relations between key variables. • Relate the situation to similar ideas and constructs previously experienced. • Represent ideas externally.

Specifically, as indicated in Table 4.3, the findings of this research demonstrated that the structuring sub-competencies of *innovative planning and design* and *constructing relations,* provided by de Villiers (2018), emerged as sub-codes. When the elementary gifted students engaged in engineering-based MEAs, the indicators of the structuring competency in terms of engineering and mathematical modeling sub-competencies that emerged are presented in Table 4.3.

First of all, the findings from groups A and B regarding the Bridge Construction activity were provided as an example of the structuring competency in the engineering-based MEAs. In addition, the extent to which elementary gifted students displayed the structuring modeling competency and its sub-modeling competencies of *innovative* *planning and design* and *constructing relations* when engaging in the Bridge Construction activity was illustrated in Figure 4.18. Using the group modeling competency observation guide suggested by de Villiers (2018), the sub-modeling competencies of the groups for structuring competency were classified into four categories: 0 *(unsatisfactory)*, 1 *(emerging/developing)*, 2 *(proficient)*, and 3 *(exemplary)*.

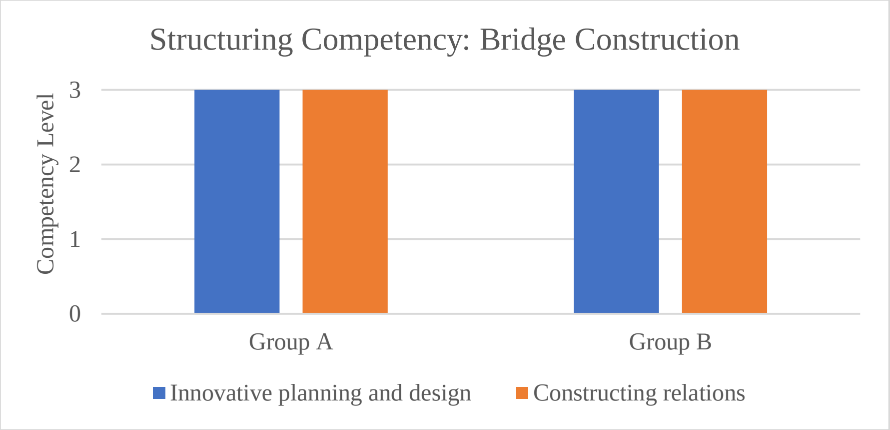


Figure 4.18. Structuring Competency: Bridge Construction

As is shown in Figure 4.18, the findings of the current study revealed that the elementary gifted students in groups A and B offered an *exemplary* display of the modeling sub-competencies of structuring when engaged in the Bridge Construction activity. The following provides examples of this competency related to the sub-categories indicated above.

Innovative planning and design, and constructing relations. The findings of the data showed that elementary gifted students consider the impact of decisions in constructing their bridges. In particular, the members of group A substantially conformed to their initial design and sketch. While constructing the first bridge design, the group changed the measurement of abutments. They commented as in the following:

> *Student 3*: Now, I cut the 25 cm for abutment. It is difficult to cut cardboard. There will not be exactly 25 cm because of cutting.
> *Student 2* and *Student 1*: We pasted the parts you cut. But 25 cm is too long for height. Can we change it to 15 cm?
> *Student 3*: We can. The suspension bridge was 10 meters high from the stream. We should also change the width. Do you agree?

> *Student 1*: 2.5 cm can be better. Is it ok for you?
> *Student 2 and 3*: Ok.

This excerpt indicates that the group tried to create a realistic representation of the original situation (*innovative planning and design)*. The above excerpt also showed that group members related the situation to real life in the decision-making process. Hence, this could be *exemplary* evidence for the *innovative planning and design* since the students differentiated the plan considering the original real-world situation. On the other hand, they received approval for group members to justify their design solutions. This showed evidence that they represent ideas externally to group members *(constructing relations)*. In addition, they stated the following:

> *Group A*: In the preliminary plan, we plan to use pet bottle caps for those who go to one side and soda caps for those who go to the other side of the road. However, we gave up on using them. Instead, we decided to use an arrow for pedestrian crossing. Also, we prefer to make the middle abutment bigger than the other abutments to make the bridge stronger.

Student A disagreed with the other group members, stating that "*I prefer to make one sided road. I think pedestrians must go as they want, making pedestrian crossing unnecessary*". However, group members convinced student A to make it to avoid confusion. This showed evidence that they valued each other's perspectives and worked collaboratively. On the other hand, the dialogues above indicate that they maintained a balance between the effectiveness of the solution process and the time involved, an indication of the engineering sub-competency *(constructing relations).* Consequently, they constructed an arch bridge. They stated the reason of why they construct an arch bridge as:

> *Student 1*: If we build an arch bridge, we can also support the bridge from the other sides. If the abutments of the bridges collapse, the bridge can be used longer with the support of an arch.
> *Student 2*: We used 2 small straws and a square cardboard to fix the arch. It can perform the same function as the abutments of the bridge.

Such reflections show that the students considered social, safety and environmental implications of their decisions, an indicator of the engineering sub-competency of *constructing relations*. In addition, they considered the interdependence, interactions, and relative importance of various factors as stated in the above dialogue. Thus, these could be understood as *exemplary* indicators of the *constructing relations* sub-modeling competency for group A. They tried to make alternative support mechanisms in the case of the collapse of the bridge abutments. As stated in their initial plan, they made two pedestrian roads and two intersections. Although the height of the abutments is equal in the drawing, the final version made the bridge more durable in their opinion, as can be seen in Figure 4.19. Taken together, the members of group A recognized and connected essential concepts about the Bridge Construction problem when setting up a situational model *(constructing relations).*

Figure 4.19. The structuring of bridge for group A

Similar to group A, group B also changed the abutments of the bridge. They commented:

> *Group B*: In the initial planning, we thought the width of the abutment would be 5 cm. However, the amount of the material was not enough to make the road. So, we made 7 abutments thinner than we planned. So, our bridge is not the same as the sketch of the bridge. There are several arches in our sketch. Other parts are slightly similar. The abutments are connected together by wires. But we prefer to use more abutments because our wires are not very strong for connection. We think that more abutments can be stronger than our initial planning.

Their reflections indicated that the members of group B considered the impact of decisions on safety aspects, which is an indication of the engineering sub-competency *(constructing relations).* When constructing their bridge, they had difficulty applying their plan and changed their plan because of the nature of the material. Due to the elementary gifted students' differentiation of the plan, this might be considered an exemplary indication of *innovative planning and design*. For example:

> *Student 4*: We were not able to realize all our intentions. We wanted to build a wire arch over the girder bridge.
> *Student 5*: However, we could not cut the wire, instead we decorated the bridge with LED lights and beads.

As can be understood from the dialogue above, the group differentiated the design as shown in Figure 4.20 when setting up the situational model, taking not only essential concepts but also aesthetic aspects in mind. Thus, group B also exhibited an exemplary display of the constructing relations sub-modeling competency, since they generated the prototype to solve the *Bridge Construction* problem considering the interdependence, interactions, and relative importance of factors. The dialogues above show evidence from both groups related to the engineering sub-competency of *innovative planning and design*, in that they are able to solve the design problem using techniques that they developed.

Figure 4.20. The structuring of bridge for group B

Secondly, the structuring competency of group C and group D in the *Mars Lunarcrete* activity was investigated in terms of *innovative planning and design* as well as *constructing relations*. In addition, the extent to which these elementary gifted students exhibited the structuring modeling competency and its modeling sub-competencies when engaged in the activity is shown in Figure 4.21. In accordance with the group modeling competence observation guide proposed by de Villiers (2018), the sub-modeling competencies of the groups for structuring competency are classified into four categories: 0 *(unsatisfactory)*, 1 *(emerging/developing)*, 2 *(proficient)*, and 3 *(exemplary)*.

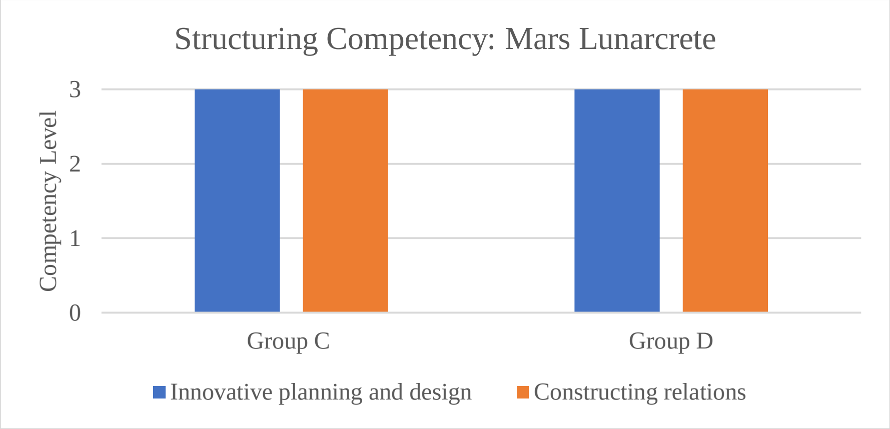


Figure 4.21. Structuring Competency: Mars Lunarcrete

As presented in Figure 4.21, when elementary gifted students in groups C and D engaged in the *Mars Lunarcrete* activity, they provided an *exemplary* display of the sub-modeling competencies of structuring. The following presents the instances of this competency in relation to the aforementioned sub-categories.

Innovative planning and design, and constructing relations. The findings from the *Mars Lunarcrete* activity show that the elementary gifted students did not strictly depend on the given data. In order to create a realistic representation of the original situation, they compared the recipes and tried to make an inference. Hence, they showed *exemplary* indicators of *innovative planning and design* for group D by differentiating the given data. For example:

> *Group D:* We chose C from data set 1 and 4:3 from the recipe. We will try it.
> *Student 11:* Actually A is the best but very expensive. It says 10.9 for regolith, but I will convert it to 11. 11 million dollars. Serious money when you include the cost of transferring it to Mars.
> *Student 12:* So we chose C. Because it ranks second among the other options in terms of wind resistance and the density is also high. The money is cheap, 4.4 million. Better than D even though it is 7.1 million.
> *Student 10:* The ratio of C is normally 2:1 but we choose 4:3. Since we thought that the materials in 4:3 could be more and their density could be more. Hence, it protects itself better.
> *Student 12:* The 2:1 ratio is smaller than 4:3 and may not be enough to build houses. The cost of sending the spacecraft increases.

> *Student 11:* The binder needs to be more, so we chose to do this. So it holds better.

As deduced from the dialogue above, elementary gifted students identified and *constructed relations* between key variables such as density, wind resistance, and cost. In addition, they considered the interdependence, interactions, and relative importance of various aspects, indicators of the engineering sub-competency, and explained the consequences of actions (*constructing relations)*. This could be accepted as *exemplary* evidence of the *constructing relations* modeling sub-competency for group D.

Consequently, the members of group D created a model to determine the ideal regolith and binder solution for the Mars colony in accordance with the conditions, constraints, and assumptions they established before (*innovative planning and design)*. Besides, they tried to reach a consensus with their group members before applying the recipe. For example:

> *Student 12:* We discussed the use of corn flour before. But we have not discussed this issue again. What do you think about it? Do you think we should add some? Student G, did you try the recipe with corn flour?
> *Student 10:* I tried and now, I have some idea about the nature of corn flour. But, we will discuss it together.
> *Student 11:* If this recipe (4:3) does not work, let us consider trying it later.
> *Student 10 and 12:* That is nice. Let us try and see.

The above dialogue shows that they reached a consensus on the recipe they followed and represented ideas externally (*constructing relations)*. This also indicates evidence that they approached the design process with the belief that products and designs may be improved and solved with their own techniques, reflecting the engineering sub-competency (*innovative planning and design)*. Correspondingly, group D prepared their models using 800 grams of flour, 200 grams of salt, and 930 ml of water. They used a beaker to measure the amount of water. Also, they put the flour and salt in the beaker and weighed it including the tare weight, with a precision balance as shown in Figure 4.22.

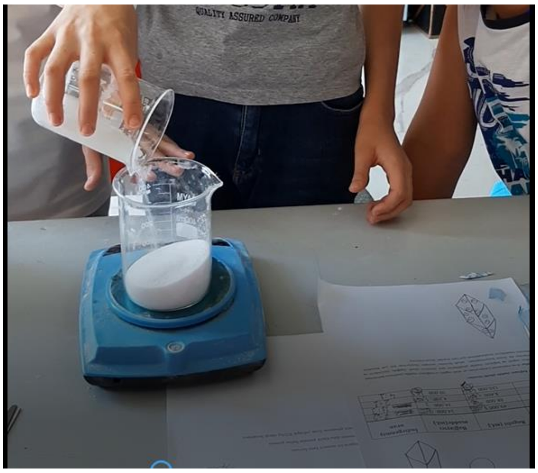


Figure 4.22. Preparation of group D’s model

In a similar vein, the members of group C did not prefer to use the ratio of the chosen regolith and binder solution. Instead, they tried to create a working model of the original situation based on their simplification. Such ideas can be accepted as evidence that group C displayed an *exemplary* demonstration of the modeling sub-competency regarding *innovative planning and design*. For example:

> *Student 8:* A is the best but very costly.
> *Student 9:* C is the second best option
> *Student 7:* But its density is very low compared to A.
> *Student 9:* We chose the 7:2 recipe, it seemed the most logical to us.
> *Student 8:* If not, we will choose A and go bankrupt. We will try 4:3 in that condition.
> *Student 9:* 400 g of flour and 250 ml of water. So do you think it makes sense? Such a recipe.
> *Student 7:* But we eliminated this recipe before because there is no salt.
> *Student 9:* Now, it does not make sense.
> *Student 7:* Lunarcrete can pass through the air.
> *Student 8:* Although it has half the durability of concrete, it can be done if the pressure and temperature differences on Mars are considered.

The above excerpt revealed that elementary gifted students set up a situation model considering the interdependence, interactions, and relative important factors such as the nature of the material, pressure, and temperature differences. Hence, this demonstrates *exemplary* evidence of the *constructing relations* sub-modeling competency for group C. Although they simplified the situation and determined particularities before structuring, they reconsidered their decisions and reached a consensus for the first model. Then, group C prepared their models using 500 grams of flour, 200 grams of salt, and 250 ml of water. Similar to group D, they used a beaker to measure the amount of water. Also, they put the flour and salt in the beaker and weighed it with tare weight with a precision balance. After that, they added the ingredients and kneaded the dough as shown in Figure 4.23. They continued the kneading process until all the ingredients were mixed together as seen in Figure 4.24.

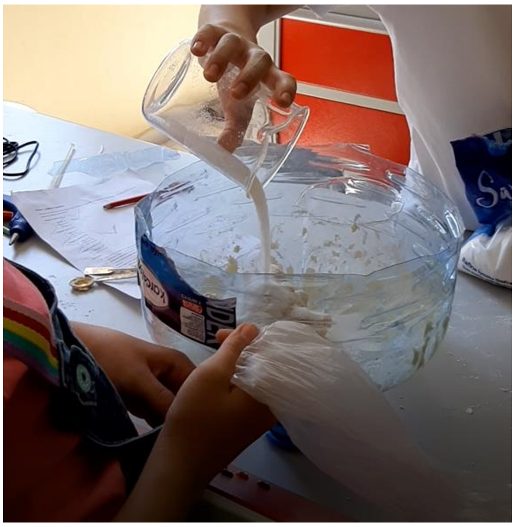


Figure 4.23. Preparation of group C’s model

Figure 4.24. Kneading process of group C

Finally, the structuring competency of groups E and F was examined in the context of the *Dr. Ahmet’s Will* activity in terms of innovative planning and design as well as constructing relations. The distribution of each sub-competency regarding the extent to which elementary gifted students in groups E and F displayed the structuring modeling competency is represented in Figure 4.25. Based on the group modeling competency observation guide proposed by de Villiers (2018), the level of sub-modeling competencies for the groups was classified as 0 (unsatisfactory), 1 (emergent/developing), 2 (proficient), and 3 (exemplary).

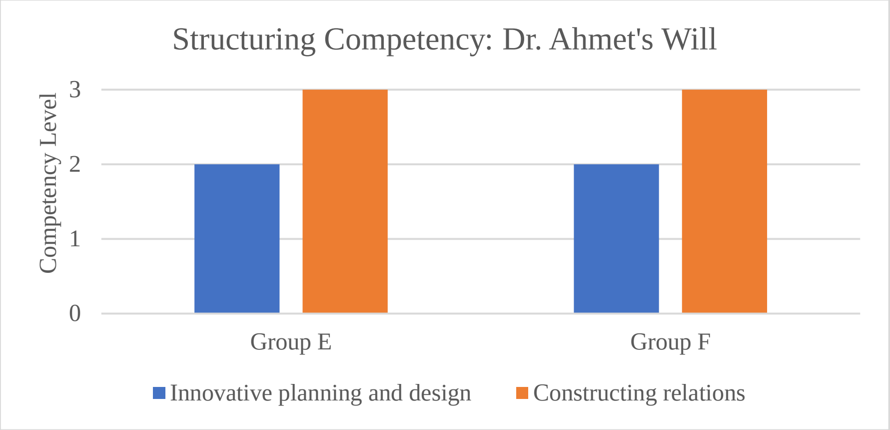


Figure 4.25. Structuring Competency: Dr. Ahmet’s Will

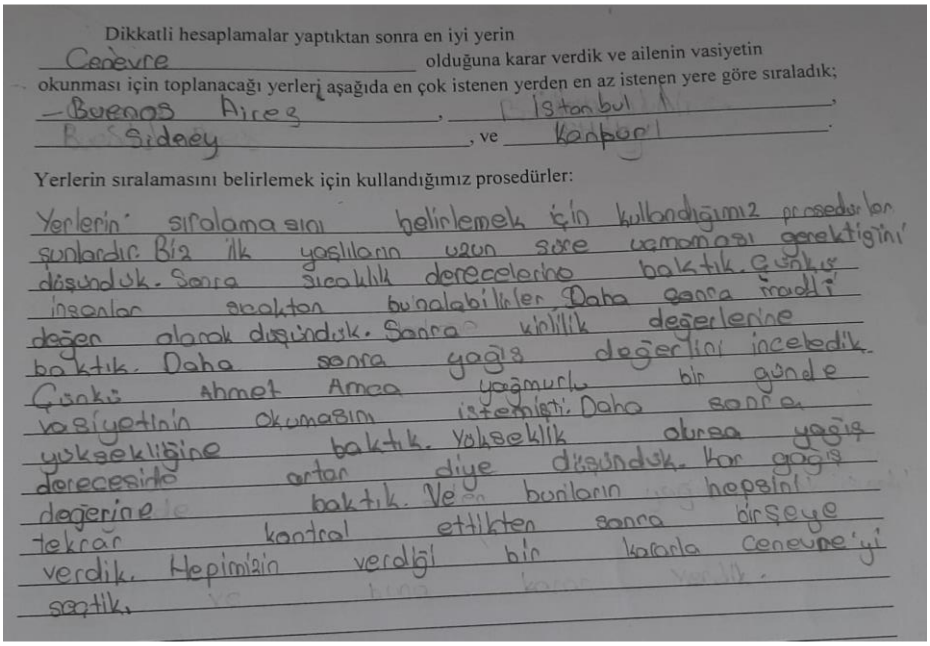
As is shown in Figure 4.25, the findings of the current study revealed that, with the exception of *innovative planning and design*, elementary gifted students in groups E and F displayed exemplary demonstrations of the modeling sub-competencies of structuring when engaged in the *Dr. Ahmet’s Will* activity. When the *innovative planning and design* modeling sub-competency is considered, both groups are seen to exhibit a proficient level of modeling sub-competency as stated in the above illustration. As a result, the following part provides examples of this competency related to the sub-categories indicated above.

Innovative planning and design, and constructing relations. The findings obtained from the data revealed that elementary gifted students generated a realistic representation of the original situation, which served as a model of the original real-world problem situation (*innovative planning and design)*. Both groups related the situation to similar ideas and constructed previously experienced (*constructing relations)*. When setting up a situational model, they used not only the given data but also the characteristics of the relatives. However, it was not regarded as exemplary evidence of *innovative planning and design* because it did not provide a novel approach to the problem. Hence, it was accepted that both groups exhibited a proficient level of the *innovative planning and design* modeling competency since their situational model was complete and accurate.

Although the first and last options of both groups were the same, the other options were different because of their preferences. As previously indicated, both groups considered Kanpur as the last option due to the lack of flights and the high level of pollution as well as the special status of aunt Belma. After critical examinations and long discussions, both groups agreed on Geneva as the first option. The next extract also highlighted the process of structuring, as the members of group F were discussing their situational model for explaining the problem:

> *Student 19:* I think the second option would be Sydney. Sydney offers moderate temperatures, average rainfall and little pollution. Let us examine it.
> *Student 18:* The highest rainfall is already there.
> *Student 19:* But Sydney's travel time is much longer than other places.
> *Student 16:* Aunt Hatice travels 10 hours.
> *Student 17:* However, uncle Nedim's travel time is 49 hours, cousin Ozan's is 47 hours, cousin Mert's is 39 hours, and aunt Belma's is 30 hours.
> *Student 16:* How long do they go to Geneva? That's right, Uncle Nedim cannot handle it.
> *Student 17:* In the second option, I think the travel times of the relatives should be less than 25 hours. If it exceeds 25, it will not be suitable.
> *Student 18:* How is Istanbul?
> *Student 19:* Let us take a look at Istanbul. I believe that it is appropriate. The degree of pollution is 3, which is still less than 5.
> *Student 17:* The rainfall is 1.9 inches
> *Student 16:* It is more than 1. Let Istanbul be the second option.
> *Student 17:* Then, let us have a look at the flight cost to Istanbul.
> *Student 16:* \$719, \$239 and so on. I think it is appropriate.
> *Student 17:* I think so.
> *Student 16:* Cousin Ozan is a bit poor, but he can easily pay \$831.
> *Student 17:* Absolutely.
> *Student 19:* The level of pollution in Istanbul is higher than the others.
> *Student 17:* We definitely sifted the Kanpura. The level of pollution is the highest.
> *Student 16:* There is no flight anyway.
> *Student 18:* So, do we put Sydney in the third option?
> *Student 19:* I think Sydney should be last, considering the travel times.

This conversation above indicates that the students identified and *constructed relations* between key variables. As an indicator of the engineering sub-competency, the members of group F maintained a good balance between the effectiveness of the solution process and the time/cost involved (*constructing relations)*. They also considered the impact of decisions on social, economic, and environmental aspects (*constructing relations)*. When determining a suitable location for the relatives, they considered the special cases of the relatives such as their age, economic situation, and familiar environment in addition to more common factors such as cost, travel time, weather conditions, and pollution level (*constructing relations)*. In the first letter of group F to the lawyer as shown in Figure 26, they created a general procedure to solve the problem.



Dikkatli hesaplamalar yaptıktan sonra en iyi yerin Cenevre olduğuna karar verdik ve ailenin vasiyetin okunması için toplanacağı yerleri aşağıda en çok istenen yerden en az istenen yere göre sıraladık;

-Buenos Aires, İstanbul,
Sidney, ve Kanpur.

Yerlerin sıralamasını belirlemek için kullandığımız prosedürler:

Yerlerin sıralamasını belirlemek için kullandığımız prosedürler şunlardır. Biz ilk yaşlıların uzun süre uçmaması gerektiğini düşündük. Sonra sıcaklık derecelerine baktık. Çünkü insanlar sıcaktan bunalabilirler. Daha sonra maddi değer olarak düşündük. Sonra kirlilik değerlerine baktık. Daha sonra yağış değerlerini inceledik. Çünkü Ahmet Amca yağmurlu bir günde vasiyetinin okumasını istemişti. Daha sonra yüksekliğine baktık. Yükseklik olursa yağış derecesinde artar diye düşündük. Kar yağış değerine baktık. Ve bunların hepsini tekrar kontrol ettikten sonra birşeye verdik. Hepimizin verdiği bir kararla Cenevre'yi seçtik.

Figure 4.26. The first letter of group F

As presented in Figure 4.26, the students in group F stated that old people should not travel for a long time. Also, they supported the idea that people may be overwhelmed by the high temperature. Then, they reported that they examined flight cost, pollution level, and rainfall, respectively. This showed evidence that they considered the specific conditions of the problem after determining a general procedure for solving related problems. Thus, this could be accepted as exemplary evidence of *constructing the relations* for group F because they generated a general procedure to solve the problem, taking the interdependence, interactions, and relative importance of factors into account.

Regarding the students' own reflections, the members of group E examined the travel time and flight cost initially. Then, they tried to find a strategy by making the problem specific to individuals (*innovative planning and design)*. As stated in the dialogue in the interpreting competency section above, group E examined the living conditions of each relative in terms of weather conditions and altitude and compared the location where they live with the location they will choose. Different from group F, they created a realistic representation of the original situation for this specific situation (*innovative planning and design)*. For example:

> *Student 14:* Have you seen that everyone is going to Geneva in a short time? 11 hours, 1 hour and the remaining 23 hours, 16 hours, and 12 hours.
> *Student 15:* I think this is suitable for all. The temperature is also manageable.
> *Student 13:* Do you believe that 23 hours of travel time is reasonable?
> *Student 14:* I think it is appropriate, compared with the others.
> *Student 15:* For instance, if we chose Kanpur, how will Aunt Belma manage 54 hours of travel with seven children?
> *Student 14:* But, let us have a look at the flight cost to Geneva.
> *Student 15:* $723, $134, $1262, $1407, $1155
> *Student 13:* However, Aunt Hatice is 85 years old.
> *Student 14:* Yes.
> *Student 13:* But, how is she going to travel 23 hours? I doubt she can.
> *Student 15:* She is going to go by plane anyway. Then, how can Uncle Nedim travel to Sydney for 49 hours?
> *Student 13:* I am not saying he should go to Sydney.
> ...
> *Student 14:* Our first choice is Geneva because it receives more rain in April than other locations.
> *Student 13:* Since the rainfall of Istanbul is slightly lower, we put it in second place.
> *Student 15:* When we compare Buenos Aires and Sidney, we prefer to put Buenos Aires in the third choice because travel time is shorter than Sidney.

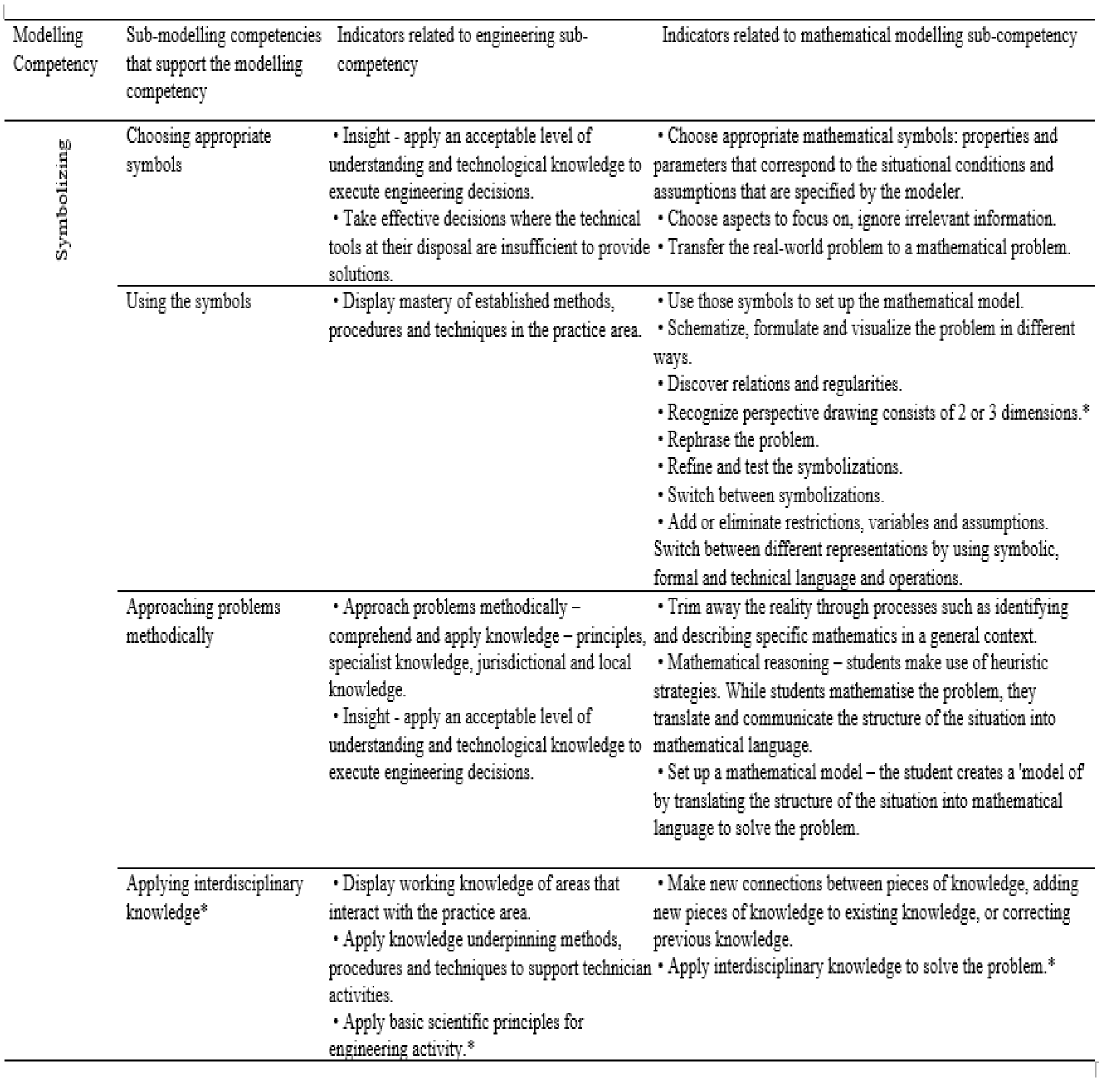
As is shown in the above extract, elementary gifted students identified and constructed relations between key variables based on the conditions and constraints in the problem situation *(constructing relations)*. They maintained a good balance between the effectiveness of the solution process and the time/cost involved as an engineering sub-competency *(constructing relations)*. Additionally, they considered the social, economic, and environmental consequences of decisions *(constructing relations)*. Hence, such ideas stated above could be accepted as *exemplary* indicators of *constructing relations* for group E since they generated a strategy for solving the problem considering the interdependence, interactions and relative importance of factors. Consequently, both groups established an engineering sub-competency in that the design problem is amenable to solution using their own procedures.

Overall, in this section, the findings from elementary gifted students' engagement in engineering-based MEAs were investigated in terms of the *innovative planning and design* and *constructing relations* sub-competencies of structuring. The analysis of the data revealed that the students exhibited *exemplary* modeling competency in structuring competency using *innovative planning and design* to set up situational models when they engaged in some of the engineering-based model eliciting activities. In addition, they constructed relations considering the interdependence, interactions, and relative importance of factors in an *exemplary* demonstration of the sub-modeling competency of *constructing relations*. Hence, the revision was made in the group modeling competency observation guide proposed by de Villiers (2018). The stated explanations above were added to the relevant dimensions in accordance with the findings of the current study

4.4. Modeling Competency 4: Symbolizing

The findings of the fourth modeling competency, symbolizing, are provided in this section of the chapter based on the classification of cognitive modeling competencies suggested by de Villiers (2008). The symbolizing competency was analyzed with respect to the sub-modeling competencies: *choosing appropriate symbols*, *using the symbols*, *approaching problems methodically*, and *applying interdisciplinary knowledge*. Moreover, the extent to which elementary gifted students in groups displayed the symbolizing modeling competency and its sub-modeling competencies when they are engaged in *Bridge Construction*, *Mars Lunarcrete* and *Dr. Ahmet's Will* activities are investigated in this section. The findings for each activity present examples of two distinct groups demonstrating the elementary gifted students' sub-modeling competencies. These sub-modeling competencies, as well as their related engineering and mathematics modeling sub-competency, as indicated in Table 4.4, are used to examine the students' symbolizing competency in more detail. New codes that arose as a result of the analysis of the current study are marked by the symbol '*' in the table below.

Table 4.4. Classification of symbolizing competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Symbolizing	Choosing appropriate symbols	• Insight - apply an acceptable level of understanding and technological knowledge to execute engineering decisions. • Take effective decisions where the technical tools at their disposal are insufficient to provide solutions.	• Choose appropriate mathematical symbols: properties and parameters that correspond to the situational conditions and assumptions that are specified by the modeler. • Choose aspects to focus on, ignore irrelevant information. • Transfer the real-world problem to a mathematical problem.
	Using the symbols	• Display mastery of established methods, procedures and techniques in the practice area.	• Use those symbols to set up the mathematical model. • Schematize, formulate and visualize the problem in different ways. • Discover relations and regularities. • Recognize perspective drawing consists of 2 or 3 dimensions.* • Rephrase the problem. • Refine and test the symbolizations. • Switch between symbolizations. • Add or eliminate restrictions, variables and assumptions. Switch between different representations by using symbolic, formal and technical language and operations.
	Approaching problems methodically	• Approach problems methodically – comprehend and apply knowledge – principles, specialist knowledge, jurisdictional and local knowledge. • Insight - apply an acceptable level of understanding and technological knowledge to execute engineering decisions.	• Trim away the reality through processes such as identifying and describing specific mathematics in a general context. • Mathematical reasoning – students make use of heuristic strategies. While students mathematise the problem, they translate and communicate the structure of the situation into mathematical language. • Set up a mathematical model – the student creates a 'model of' by translating the structure of the situation into mathematical language to solve the problem.
	Applying interdisciplinary knowledge*	• Display working knowledge of areas that interact with the practice area. • Apply knowledge underpinning methods, procedures and techniques to support technician activities. • Apply basic scientific principles for engineering activity.*	• Make new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge. • Apply interdisciplinary knowledge to solve the problem.*

As presented in Table 4.4, the analysis of this study revealed that in addition to the symbolizing sub-competencies, suggested by de Villiers (2018), of choosing appropriate symbols, using the symbols, and approaching problems methodically, applying interdisciplinary knowledge emerged as a sub-code based on the findings of the current study. Accordingly, the indicators of symbolizing sub-competencies in terms of engineering and mathematical modeling sub-competencies when elementary gifted students are engaged in engineering-based MEAs are provided in Table 4.4.

As the first example of engineering-based model eliciting activities, the findings for the Bridge Construction activity related to symbolizing competency of groups A and B were presented regarding the sub-competencies of *choosing appropriate symbols*, *using the symbols*, *approaching problems methodically* and *applying interdisciplinary knowledge*. In addition, the extent to which elementary gifted students exhibited the symbolizing modeling competency and related sub-modeling competencies when engaging in the Bridge Construction activity was shown in Figure 4.27. Based on the group modeling competency observation guide proposed by de Villiers (2018), the level of sub-modeling competencies for the groups was classified as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)*, and 3 *(exemplary)*.

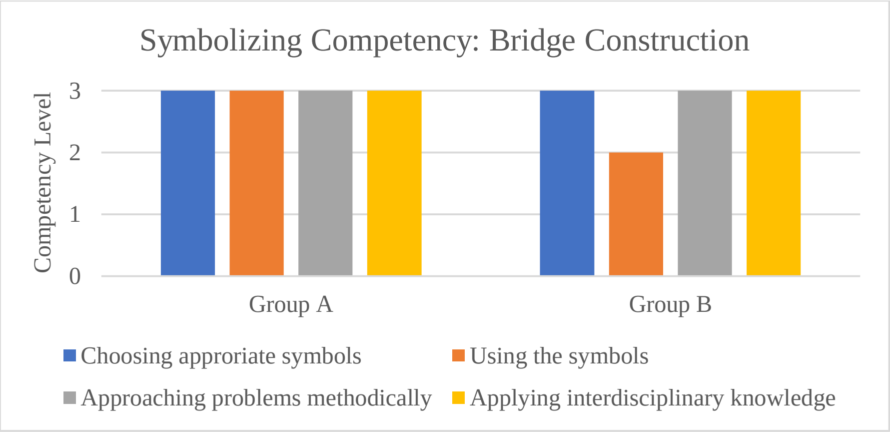


Figure 4.27. Symbolizing Competency: Bridge Construction

As presented in Figure 4.27, elementary gifted students in groups A and B generally displayed *exemplary* modeling sub-competencies of symbolizing when they are engaged in the *Bridge Construction* activity, with the exception of the *using the symbols* modeling sub-competency. Group B exhibited a proficient level of the *using the symbols* modeling sub-competency as demonstrated in the figure above. Thus, the next part provides specific instances of this competency in relation to the preceding sub-categories.

Choosing appropriate symbols and using the symbols. The findings of the data showed that elementary gifted students not only chose appropriate mathematical tools but also scientific and engineering tools in constructing their bridges. As stated in the interpreting competency section of this chapter, the members of group B *chose* *appropriate mathematical symbols* in their drawing. They generally gave information about the height and width of the bridge at first by using units that correspond to the situational conditions (*choosing appropriate symbols)*. For instance:

> *Group B*: There are two roads here. So, we thought of it as a double-sided road. It will be 40 cm tall. Since there is 12 cm, here is 5 cm. It is 40 cm tall. Based on our calculations, this is 12,5 cm. If these are 5 cm, then 5+5+5=15 cm. If we subtract 15 from 40, it becomes 25. If we divide it by 2, it becomes 12,5 cm. This is how we plan to construct our bridge (Student 5).

As presented in the above extract, the members of group B used mathematical symbols to set up the mathematical model *(using the symbols)*. In a similar vein, the excerpt shown in the structuring competency section of this chapter related to group A indicated that elementary gifted students *choose appropriate mathematical symbols* in order to transfer the real-world problem to a mathematical problem in their drawing. Considering all these instances, both groups used symbolic, formal, and technical language, and operations to switch between different representations of their bridges *(using the symbols)*. Moreover, both groups used geometric shapes in their planning and constructing their bridges *(using the symbols)*. The students in group B expressed their chosen shapes as follows:

> *Student 6*: The reason we use a rectangle is to be fixed to the ground.
> *Student 5*: We also use cross bracing to fix the abutments to each other. So that the abutments are not broken.
> *Student 6*: We made an arch to connect these parts together.

In addition, the students in group A commented on their chosen shapes as follows.

> *Student 3*: Here we will use the rectangle to fix the infrastructure. We thought if people fell, they would not be harmed.
> *Student 1*: We made a half circle here. We thought that the whole road should hold when it is broken so that the ropes are fixed more.

As deduced from the dialogue above, both groups made effective decisions where the technical tools at their disposal are insufficient to provide solutions, an indicator of the engineering sub-competency (*choosing appropriate symbols)*. The reflections of students revealed that elementary gifted students use shapes to make their bridge stronger. Specifically, both groups preferred to use rectangles to make the ground more stable. This showed evidence that they displayed mastery of established methods, procedures, and techniques in the practice area, reflective of the engineering sub-competency *(using the symbols)*.

In addition to choosing and using the symbols in mathematics, elementary gifted students also chose and used the appropriate materials and methods to construct their bridges. For example, the members of group B expressed why they chose to use such materials as in the following:

> *Group B*: We will use wire in the places to fix these two arches, wire in here and wire in the places to connect these together (Student 4).

The above excerpt showed that they determined the materials to provide their predetermined conditions (*choosing appropriate symbols)*. Similarly, the members of group A commented as follows:

> *Student 2*: We will also make the abutments with very thick cardboard. We will also support them with tongue sticks so that it does not fall out and open.
> *Student 3*: We will silicon the straws and paste them to the side, we will make a rope from here so that we support it from the height. We will also stop the bridge from above with thick cardboard.
> *Student 1*: Copper wire will be used. Rope will also be used. We think that rope is enough because we sufficiently support from the bottom and it is enough to support the rope from the top.

As it is presented in the above dialogue, elementary gifted students explained and described the tools used in their model as well as alternative methods for working with the problem. Hence, this could be accepted as *exemplary* evidence of the *using the symbols* modeling sub-competency for group A. However, the members of group B explained and described the symbols used in their model accurately but did not identify alternative methods and so showed *proficient* evidence of *using the symbols*. On the other hand, the excerpts of both groups showed that they preferred extra materials in order to make their bridge stronger. They used original ideas not directly from an external source to develop solutions to the engineering problems. This showed evidence regarding the engineering sub-competency, in that elementary gifted students took effective decisions where the materials at their disposal were insufficient to provide solutions (*choosing appropriate symbols)*. Accordingly, both groups exhibited *exemplary* demonstration of the *choosing appropriate symbols* sub-modeling competency since they chose mathematical, scientific and engineering tools that would lead to an elegant solution of the problem.

Approaching problems methodically and applying interdisciplinary knowledge. In addition to all these, both groups *approached problems methodically* by comprehending and applying knowledge. Throughout the engineering design process, group A applied science, mathematics, and engineering knowledge regarding materials and properties according to which different bridge types support load in science, use of certain shapes, estimation and measurement skills, cost efficiency in mathematics, methods of strengthening the abutments and increasing the stability and strength of the bridge *(applying interdisciplinary knowledge)*. The first prototype of group A was shown in Figure 4.28.

Figure 4.28. The first prototype of group A

On the other hand, group members reflect their views regarding the processes by which they apply science, technology, and engineering knowledge. For example:

> *Student 2*: If any part does not support the bridge, we form the alternative to support the bridge. The water removal system is also (science);
> *Student 3*: (engineering) in the whole process because we draw, design, and think about how to design;
> *Student 1*: We estimate, calculate, and try the measurements of abutments, try to adjust the diameter of the semicircle and think about how to fix it. We consider the angles while fixing the bridge from the top. We also use (mathematics) to adjust the size of signboards. While constructing our bridge we applied science, engineering, and mathematics in each phase and decision.

As shown in the above excerpt, elementary gifted students employed their knowledge of the processes and procedures underpinning science to support their decisions in accordance with mathematical knowledge *(applying interdisciplinary knowledge)*. Similar to the other group, Group B also demonstrated science, technology and engineering knowledge and applied them in the design process. The first prototype of group B was presented in Figure 4.29.

Figure 4.29. The first prototype of group B

Differently, they used the nature of the material to construct new bridges. They indicated knowledge of material use and force (science), measurement, geometry, and cost effectiveness (mathematics), and design process and strategies to construct stronger and more stable bridges (engineering). Elementary gifted students also reflected how they apply science, technology, engineering, and mathematics knowledge to the nature of spaghetti in science; adjusting the measurement of cardboard and spaghettis, placing beads in equal intervals in mathematics and the whole design process in engineering *(applying interdisciplinary knowledge)*. Such reflections showed that they applied their engineering and mathematics knowledge to increase the strength and stability of their bridge. This provided evidence that both groups demonstrated working knowledge of areas that interact with the area of practice, an indicator of the engineering sub-competency of *applying interdisciplinary knowledge*. Taken together, both groups displayed *exemplary applying interdisciplinary knowledge* sub-modeling competency by enabling in-depth investigation integrating many different disciplines in order to solve the problem. It can be concluded that the design processes of both groups involved a step-by-step consideration of facts and evidence, as well as the logical conclusions that could be drawn from this consideration of facts and evidence *(approaching the problem methodically)*. Hence, all aspects of their reasoning resulted in satisfactory solutions and both groups exhibited *exemplary* demonstration of the *approaching problems methodically* sub-modeling competency.

As the second example of an engineering-based model eliciting activities, the findings of the *Mars Lunarcrete* activity related to the symbolizing competency of groups C and D in the activity were investigated under the sub-competencies of *choosing appropriate symbols, using the symbols, approaching problems methodically*, and *applying interdisciplinary knowledge*. Moreover, the extent to which elementary gifted students exhibited the symbolizing modeling competency and related sub-modeling competencies when engaging in the *Mars Lunarcrete* activity is illustrated in Figure 4.30. Based on the group modeling competency observation guide proposed by de Villiers (2018), the level of sub-modeling competencies for the groups was classified as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)* and 3 *(exemplary)*.

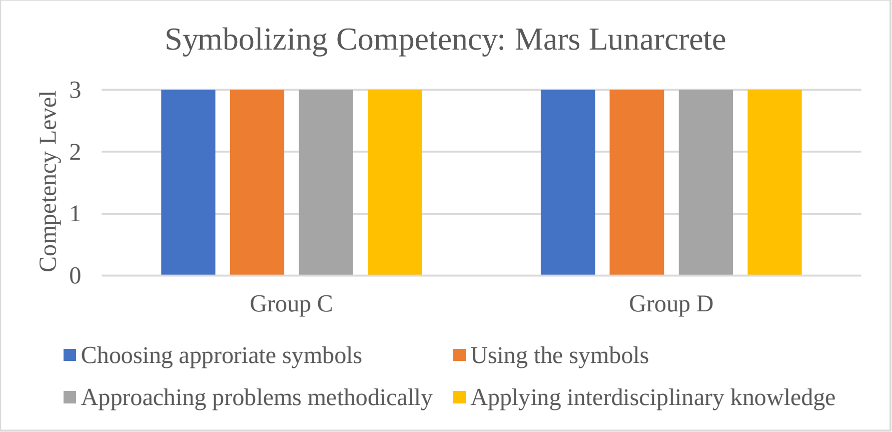
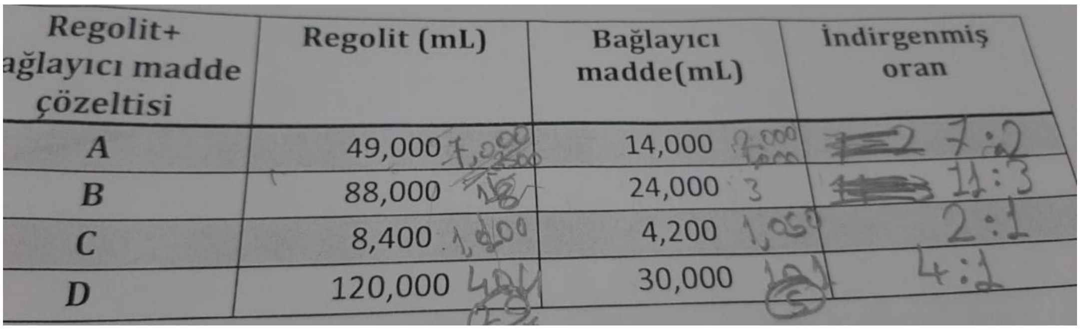


Figure 4.30. Symbolizing Competency: Mars Lunarcrete

As shown in Figure 4.30, the findings of the current study indicate that the elementary gifted students in groups C and D, when they engaged in the *Mars Lunarcrete* activity, exhibited *exemplary* modeling of the sub-competencies of symbolizing. To clarify and expand on the aforementioned categories, sample excerpts representative of each category are provided below.

Choosing appropriate symbols and using the symbols. The findings from the Mars Lunarcrete activity related to the competency of symbolizing revealed that both groups *chose appropriate mathematical symbols* in their initial planning and structuring of their model. First of all, the groups found the reduced ratios. As an example, the worksheet of group C was shown in Figure 4.31



Regolit+ ağlayıcı madde çözeltisi	Regolit (mL)	Bağlayıcı madde(mL)	İndirgenmiş oran
A	49,000 7,000	14,000 2,000	7:2
B	88,000	24,000 3	11:3
C	8,400	4,200 1,050	2:1
D	120,000	30,000	4:1

Figure 4.31. The worksheet of group C

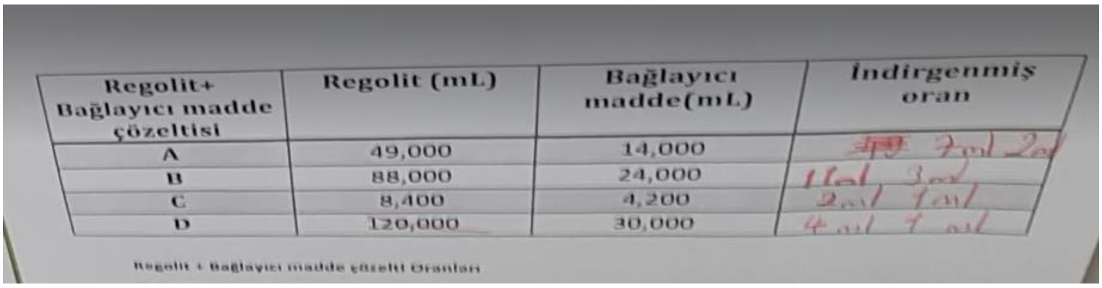
The above Figure 4.31 and the excerpt below showed evidence that the members of group C used symbolic, formal, and technical language and operations to switch between multiple representations *(using the symbols)*. For instance:

> *Student 9:* We discard the zeros first. How many can we divide 49 by?
> *Student 8:* It is divided by 7. Yes. 7 times 7 is equal to 49. Can we divide more?
> *Student 7:* We cannot divide.
> *Student 9:* If we divide 7000 by 2, this will be 3500. Then, it is 1 to 3.5.
> *Student 7:* But, we will write using integers. You can write 7:2.
> *Student 8:* If we divide this by 11, we get 8. This is 3.
> *Student 7:* If we divide C by 4. It is 2100. If we divide this by 4, it is 1050. The reduced ratio is 2:1
> *Student 9:* That is going to be 4:1. It happens at 20:5.
> *Student 8:* But we should use the simplest form. If we divide by 30, 4:1.

As indicated in the above dialogue, members of group C chose appropriate mathematical symbols, properties, and parameters that corresponded to the situational conditions *(choosing appropriate symbols)*. In addition, they focused on the reduced ratio to more easily see the relationship between regolith and binder *(choosing appropriate symbols)*. The above dialogue also shows *exemplary* evidence of the *using* *the symbols* sub-modeling competency for group C, in that they explained the symbols used in their model as well as the alternative methods of dealing with the situation.

Approaching problems methodically and applying interdisciplinary knowledge. Moreover, the above dialogue indicated that elementary gifted students used heuristic strategies such as discarding the zeros *(approaching problems methodically)*. When they mathematized the problem, they also transferred the structure of the situation into a mathematically comprehensible language *(approaching problems methodically)*. In addition to this, group D used units and described the meaning of the ratio in the context of the situational condition, as illustrated in Figure 4.32. For instance:

> *Student 12:* Let us find the reduced ratios first. It can be divided by 7000.
> *Student 10:* If we are going to use 7 items of regolith, we will use 2 items of binder solution.
> *Student 11:* Now let us look at this, if we are going to use 11 ml of regolith when divided by 11,000, 3 ml of binder.
> *Student 10:* We should use something liquid in order to make the binder a solid. Something like water or liquid glue. For example, we mix water to make concrete.
> *Student 12:* 4:2 but there is a simpler version 2:1
> *Student 11:* It is 20:5. Then it becomes 4 to 1.



Regolit+ Bağlayıcı madde çözeltisi	Regolit (mL)	Bağlayıcı madde(mL)	İndirgenmiş oran
A	49,000	14,000	7ml 2ml
B	88,000	24,000	11ml 3ml
C	8,400	4,200	2ml 1ml
D	120,000	30,000	4ml 1ml

Regolit + Bağlayıcı madde çözelti Oranları

Figure 4.32. The worksheet of group D

As deduced from the dialogue and figure above, elementary gifted students in group D trimmed away the reality through procedures such as identifying and describing mathematics in a broad perspective *(approaching problems methodically)*. Moreover, they made new connections between pieces of knowledge to correct previous knowledge *(applying interdisciplinary knowledge)*. This showed evidence that they demonstrated a comprehensive understanding of established methods and procedures in the practice area, indicative of the engineering sub-competency of *using the* *symbols*. As shown in the drawings of both groups before, they schematize, formulate and visualize the problem in different ways *(using the symbols)*. To illustrate, in the extract below group C explained why they use such visualizations by applying science knowledge:

> *Researcher:* Which geometric shapes does it have?
> *Student 8:* Pyramid on top, prisms and triangles on other parts
> *Researcher:* Why did you choose them?
> *Student 9:* Because they are more convenient when they receive sunlight. They are more advantageous in benefiting from sunlight, so we used it.

The above excerpt demonstrates that the elementary gifted students displayed working knowledge of science interacting with the area of practice, indicative of the engineering sub-competency of *applying interdisciplinary knowledge*. The above dialogue also indicates that they applied an acceptable level of understanding to execute engineering decisions to use the geometric shapes in an effective way *(choosing appropriate symbols)*. In addition, the members of group C recognized the type of drawing and perspective drawing. For instance:

> *Student 8:* Are you drawing in 2D?
> *Student 9*: Yes. Should we draw in 3D?
> *Student 8:* No, I just asked. It will be easier to draw in 2D.
> *Student 9:* If appropriate, I will continue.
> *Student 7:* Are you drawing the view from above? (See Figure 4.11)
> *Student 9:* Yes.
> *Student 7:* For example, let me draw a side view of this.

As shown in the dialogue above, elementary gifted students used different perspectives of shapes in their drawings *(using the symbols)*. On the other hand, the excerpt in interpreting related to group D indicated that they sketched the garage with a forward sloping door and explained the reason to be more resistant to wind. This showed evidence that elementary gifted students transferred the real world situation to their drawing *(approaching problems methodically)*. In addition, they applied an adequate level of technological and scientific knowledge to perform engineering judgements *(applying interdisciplinary knowledge)*. In addition, as shown in the competency of structuring, the members of group D compared the ratio of 2:1 and 4:3 in terms of the amount of regolith and binder solution they have. Hence, they applied mathematics to support engineering activities by using the knowledge that underpins methodologies and approaches *(applying interdisciplinary knowledge)*.

Furthermore, both groups used the beaker correctly to measure the amount of water they used and performed the measurement by paying attention to the units. They also put the substances they use, such as flour and salt, into the beaker. Then, they calculated and noted the tare weight of the beaker, and then weighed on a precision balance and recorded the result. Lastly, they subtracted the recorded result from the tare weight of the beaker and found the amount of substances. This showed evidence that elementary gifted students applied knowledge of measurement to set up the mathematical model *(approaching problems methodically)*.

In accordance with knowledge of mathematics, elementary gifted students applied knowledge of the methods and procedures underpinning science to support their sketches *(applying interdisciplinary knowledge)*. Group C stated that they use plastic, glass, and cardboard to create the design of their sketch *(choosing appropriate symbols)*. On the other hand, group D explained the materials they plan to use for their sketch in the excerpt below:

> *Student 12:* We can use insulated materials for thermal insulation.
> *Student 11:* I think we can use double wool and Styrofoam. That is why we use Styrofoam to make the edges heat resistant.
> *Student 12:* We probably make things like the garage and basement which is important there using Styrofoam.
> *Student 10:* There is something like an aluminum jacket inside the first aid kits that I know of. The burnt jacket is that kind of thing; it provides thermal insulation. That way, aluminum things can be used for thermal insulation of the edges.
> *Student 11:* Other than that, we can use it to fasten plastic, cardboard, wire, copper wire, etc. to other places.

As deduced from the dialogue below, Student 10 made new connections between pieces of knowledge, adding new pieces of scientific knowledge based on one's previous experience *(applying interdisciplinary knowledge)*. Group D also incorporated the temperature differences on Mars in the real-world problem to choose the appropriate material *(choosing appropriate symbols)*. In addition, group D explained and described the tools used in their model as well as alternative methods for working with the problem and showed *exemplary* evidence of the *using the symbols* sub-modeling competency. The dialogues above showed *exemplary* evidence among both groups related to the *choosing appropriate symbols* sub-modeling competency in that they chose mathematical and scientific tools to arrive at the desired solution with the least amount of material and effort wasted.

Besides, both groups approached the problem methodically by comprehending and applying knowledge. During the engineering design process, both groups applied science, mathematics, and engineering knowledge regarding materials and properties, how density, wind resistance, and temperature differences affect building on Mars, interpretation of data, calculation of reduced ratios, how to use certain shapes, estimation, and measurement skills, cost efficiency in mathematics, and also how to increase the stability and durability of the building in engineering. Taking all of the aforementioned into account, both groups displayed *exemplary* evidence of the *applying interdisciplinary knowledge* sub-modeling competency by integrating many different disciplines to allow in-depth investigation and to ultimately solve the problem. In conclusion, it can be stated that the modeling processes of both groups involved a step-by-step examination of facts and evidence, as well as the logical inferences that could be taken from this examination of facts and evidence *(approaching the problem methodically).* Thus, both groups demonstrated *exemplary* evidence *approaching problems methodically* sub-modeling competency since all aspects of their reasoning were accurate and translated the structure of the situation into satisfactory solutions.

As the last example of an engineering-based model eliciting activity, the findings of the *Dr. Ahmet's Will* activity related to the symbolizing competency of group E and F in the activity were investigated with respect to sub-competencies: *choosing appropriate symbols*, *using the symbols*, *approaching problems methodically,* and *applying interdisciplinary knowledge*. The distribution of each sub-competency regarding the extent to which elementary gifted students in groups E and F displayed the symbolizing modeling competency is represented in Figure 4.33. Based on the group modeling competency observation guide proposed by de Villiers (2018), the level of sub-modeling competencies for the groups was categorized as 0 *(unsatisfactory),* 1 *(emergent/developing),* 2 *(proficient)* and 3 *(exemplary).*

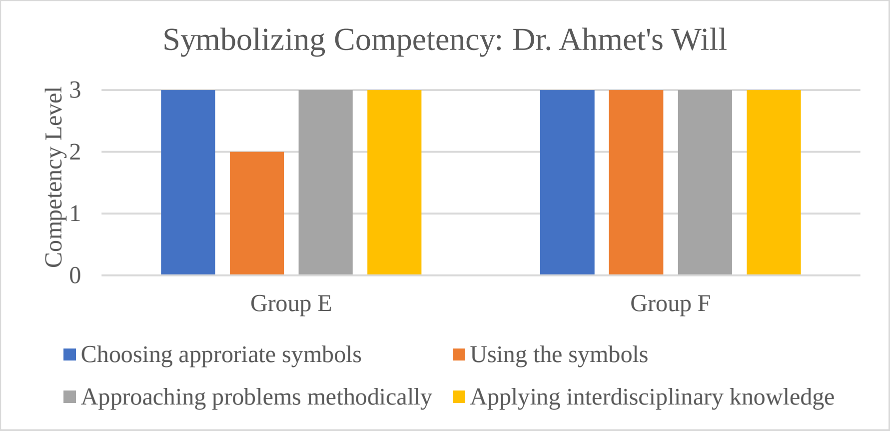


Figure 4.33. Symbolizing Competency: Dr. Ahmet’s Will

As shown in Figure 4.33, when elementary gifted students in groups E and F engaged in the *Dr. Ahmet’s Will* activity, they generally exhibited *exemplary* modeling of the sub-competencies of symbolizing. From the graph above, it can be asserted that group E displayed a *proficient level of using the symbols* sub-modeling competency engaging in the activity. In order to explain and expand on the components of symbolizing competency, sample extracts which are representative of each sub-competency are given below.

Choosing appropriate symbols and using the symbols. As stated in the dialogues in previous sessions, the findings from the *Dr. Ahmet’s Will* activity showed that elementary gifted students chose appropriate mathematical symbols, properties and parameters that correspond to the situational conditions in their planning and structuring *(choosing appropriate symbols)*. In particular, both groups discovered relations and regularities by comparing many variables *(using the symbols)*. They compared flight costs of each relative to the selected location in dollars. However, they preferred to use hours excluding minutes. Thus, they refined and tested symbolization *(using the symbols)*. On the other hand, both groups schematized, formulated, and visualized the problem in different ways *(using the symbols)*. For example, the members of group F categorized flight cost as cheap and expensive and travel times as less and more to formulate the data, but concluded that this was not appropriate to reach the solution of the problem. In a similar way, group E calculated the total travel time of the relatives for each option in order to simplify the problem of the travel time of each relative to the location of the office as stated before but it did not work. Hence, they chose aspects to focus on and ignored irrelevant information *(choosing appropriate symbols).* Considering all these instances, both groups exhibited *exemplary* evidence of *choosing appropriate symbols* by choosing mathematical and scientific tools to reach the desired solution with the least amount of time and energy.

Besides, they switched between different representations by using symbolic, formal, and technical language and operations *(using the symbols).* The next extract details how the members of group F converted temperatures from Fahrenheit to Celsius:

> *Student 18:* There are those that travel to locations of extreme temperatures they have never experienced.
> *Student 17:* They are capable of surviving in extreme cold.
> *Student 19:* How do you survive in temperatures of 83 degrees? Do you prefer the cold or the heat?
> *Student 16:* Do we maintain ice at -10 degrees Celsius during the winter?
> *Student 19:* There is no possible way to travel to Siberia.
> *Student 17:* Humans are more prone to living in the cold.
> *Student 19*: We can withstand -10 to -20 degrees Celsius here.
> *Student 16:* Look at the lowest temperatures now, can you not stand 30 degrees?
> *Student 18:* We can stand it because we experience that temperature.
> *Student 16:* The highest temperature in Geneva is 56 degrees. Also, Honolulu has the highest temperature.
> *Student 18:* I say the lowest temperature.
> *Student 16:* What is between 83 and 68? Let us write. (See Figure 4.34)
> *Student 17:* All of these temperatures are already occurring. They experienced it all.
> *Student 18:* Do you have 38 and 39 here? (Crying). No. Is 38 degrees between 83 and 68 degrees. How can an 84-year-old woman handle living in a hot city?
> *Student 19*: A change of 32 degrees Fahrenheit is equivalent to a change of 0 degrees. Look at the thermometer on the wall.
> *Student 18*: You are looking at a high temperature; I am looking at a low temperature.
> *Student 16*: There is no better place than Geneva.
> *Student 18*: I am not saying there should be a place other than Geneva. I just want to discuss this. Where is 38 degrees Fahrenheit by the way?
> *Student 19*: Also, consider this. 83 degrees Fahrenheit is equal to 28 degrees Celsius
> *Student 17*: 3 meters, 1 foot in height. We can think so.

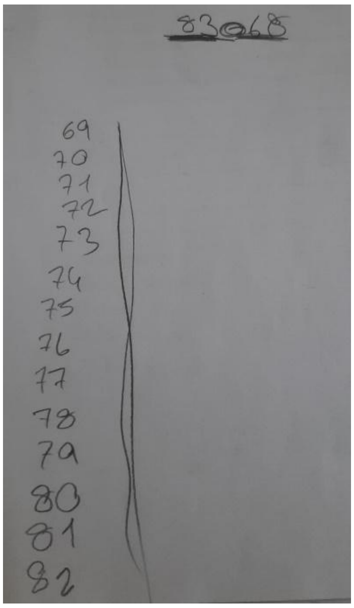


Figure 4.34. The difference in temperature between the highest and lowest temperatures in Honolulu.

The conversation above indicates that the members of group F tried to use a common language by converting temperature to the unit they often use in their daily lives to resolve the conflict between them. Hence, they showed an engineering competency by displaying mastery of established methods, procedures, and techniques in the practice area (using the symbols). In addition, both groups made accurate decimal comparisons when comparing the amount of rainfall at the possible locations. As is presented in the above dialogue, elementary gifted students explained and described the tools they utilized in their model as well as alternative approaches to deal with the problem situation. Hence, this could be accepted as *exemplary* evidence of the *using the symbols* modeling sub-competency for group F. On the other hand, the members of group E explained and described the symbols used in their model accurately but did not identify alternative methods and so showed *proficient* evidence of the *using the symbols* modeling sub-competency.

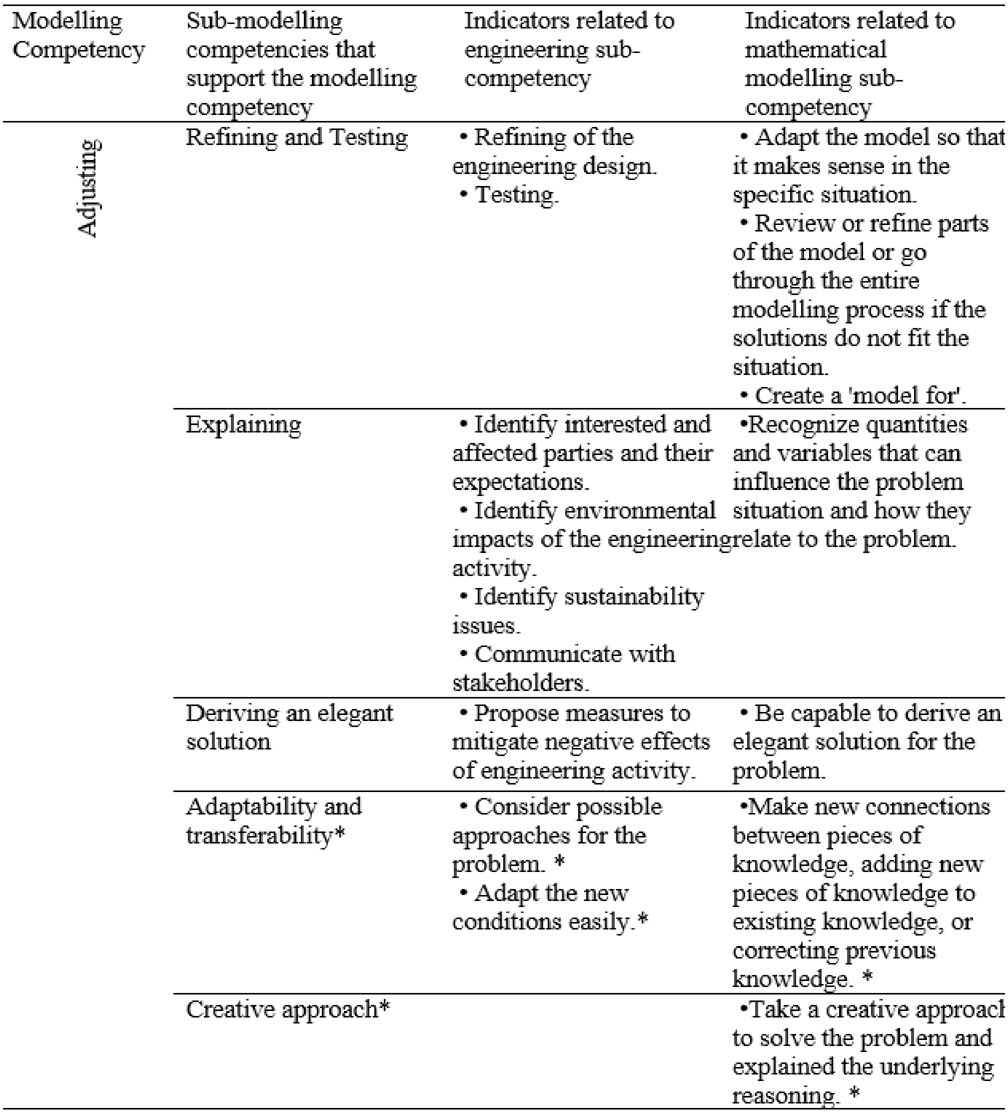
Approaching problems methodically and applying interdisciplinary knowledge. The above dialogue also indicated that elementary gifted students in group F made new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge and correcting previous knowledge *(applying interdisciplinary knowledge)*. They also displayed working knowledge of areas that interact with the practice area, which is an indicator of the engineering sub-competency *(applying* *interdisciplinary knowledge)*. Furthermore, both groups approached the problem methodically by comprehending and applying knowledge. Throughout the engineering design process, both groups applied knowledge of science, mathematics, and engineering. They applied data interpretation, computation, estimation, and measurement skills and cost/time efficiency in mathematics. They applied knowledge of how rainfall, pollution level, and temperature differences affect travel in science and the design process in order to meet well defined requirements and make arrangements that fulfill all requirements as efficiently and economically as possible in engineering *(approaching problems methodically)*. Taking all the aforementioned into account, both groups displayed *exemplary* evidence for *applying interdisciplinary knowledge* sub-modeling competency by in-depth investigation of the problem. As a concluding point, it can be asserted that both groups displayed *exemplary* indicators of *approaching problems methodically* because all aspects of their reasoning resulted in satisfactory solutions using step-by-step investigation of the evidence.

To sum up, this section investigated the elementary gifted students' symbolizing competency through the lens of *choosing appropriate symbols*, *using the symbols*, *approaching problems methodically*, and *applying interdisciplinary knowledge* when they are engaged in engineering-based model eliciting activities. The findings from this study revealed that elementary gifted students exhibiting symbolizing modeling competency science and engineering knowledge beyond their knowledge of mathematics. Hence, *applying an interdisciplinary knowledge* dimension was added to the group modeling competency observation guide proposed by de Villiers (2018). In accordance with this, the other dimensions that are related only to mathematics were revised. In addition, the inclusion of applying in-depth interdisciplinary knowledge into the situational condition was also unique for elementary gifted students.

4.5. Modeling Competency 5: Adjusting

De Villiers (2008) proposed a categorization of cognitive modeling competencies that included seven modeling competencies. In accordance with the aim of the current study, the findings of the fifth modeling competency, adjusting, were presented in this section of the chapter. In particular, the adjusting competency was analyzed with respect to the sub-modeling competencies of *refining and testing, explaining, deriving* *an elegant solution, adaptability and transferability* as well as *creative approach.* Additionally, the extent to which elementary gifted students in groups displayed the adjusting modeling competency and its sub-modeling competencies when engaging in the Bridge Construction, Mars Lunarcrete, and Dr. Ahmet's Will activities were examined in this section of the research. The findings for each activity provided examples of two distinct groups exhibiting the sub-modeling competencies. Specifically, the adjusting competency of elementary gifted students was analyzed through the aforementioned sub-modeling competencies and their related engineering and mathematical modeling sub-competency as presented in Table 4.5. In the table below, new codes that emerged as a result of the analysis of the present study are represented by the symbol '*'.

Table 4.5. Classification of adjusting framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Adjusting	Refining and Testing	• Refining of the engineering design. • Testing.	• Adapt the model so that it makes sense in the specific situation. • Review or refine parts of the model or go through the entire modelling process if the solutions do not fit the situation. • Create a 'model for'.
	Explaining	• Identify interested and affected parties and their expectations. • Identify environmental impacts of the engineering activity. • Identify sustainability issues. • Communicate with stakeholders.	•Recognize quantities and variables that can influence the problem situation and how they relate to the problem.
	Deriving an elegant solution	• Propose measures to mitigate negative effects of engineering activity.	• Be capable to derive an elegant solution for the problem.
	Adaptability and transferability*	• Consider possible approaches for the problem. * • Adapt the new conditions easily.*	•Make new connections between pieces of knowledge, adding new pieces of knowledge to existing knowledge, or correcting previous knowledge. *
	Creative approach*		•Take a creative approach to solve the problem and explained the underlying reasoning. *

As shown in Table 4.5, in addition to the adjusting sub-competencies of *refining and testing*, *explaining*, and *deriving an elegant solution* proposed by de Villiers (2018), the analysis of the current study's findings revealed that *adaptability and transferability*, as well as *creative approach* also emerged as sub-codes. The indicators of symbolizing sub-competencies in terms of engineering and mathematical modeling sub-competencies when elementary gifted students engaged in engineering-based MEAs are presented in Table 4.5.

As the first example of engineering-based model eliciting activities, the findings for the Bridge Construction activity related to the adjusting competencies of group A and B are presented regarding the sub-competencies of *refining and testing*, *explaining*, *deriving an elegant solution*, *adaptability and transferability*, and *creative approach*. Moreover, the extent to which elementary gifted students showed the adjusting modeling competency and related sub-modeling competencies when engaging in the *Bridge Construction* activity was illustrated in Figure 4.35. Based on the group modeling competency observation guide proposed by de Villiers (2018), the level of sub-modeling competencies for the groups was classified as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)* and 3 *(exemplary)*.

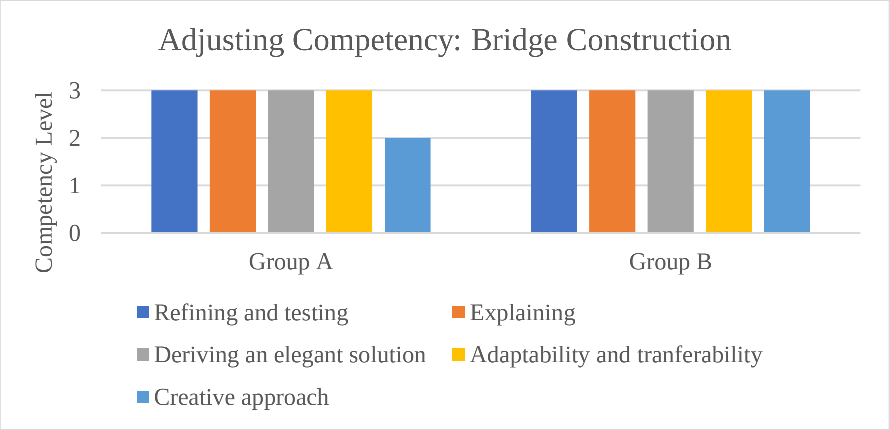


Figure 4.35. Adjusting Competency: Bridge Construction

As presented in Figure 4.35, when elementary gifted students in groups A and B engaged in the Bridge Construction activity, they generally displayed *exemplary* sub-modeling competencies of adjusting except *creative approach*. Regarding the level of creative approach sub-modeling competency, group A demonstrated a *proficient* level of the *creative approach* sub-modeling competency, as shown in Figure 4.35. The following part presented the instances of this competency in relation to the aforementioned sub-categories.

Refining and testing, and explaining. The analysis of the data revealed that while group A refines the part of the model, group B creates a new model in refining the engineering design as the engineering sub-competency of *refining and testing*. After group A assessed their design, they made some changes in reconstructing the bridge. For example, they named the bridge CDA because they thought that the bridge should carry the initial names of the builders as shown in Figure 4.36.

Figure 4.36. The second prototype of group A

In addition, they expressed their changes for the second prototype as in the following excerpt:

> *Student 2*: We place lamps outside the bridge. If we place it indoors, it can blind the pedestrians and drivers. We put two signboards in the entrance (Welcome to Harsit Stream) and exit of the bridge (We look forward to seeing you again).
> *Student 1*: We think that people come to the bridge and say what a beautiful and nice village and bridge there is. So, we make advertisements for the village and bridge without any charge.
> *Student 3*: We discussed how we can ensure that the bridge is affected much less by rainfall. For the second design, we build a shed to prevent getting water on rainy days.

The reflections of the students revealed that the group adapted the model so that it makes sense for the specific situation for the village *(refining and testing).* When testing their model, the members of group A expressed that the new situation and material did not require their model totally *(refining and testing)*. As shown in the above excerpt, they mostly focused on representation of the bridge. This can be an accepted indicator related to the engineering sub-competency that they identified interested and affected parties and their expectations *(explaining)*. In addition, they associated the underlying structure of the problem with other similar problems and thus decided to develop a mechanism to prevent the second prototype from being affected by rain. This was an *exemplary* demonstration of the *refining and testing* modeling sub-competency. Correspondingly, the members of group A gave an in-depth explanation of their reasoning and showed *exemplary* instances of the *explaining* modeling sub-competency. Identifying the environmental impacts of engineering activity and associated sustainability issues, which is an indicator of the engineering sub-competency in the above extract, could be accepted as evidence of exhibiting an *exemplary explaining* modeling sub-competency.

Furthermore, the members of group B assessed their first bridge construction *(refining and testing).* Group members had a consensus that their first bridge prototype was more suitable for the city and more expensive. They stated that decoration of the bridge such as using more silicone to paste beads and colorful tongue sticks increases its cost. Hence, they identified interested and affected parties and their expectations, an indicator of the engineering sub-competency of *explaining.* Correspondingly, they changed their strategy for the second construction of the bridge. Group members allocated more time to plan the reconstruction of their bridge. They decided to construct a totally new bridge for the new situation using new material, spaghetti, as shown in Figure 4.37. Hence, they created a new model to deal with the changing circumstance *(refining and testing).*

Figure 4.37. The second prototype of group B

This showed that the group goes through the entire engineering design process if the prototype does not fit the situation *(refining and testing).* The extract below presents reflections of the members of group B on their second prototype of the bridge:

> *Student 6*: In the science lesson, we learned that spaghetti remained above the water so we constructed a floating bridge in the second design. We only use cardboard, spaghetti and tape as the materials of this bridge. If one land is here and the other land is here, you can use it when you place the bridge in between.
> *Student 5*: We also have a pedestrian path on the bridge. The peasantry can use the floating bridge whenever they want, it is less costly and more useful.
> *Student 4*: For instance, we never used silicon in the second bridge and we use the tape to cover the surface of the bridge since the cost of the tape is low.
> *Student 6*: We also thought that silicon might not hold the cardboard, as cardboard is heavy. There is almost 1 package of spaghetti on this bridge, so even if you want to break it, you cannot break it, we think it is stable.

The above conveys that the members of group B found multiple solutions based on various interpretations of the problem and refined the engineering design *(refining and testing)*. It is clear from the conversation above that the students in group B associated the underlying structure of the problem to other similar problems involving bridge situations over streams, rivers, and the sea. Hence, they showed *exemplary* evidence of the *refining and testing* modeling sub-competency. After reconstruction, they thought that their first design was more aesthetic and expensive but the second design was cheaper and stable and also more available for the village condition. As is shown in the reflections of group B, they explained their reasoning extensively and provided *exemplary* instances of the *explaining* sub-modeling competency.

Deriving an elegant solution, adaptability and transferability, and *creative approach.* The findings of the data revealed that elementary gifted students were capable of deriving *an elegant solution* to the *Bridge Construction* problem. In the above excerpt from group B's collaborative communication, they sought the most elegant solution to construct the second prototype with the least amount of materials and effort and so showed *exemplary* evidence of the *deriving an elegant solution* sub-modeling competency. On the other hand, group A reviewed only some parts of their first prototype, demonstrating that an *exemplary deriving an elegant solution* sub-modeling competency could be exhibited through the use of appropriate methods and materials. As deduced from the above dialogues, both groups easily adapted to the new condition and considered possible approaches for the problem situation as engineering sub-competency of *adaptability and transferability*. While group A applied a water removal system to construct their bridge in case of rainfall, group B transferred their knowledge of science into the problem solution. This means that both groups exhibited *exemplary* evidence of *adaptability and transferability* by transferring the knowledge both inside and outside of the school setting. On the other hand, the idea of constructing the floating bridge could be accepted as an *exemplary* instance of the *creative approach* sub-modeling competency for group B since they came up with a new, unique, unexpected, and useful idea as well as appropriate and adaptive in terms of existing constraints of the problem. The members of group B made proposals about how the prototype could function as well as the nature of the components and materials that would be necessary to accomplish this. However, group A substantially took creative approaches and explained the underlying reasoning and so showed *proficient* evidence of *creative approach.* The ideas of placing a lamb outside the bridge and making advertisements without any charge could be accepted as *proficient* instances of *creative approach* within the restrictions of the problem. Different from the members of group B, the members of group A presented a proposal mostly about the features of the prototype.

As the second example of an engineering-based model eliciting activity, the findings for the *Mars Lunarcrete* activity related to the adjusting competency of groups C and D were investigated with respect to the sub-competencies: *refining and testing, explaining, deriving an elegant solution, adaptability and transferability* as well as *creative approach*. In addition, the extent to which elementary gifted students exhibited the adjusting modeling competency and related sub-modeling competencies when engaging in the *Mars Lunacrete* activity is shown in Figure 4.38. According to the group modeling competence observation guide, the sub-modeling competencies of the groups were classified as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)*, and 3 *(exemplary)* proposed by de Villiers (2018).

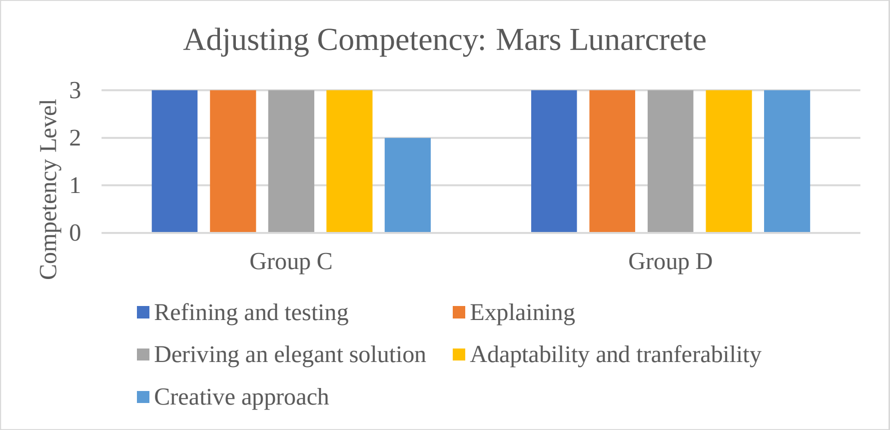


Figure 4.38. Adjusting Competency: Mars Lunarcrete

As illustrated in Figure 4.48, the findings of the current study revealed that the elementary gifted students in groups C and D, when engaging in the *Mars Lunarcrete* activity, generally exhibited *exemplary* modeling sub-competencies of adjusting with the exception of *creative approach*. When the sub-modeling competency of creative approach was considered, the students in group C exhibited *proficient evidence of the creative approach* modeling sub-competency. To clarify and expand on the aforementioned categories, sample excerpts representative of each category are provided below.

Refining and testing. The findings from the *Mars Lunarcrete* activity related to the competency of adjusting revealed that both groups tested their models. After they assessed their first design, their first model did not fit the situation totally (*refining and testing)*. The group C reflected on their first model as shown in Figure 4.39.

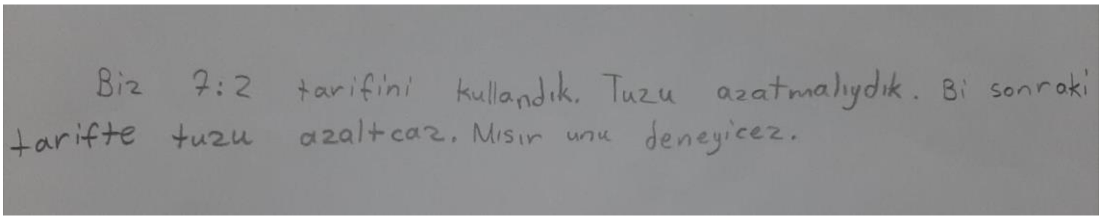
Figure 4.39. The first model of group C

They commented "*if we build a structure from such a material that can be friable very easily when it dries a little more, it will be unstable"*. This statement showed that the elementary gifted students' model was not suitable for the conditions and constraints of the problem situation. Consequently, because their first model was inadequate they went through the entire modeling process to address the particularities of the problem (*refining and testing)*. On the other hand, group D stated that their model as shown in Figure 4.40 may be adaptable to new situations.

Figure 4. 40. The first model of group D

For instance, "*Let us wait until next week, maybe we can put the substances we think about for the new situation (see Figure 4.40). We can test its density and durability once again*". Their comments showed evidence that elementary gifted students approached the design process with the belief that their models may be improved. This indicated that if their model fits the situation they adapt their model to make sense in that specific situation (*refining and testing).* Consequently, both groups found multiple solutions using different interpretations of the problem and refined parts of the model or went through the entire modeling process when their models did not fit the situation. Beyond that, together these findings provided *exemplary* evidence regarding *refining and testing* modeling sub-competency since they related the underlying structure of the problem to other similar problems in terms of density, durability, and property of substance.

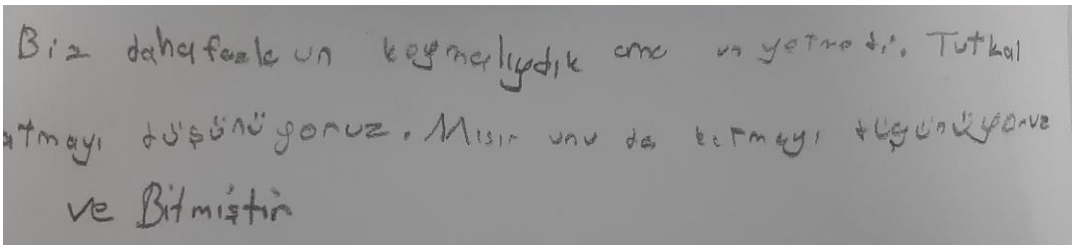
Explaining. Furthermore, both groups communicated with stakeholders, reflecting the engineering sub-competency of *explaining*. The elementary students gave brief information about the refining and testing process of their models by writing. For instance, the reflections of group C members regarding their first model are presented in Figure 4.41.



Biz 7:2 tarifini kullandık. Tuzu azatmalıydık. Bi sonraki tarifte tuzu azaltcaz. Mısır unu deneyicez.

Figure 4.41. The reflections of group C on their first model

As shown in Figure 4.41, the members of group C explained which recipe they used in their first model and which method did not work. In addition, they expressed which substance they plan to use (*explaining).* Similarly, the reflections of group D on their first model are shown in Figure 4.42.



Biz dahafazla un koymalıydık ama un yetmedi. Tutkal atmayı düşünüyoruz. Mısır unu da katmayı düşünüyorve ve Bitmiştir

Figure 4.42. The reflections of group D on their first model

As presented in Figure 4.42, the group members of group D refined the model entirely because it did not work in some way and planned to use new substances for the second model (*explaining).* The above reflections from members of both groups also shows evidence related to the engineering sub-competency of *explaining* in that they identified interested and affected parties and their expectations to test their first model and refine them.

Deriving an elegant solution, adaptability and transferability, and *creative approach.* In addition to their initial request, the corporation asked students to shorten setting time to reduce the cost. Hence, the students reexamined their plans for the second model considering their reflections for the first model and the new situation. Both groups changed their strategy and determined the new recipe after long discussions with group members. They predicted the consequences of their actions, evidence of the engineering sub-competency, given that they adapted the new conditions easily and considered possible approaches for the problem *(adaptability)*. Different from the process of creating the first model, both groups created their second model determining the minimum amount of substances in the recipes. Hence, they made new connections between pieces of knowledge by using their experiences when constructing their first model *(transferability)*. They compared the amount of substances in the recipes and the consequences of actions in the first model. For instance, the extract below presented the decision-making process of group C for the second model:

> *Student 7:* Now, when we look at the 7:2 and 3:1 recipe, 7:2 has a little more flour. If we reduce the salt, which I think we should make at least 100 gr., then we will also reduce the water, because they are the same amount in both recipes.
> *Student 8:* When we tried it, we saw that it does not work when there is too much salt. Maybe 100 gr or 150 gr. We can look at it by adding little by little. Let us decide on the mix after we have agreed on something minimal. 125 grams is fine.
> *Student 9:* I think we need to reduce the water a little.
> *Student 7:* We can also add glue to create a stronger structure.
> *Student 8:* What do you think, maybe this will reduce the setting time?
> *Student 9:* Nice, let us try.

Based on the discussion snippet above, the elementary gifted students tried to *derive an elegant solution* for the new problem situation. Group C also made an in-depth investigation about comparing the ratios in recipes in terms of the ingredients as well as the reflections stated above and so showed *exemplary* evidence of *explaining*. Besides, they made an inference to determine the minimum amount of substance to be used in the recipe applying their experiences in the first model. This could be accepted as exemplary evidence of *adaptability and transferability* since they not only adapted their model easily to the new situation but also transferred the previous knowledge of modeling into the problem situation. Due to their efforts to reduce the amount of materials and processes required to construct their second prototype, the members of group C provided *exemplary* instances of the *deriving an elegant solution* sub-modeling competency. In addition, the ideas stated above for determining the minimum amount of substance and using glue to reduce setting time can be accepted indicators of a *proficient* level of *creative approach*.

Interestingly, group D differentiated the recipe they would use for their model. The discussion among group members unfolded as follows:

> *Student 10:* I do not think we should add corn flour, we added water to corn flour in science class. Then a liquid substance emerges. If we hit it hard, it becomes like a stone, so if you put your hand flat, something liquid comes out.
> *Student 12:* I know that but wasn't it cornstarch instead of cornmeal? For example, when you try to sink your hand so quickly, it does not sink, but when you do it slowly like that, it sinks into your hand.
> *Student 10:* Yeah right, I said it wrong. Actually, I thought we could use it as a binder instead of water. We can use an oobleck substance made with cornstarch and water as a binder. In that case, we add the cornstarch to the recipe. For example, in science lessons, we added 400 or 200 ml of water little by little, and we got such a slightly liquid but agglomerated substance.
> *Student 11*: How much water will we use? We will add it little by little.
> *Student 10:* I think we should do it with cornstarch. Let's use the oobleck substance instead of water. It will also be something harsh. We can also use water. I don’t know.
> *Student 11*: How large of a substance did it become when you prepared it with 400 ml, for example?
> *Student 10:* [Shows the size with hand]
> *Student 11:* It is huge. Let us try half of it with 200 ml.
> *Student 10:* I think if we mix it with wheat flour, it will be doughier, there will be no lumps in it.
> *Student 12:* We determine the ratio of our own recipe. We also get ideas from the recipes here.
> *Student 10:* There will be water in the oobleck substance, we cannot say for sure that it will be this much right now. We will add it according to its consistency. Only the water in the oobleck substance will be enough.

This dialogue segment could be accepted as evidence that the students offered an extraordinary, original, and appropriate solution to the problem and so showed *exemplary creativite approach* modeling sub-competency. In particular, student 10 transformed their previous knowledge of science and experiences into the problem situation. The above discussion allowed elementary gifted students in-depth investigation of their thinking, leading to *exemplary* evidence of *explaining*. In the end, they used 460 ml of water, 100 g salt, 107 gr cornstarch, and 300 g flour. This showed evidence that they did not stick to the predetermined conditions they set. Hence, they differentiated the strategies and adapted the model based on the changing conditions they encountered. Quite interestingly, group D exhibited *exemplary* indicators of *adaptability and transferability* by adapting their model to the new situation and transferring their knowledge of science and mathematics into making their second model. As stated above, extract of group D indicated that they also considered possible approaches for the problem, an undertaking indicative of the engineering sub-competency *(deriving an elegant solution).* As a concluding point, it can be asserted that the members of group D displayed *exemplary* evidence of *deriving an elegant solution* by achieving maximal effect with appropriate methods and materials as a result of their efforts.

As the final example of an engineering-based model eliciting activity, the findings of the *Dr. Ahmet's Will* activity regarding the adjusting competency of groups E and F are provided. The sub-competencies of adjusting are categorized under the subtitles of *refining and testing*, *explaining*, *deriving an elegant solution*, *adaptability and transferability* as well as *creative approach.* In addition, the extent to which elementary gifted students displayed the adjusting modeling competency and related sub-modeling competencies when engaging in the *Dr. Ahmet's Will* activity is presented as shown in Figure 4.43. According to de Villiers's (2018) group modeling competency observation guide, the sub-modeling competencies of the groups for the adjusting competency were classified as follows: 0 *(unsatisfactory)*, 1 *(emerging/developing)*, 2 *(proficient)*, and *3 (exemplary).*

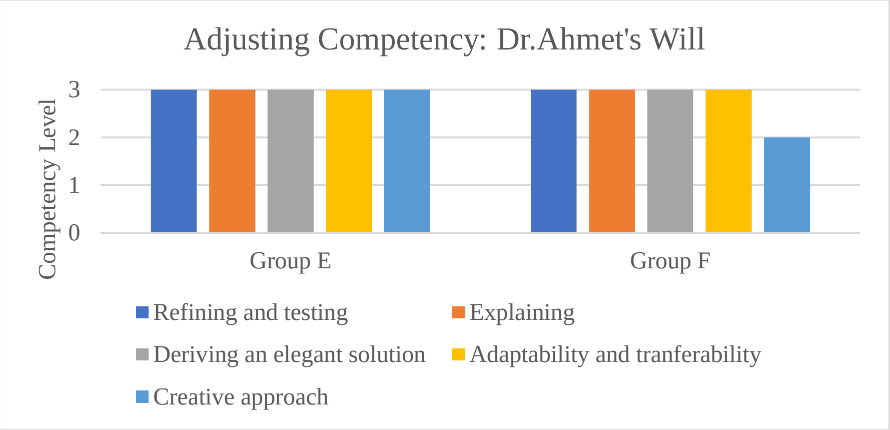


Figure 4.43. Adjusting Competency: Dr. Ahmet's Will

As seen in Figure 4.43, when the students in groups E and F engaged in the *Dr. Ahmet's Will* activity, they generally exhibited exemplary sub-modeling competencies of adjusting. However, the students in group F displayed a proficient level of the *creative approach* sub-modeling competency when engaging the activity. Thus, examples of this competency in relation to the aforementioned sub-categories are presented below.

Refining and testing, and explaining. The analysis of the data revealed that both groups adapted the model to the new situation *(refining and testing)*. According to the current case, three of the relatives were diagnosed with asthma, lymphoma, and albinism, and the lawyer was asked to reconsider the choices of elementary gifted students in light of the new information. First of all, the students determined the kinds of environments in which individuals with such diseases lived. Then, they compared the conditions they determined with the conditions suitable for people with these diseases. Thus, these might be acceptable indicators of the engineering sub-competency because both groups identified interested and affected parties and their expectations and the environmental impacts of the engineering activity as well as sustainability issues *(explaining)*.

Furthermore, they stated that they do not intend to change their choices if appropriate. When reviewing the models, the groups did not change their first and last preferences for the new situation, but they changed other options *(refining and testing)*. The reflections of the members of group F on their models were as follows:

> *Student 18:* To begin, we decided which office in which city would be the most suitable. We looked at the weather conditions where the relatives live, and then

> we looked at the weather conditions where the offices are located. According to us, it was Geneva that suited both.
> *Student 19:* We reviewed all of the precipitation, temperature degrees, and altitude, etc. Then, we discussed diseases.
> *Student 16:* We wanted to decide again which is the most appropriate, including diseases.
> *Student 17:* Geneva seemed like a pleasant place, the degree of pollution is low and other conditions are also suitable.
> *Student 19:* We paid attention to the characteristics of each person when identifying diseases.
> *Student 17:* We put Istanbul in the last place before Kanpur. Then, we moved one row.
> *Student 18:* We assumed as much because Aunt Hatice would be unable to travel for an extended period of time due to her old age. Additionally, Aunt Belma also had seven children. Personality characteristics and rainfall were important factors in our decision.

The dialogue here shows evidence that group F adapted parts of their model to make sense in the new situation *(refining and testing).* Although their solutions partially fit the problem situation, they reconsidered all the conditions again. Together these findings provided *exemplary* evidence of *refining and testing* because both groups related the underlying structure of the problem with the new case and created a new model for the situation by adjusting parts of their initial model.

Deriving an elegant solution, adaptability and transferability, and creative approach. When the elementary gifted students' reflections were examined, they offered strategies to minimize the negative consequences of the engineering activity - an indicator of the engineering sub-competency *(deriving an elegant solution).* That is to say, they developed a new method to reduce the negative consequences of many factors such as the illnesses of the relatives, age, economic situation, and weather conditions. As stated in the below extract, the members of group F eliminated some variables in the same way that they followed for their first model:

> *Student 16:* We validated that all the conditions required from us were met.
> *Student 18:* We went through all of them in turn. We eliminated them one by one.
> *Student 17:* We eliminated Kanpur first. The heat and pollution were too much. There was no flight.
> *Student 19:* We continued like that, our opinion did not change for the second time.
> *Student 16:* We found out which of the cards with the characteristics of the people has which disease.
> ….

> *Student 17:* Our first option did not change when the situation occurred, but we changed the others.
> *Student 18:* In my opinion, the first place we chose has not changed since we examined the diseases from the character cards. For both, we actually paid attention to them.
> *Student 19:* I agree.
> *Student 17:* Our first choice is always the best, but our other preferences have changed because of diseases and our perspective on them has changed as well.

This conversation above indicates that group F flowed logically from one step to the next to explain their process as they considered all the conditions. Beyond that, they showed *exemplary* evidence of the *explaining* sub-modeling competency since they explained their reasoning in-depth. As a result of long discussions, the members of group F assumed that Uncle Nedim suffered from asthma, Cousin Ezgi had albinism, and Cousin Ozan had lymphoma. To reach this assumption, they matched the characteristics of the relatives, the location where they live, the appropriate conditions where patients live and diverse characteristics of the disease. For example, they concluded that cousin Ezgi generally works on the computer and she does not need to go out much. They stated that she is not exposed to the sun and so she could have albinism. This showed *exemplary* evidence that the students in group F adapted their solution easily and transferred the previous knowledge, knowledge of science, and medicine to the problem situation when adjusting the problem *(adaptability and transferability)*. On the other hand, the above statements indicated that group F substantially took a creative approach to solve the problem and explained the underlying reasoning. This could be considered *proficient* evidence of the *creative approach* sub-modeling competency because their proposals were based on the data that were given for the problem, and none beyond it. Furthermore, the following dialogue is given as an example of how the members of group E discussed their illness:

> *Student 14:* For example, aunt Hatice cannot have albinism. She is always out in the sun. Someone who suffers from asthma requires fresh air, which should not be too hot.
> *Student 15:* Let us examine Geneva from these perspectives. First we need a clean place.
> *Student 14:* We do not know who possesses which diseases. Let us find this first.
> *Student 13:* It is not necessary, instead let us select a location that fits these characteristics. How can the relatives with these diseases travel to a common place?

As shown in the above extract, elementary gifted students investigated each relative and examined their suitability for the situation. On the other hand, Student 13 gave an in-depth explanation of one's reasoning. However, the other group members did not support that idea and examined all the relatives to determine which diseases they have:

> *Student 14:* We need to find who has each disease. Cousin Mert does not suffer from asthma because he is interested in plants.
> *Student 15:* Uncle Nedim cannot suffer from asthma either.
> *Student 13:* We have three diseases. Has cousin Mert been diagnosed with albinism?
> *Student 15:* How old is he? If tulips are in the garden, he should take care of them.
> *Student 14:* Therefore, he cannot have albinism. But I'm not so sure. It is acceptable if he goes out every week.
> *Student 13:* He can go out wearing a hat. He lives in Amsterdam. Amsterdam is located to the north of us, and it is not very sunny where he lives.
> *Student 15:* My grandfather is also bald and does not go out much because he has no hair.
> *Student 14:* Much evidence contradicts your claims. That is what we are trying to convey to you.
> *Student 13:* Cousin Mert and uncle Nedim are taking care of plants. I do not think they suffer from asthma.
> …

The extract above presents a discussion highlighting how elementary gifted students identify interested and affected parties and their expectations as well as the environmental impacts of the engineering activity *(explaining)*. These could be accepted as *exemplary* indicators of *explaining* for group E because of the in-depth investigation of their reasoning. In addition, the dialogue above indicates that elementary gifted students in group E easily adapted to new circumstances and transferred their previous knowledge and other disciplinary knowledge into the solution process of the problem and so showed *exemplary* evidence of the *adaptability and transferability* sub-modeling competency. On the other hand, they were requested to learn the degree of pollution of the place where the relatives live. The researcher replied to this request by saying that if you need this information, you can search for it on the internet which is available for you. Hence, they manifested *exemplary* evidence of the *creative approach* sub-modeling competency in their attempt to solve the problem since they came up with a new, unique, and effective idea as well as one that is suitable regarding the restrictions of the problem. In the following extract, they discuss the degree of pollution considering the conditions of the location:

> *Student 14:* Cleaning and fresh air are important for Lymphoma.
> *Student 15:* The weather condition is as well.
> *Student 14:* It could be Buenos Aires or Sydney. But, I thought travel time is too much and the cost of travel is expensive.
> *Student 13:* Now, we look at the snowfall for cleaning. Let us take a look at where they live.
> *Student 14:* Kenya is in Africa, so uncle Nedim has not definitely albinism.

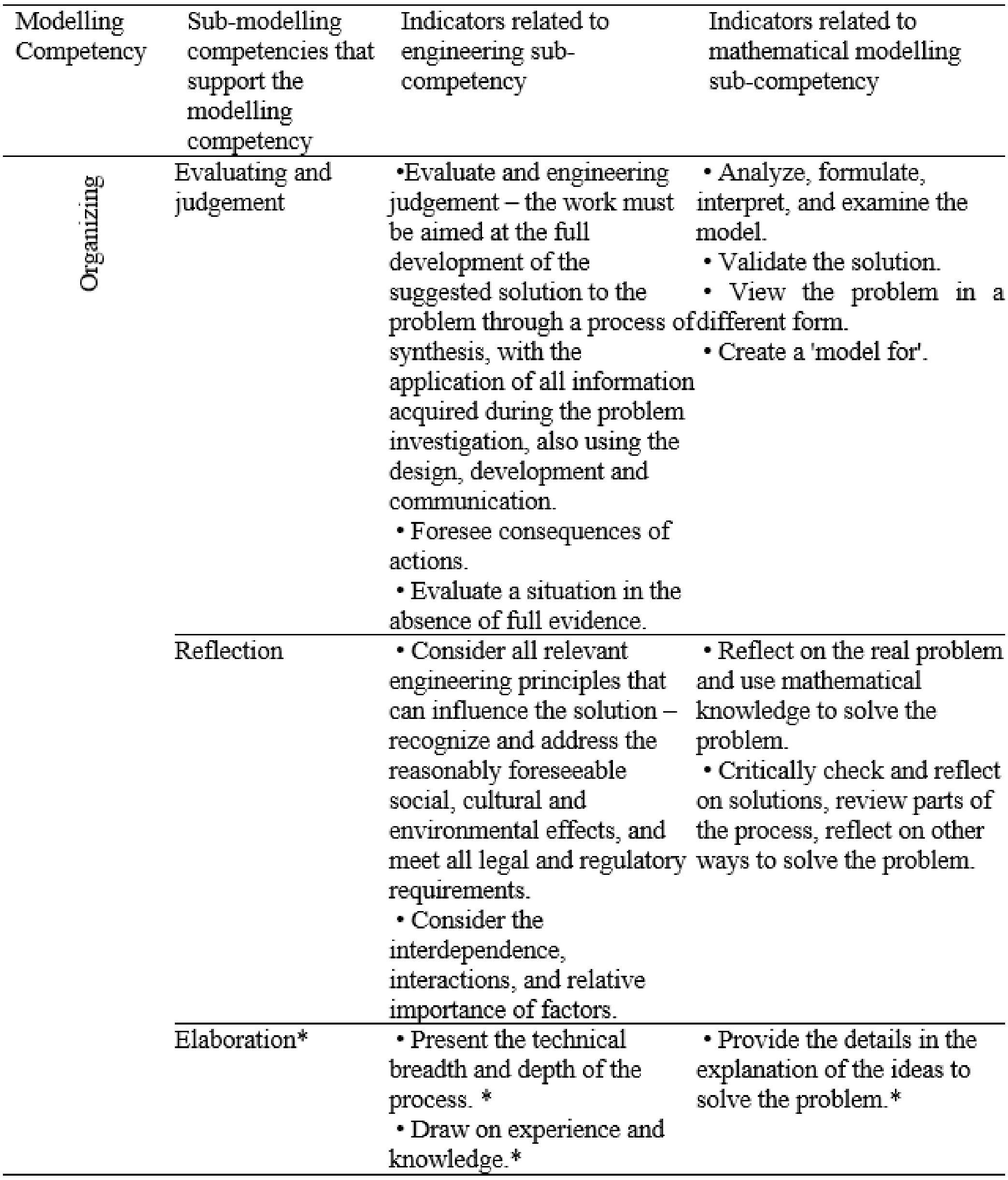
In this example, students also associated snowfall with cleaning. On the other hand, they took into consideration travel time and cost. Taking all of the aforementioned into account, elementary gifted children enhanced and adjusted their models through integration. Hence, both groups were capable of deriving *an exemplary elegant solution* to the problem by providing appropriate approaches for obtaining the maximum desired effect. Additionally, both groups assessed the appropriateness of solutions by examining, thinking on, and reconciling them with the original problem.

To summarize, the adjusting competency of elementary gifted students when they are engaged in engineering-based model eliciting activities were categorized under the sub modeling competencies of *refining and testing*, *explaining*, *deriving an elegant solution*, *adaptability and transferability*, as well as *creative approach*. In addition to the group modeling competency observation guide proposed by de Villiers (2018), *adaptability and transferability*, and *creative approach* sub-modeling competencies emerged from the data of the current study. The findings of this study demonstrated that elementary gifted students in groups exhibited exemplary instances when they engaged in engineering-based model eliciting activities by transferring previous knowledge from inside or outside of the school setting into the new situation comfortably. In addition to the competency to transfer ideas, they took a creative approach and gave in-depth explanations of their reasoning when they are engaged in engineering-based MEAs.

4.6. Modeling Competency 6: Organizing

This section of the chapter addresses the findings for organizing, the sixth modeling competency from the classification of cognitive modeling competencies proposed by de Villiers (2018). The organizing competency was examined regarding the sub-modeling competencies: *evaluating and judgment*, *reflection* and *elaboration*. In addition, this section investigates the extent to which elementary gifted students in groups displayed the organizing modeling competency and its sub-modeling competencies when engaging in *Bridge Construction*, *Mars Lunarcrete*, and *Dr. Ahmet's Will* activities. The findings for each activity include examples of two distinct groups displaying the sub-modeling competencies of elementary gifted students. In particular, the organizing competency of elementary gifted students was analyzed through these sub-modeling competencies and their related engineering and mathematical modeling sub-competency as shown in Table 4.6. In the table, new codes that emerged from the analysis of the current study are indicated with "*".

Table 4.6. Classification of organizing competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Organizing	Evaluating and judgement	•Evaluate and engineering judgement – the work must be aimed at the full development of the suggested solution to the problem through a process of synthesis, with the application of all information acquired during the problem investigation, also using the design, development and communication. • Foresee consequences of actions. • Evaluate a situation in the absence of full evidence.	• Analyze, formulate, interpret, and examine the model. • Validate the solution. • View the problem in a different form. • Create a 'model for'.
	Reflection	• Consider all relevant engineering principles that can influence the solution – recognize and address the reasonably foreseeable social, cultural and environmental effects, and meet all legal and regulatory requirements. • Consider the interdependence, interactions, and relative importance of factors.	• Reflect on the real problem and use mathematical knowledge to solve the problem. • Critically check and reflect on solutions, review parts of the process, reflect on other ways to solve the problem.
	Elaboration*	• Present the technical breadth and depth of the process. * • Draw on experience and knowledge.*	• Provide the details in the explanation of the ideas to solve the problem.*

As seen in Table 4.6, the analysis of this study revealed that in addition to the organizing sub-competencies of *evaluating and judgment* and *reflection* suggested by de Villiers (2018), *elaboration* emerged as a sub-code based on the findings of the current study. The indicators of the organizing sub-competencies related to engineering and mathematical modeling observed when elementary gifted students engaged in engineering-based MEAs are presented in Table 4.6.

First of all, the organizing competency of groups A and B was examined in the context of the *Bridge Construction* activity in terms of *evaluating and judgment, reflection* as well as *elaboration*. The distribution of each sub-competency regarding the extent to which elementary gifted students in groups A and B exhibited the organizing modeling competency is represented in Figure 4.44. Based on the group modeling competency observation guide proposed by de Villiers (2018), the level of sub-modeling competencies for the groups was classified as 0 *(unsatisfactory),* 1 *(emergent/developing),* 2 *(proficient)* and 3 *(exemplary).*

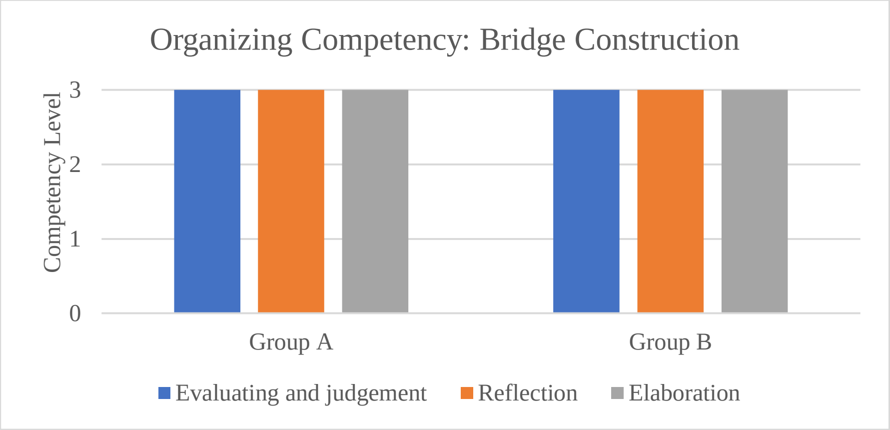


Figure 4.44. Organizing Competency: Bridge Construction

As shown in Figure 4.44, the findings of the current study indicated that when the elementary gifted students in groups A and B engaged in the *Bridge Construction* activity, they exhibited *exemplary* performance of the modeling sub-competencies of organizing. To clarify and expand on the aforementioned categories, sample excerpts representative of each category are provided below.

Evaluating and judgment. Analysis of the data revealed that both groups displayed the engineering competency in that all of the information gathered was used to produce a comprehensive solution through a process of synthesis that included design, development, and communication during the investigation of the problem *(evaluating and judgement)*. First of all, group A evaluated their final bridge prototype as steady and aesthetic. They stated that they used the bridge designs in London as an example. They also pointed out that they had a basic idea at first but they develop it later. They thought the bridge would last 180 years. This showed evidence that the students in group A analyzed, interpreted, and examined the model *(evaluating and judgements).* In addition, they evaluated a situation in which there was lack of evidence by making inferences about the durability of their bridge prototype, indicating evidence of the engineering sub-competency of *evaluating and judgements.*

Compared to group A's one bridge prototype, group B evaluated the two bridge prototypes that they made. While they thought that their first prototype was aesthetic and appropriate for the city, the final prototype was determined to be more useful, less expensive, and appropriate for village conditions. This indicated that they viewed the problem in a different form after constructing their first bridge prototype *(evaluating and judgement)* and means that the members of group B validated their solution for the problem situation and constraints. Consequently, they pointed out that they did a good job as a team and included all of the group members' decisions into their design process. They commented as follows:

> *Student 4*: If we constructed the bridge for the city, we would prefer the first prototype because it fits in better with the lighting of the city. But we do not think the final prototype will be preferred for the city. This bridge can be used for bridges over streams.
> *Student 6*: If we had the necessary materials, we could enlarge this bridge to create a double-sided road. However, we did it this way because both the conditions of the village and our materials were not convenient.

The above excerpt indicated that the members of group B foresaw the consequences of their actions, an indicator of the engineering competency of *evaluating and judgment*. Together these findings for both groups provided *exemplary* evidence regarding the *evaluating and judgment* sub-modeling competency through comprehensive, insightful analysis, synthesis, and evaluation, as well as the clear connections established in real-life and problem situations.

Reflection and elaboration. When elementary gifted students' *reflections* on their bridge prototypes and modeling processes were examined, it was revealed that they considered relevant principles that can influence the solution. Correspondingly, the members of group A expressed that the final prototype looks like their sketch. Their *reflection* about the prototypes was as follows:

> *Student 1*: We think it is appropriate for real life and problems. But the letters can be made more aesthetic and better.
> *Student 3*: We could have enlarged the roads, but this would increase the cost. The peasants might not accept this.
> *Student 2*: As it is, two cars can comfortably move. We might not have made the arrow either. So, the cost has increased. We could make the pedestrian path smaller, even if it was small, people could cross it easily.

The above excerpt indicated that they critically checked and reflected on their prototype as well as other ways to construct their bridge *(reflection)*. This showed evidence that they recognized and addressed social, cultural, and environmental consequences that are reasonably predictable, which is an indicator of the engineering sub-competency of *reflection*. In addition, the dialogues between two students (Student 1 and Student 5) on group A's final prototype of group A is shown here:

> *Student 5*: The end of the arch and the end of the road are not the same, cars can fall from here.
> *Student 1*: No here, as you said, it can cross from land to land here, like your bridge.
> *Student 5*: Okay, does this arch not block that road?
> *Student 1*: It does not prevent it because it can separate these two roads.
> …
> *Student* 5: For example, have you fixed the barrier that happens when people walk this road?
> *Student 1*: Yes, we fixed them. You can look.
> …
> *Student 5*: What if this bridge bends from wind or something? You told me the same. I think this condition is more valid for your bridge since all abutments are in one line. I think you can distribute the forces and so it can be more stable. As I said, instead of the thick foot in the middle, you could construct a smaller abutment and increase the number of abutments.
> *Student 1:* If we made it smaller, it would swing more. We had to put in the fixed holders.
> *Student 5*: I think it would be more solid if you did it smaller and more. I like your water removal system. I do not understand why you put the straws on the tip.
> *Student 1*: We could not do it from here and we could not fix the rope to it.

This conversation indicates that the elementary gifted students reflect on the social, safety, and environmental implications of their decisions, which is an indication of the engineering sub-competency of *reflection*. As deduced from the above dialogue, the students provided the details in the explanation of the ideas to construct the bridge *(elaboration)*. More precisely, they presented the technical breadth and depth of the process, which reflects evidence of the engineering sub-competency by detailing the distribution of the force, the effect of weather conditions, and safety issues (*elaboration*). In addition to this, the next extract highlights their own self-reflection:

> *Student 5*: Why did you put a stick here? Is there a double-sided road here?
> *Student 1*: Yes.
> *Student 5*: If two cars are as much as my two fingers, two people can be about one finger, right?
> *Student 1*: Yes.
> *Student 5*: Here you have made this pedestrian path 10 times the size of a finger.

This shows that Student 5 analyzed and interpreted the prototype by applying mathematics knowledge and transferred the knowledge into the real world *(reflection)*. The dialogues above and below also revealed that elementary gifted students *elaborate* on their ideas by drawing on their own experiences as well as knowledge of mathematics. For example:

> *Student 1:* We did it because we thought a lot of people could pass through the road.
> *Student 5:* Will 10 people go side by side?
> *Student 1:* No. For example, in Kızılay (one of the more crowded districts in Ankara), people pass side by side quickly. We thought that if we did this, people would pass quickly.
> *Student 5*: Well, you are constructing this bridge for a village, not for Kızılay. So, I do not think the village will be as crowded as Kızılay.
> *Student 1*: The village will not be crowded, you are right, but we thought that it should pass quickly, so that pedestrians do not get angry.

This shows that the elementary gifted students used mathematical knowledge to evaluate the prototype according to real life situations and relate the situation to similar ideas previously experienced. Thus, both groups showed *exemplary* indicators of *reflection* through recognizing the strengths and weaknesses of their own thinking, as well as different viewpoints on the situation as compared to others. They also evaluated their own thinking in the context of alternate points of views. In a similar vein, the dialogue between student 1 and 5 on both prototypes of group B was as follows:

> *Student 1*: You added more abutments. But you could make them much stronger. If you did, it would not bend from the wind.
> *Student 5*: We will strike the abutments into the sand or stone underwater. So, the abutments cannot move anyway. Our first bridge is more useful than a suspension bridge.
> *Student 1:* But in the news we saw, it became more fixed. It did not swing like yours.
> *Student 5*: If these abutments are fixed on the ground, I do not think that it will swing much because we fixed them with a stabilizer. Therefore, the probability of dislodgement is very low.
> *Student 1*: I think these decorations will exceed the cost of the village. You could choose to reinforce the costly bridge more than to decorate it.
> *Student 5*: This is why we construct the second bridge.

As indicated in the above excerpts, the students discuss whether their prototype meets the requirements in the problem. The discussions above and below also showed that they considered the interdependence, interactions, and relative importance of factors such as strength, safety, materials, and cost, which reflects the engineering sub-competency *(reflection)*. For example:

> *Student 1*: For example, what will the final bridge be on a rainy day?
> *Student 5*: We do not think there will be many problems on stormy days. Like this normal road, the only difference is that it is above the water, not the land. Just like the road, there are sidewalks and crosswalks.
> *Student 1*: Is the bridge over the water? 10 meters below that. I am telling you how these cars will go up.
> *Student 5*: This will stand 10 meters; it has an engine. It will stand in line with the land. The water does not come that much. Its meanness was more when we saw it in the news. It can vary according to the altitude and the height of the water, and the shape of the place. For example, this can be done as a slope, it can be done as a ramp. Our materials were suitable for this, so we made it straight.

These dialogues demonstrate that the students also recognized the environmental effects of the prototype. They provided the technical breadth and depth of the process by applying science, mathematics, and engineering knowledge *(elaboration)*. The above dialogues between two different group members indicate that they not only explained and justified their design for their own group members but also the other group members verbally. This showed evidence that they *elaborate* on the ideas by interacting with each other. As a concluding point, it can be asserted that both groups exhibited e*xemplary* evidence of *elaboration* by providing many details in the explanation of ideas and by integrating the technical breadth and depth of many different disciplines.

Secondly, the findings related to organizing competency of groups C and D's engagement in the *Mars Lunarcrete* activity were investigated under the sub-competencies of *evaluating and judgment, reflection* and *elaboration*. In addition, the extent to which the groups displayed the organizing modeling competency and its sub-competencies when engaging in the activity are given in Figure 4.45. According to the group modeling competence observation guide proposed by de Villiers (2018), the sub-modeling competencies of the groups for organizing competency were categorized as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)*, and 3 *(exemplary)* proposed by de Villiers (2018).

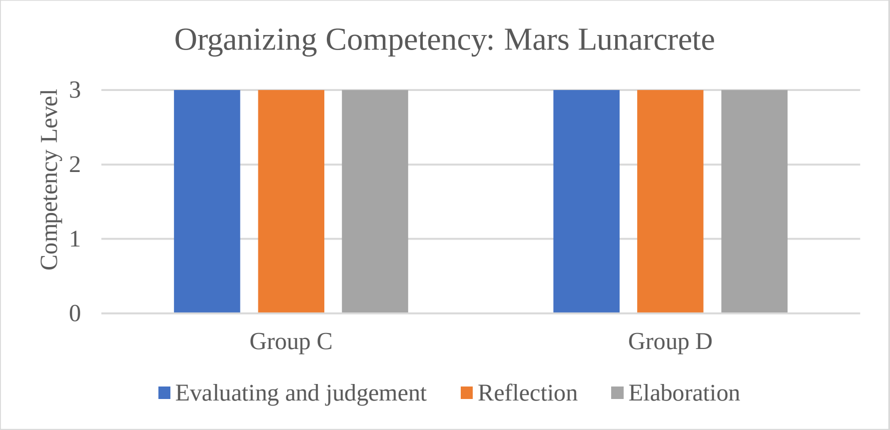


Figure 4.45. Organizing Competency: Mars Lunarcrete

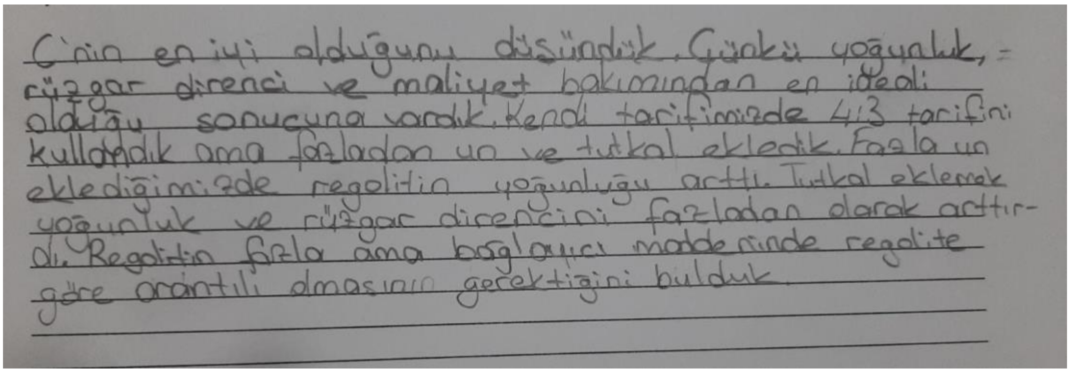
As illustrated in Figure 4.45, elementary gifted students in groups C and D displayed exemplary sub-modeling competencies of organizing when they engaged in the *Mars Lunarcrete* activity. In order to clarify and elaborate on the aforementioned categories, some specific instances of the organizing competency for each category are presented below.

Evaluating and judgment. The findings of the current study revealed that elementary gifted students analyzed, formulated, interpreted, and examined their models *(evaluating and judgment).* For instance, group C evaluated their first and second model. As stated in the adjusting competency section of this chapter, their model was easily friable and unstable since the amount of salt included was high. Hence, this showed evidence of the engineering sub-competency in that they foresaw the consequences of their action by making inferences that the structure of the model resulted from the increasing amount of substance *(evaluating and judgment).* For the second model, the members of group C pointed out that their model was not rigid and they had difficulty in shaping it because of the glue they used. Although they did not create a thorough working model, they made an inference based on their experiences. This indicated evidence of the engineering sub-competency of *evaluating and judgment* in that they examined the problem situation in the absence of full evidence. Besides, they made an engineering judgment on the suggested solutions to the problem through a process of synthesis *(evaluating and judgment).* On the other hand, the students in group C wanted to make sure that the precision balance was weighing accurately when they built the model. In order to check, they asked the researcher to bring some items whose weight was known. They ended up using 50-gram brass weights to validate their measurements *(evaluating and judgment).* For example, they weighed the flour by filling the beaker completely, and then made their measurements by comparing the exact amount of flour they wanted to add. Taking all of these into account, the members of group C exhibited *exemplary* evidence regarding the *evaluating and judgment* sub-competency by providing comprehensive, insightful analysis, synthesis and evaluation, as well as clear connections to real-life situations and previous experiences. On the other hand, group D evaluated their two models by comparing them with each other in terms of suitability for the conditions *(evaluating and judgment)*:

> *Student 12:* I think this model will be more resistant than the first model when it dries. What are you thinking?
> *Student 11:* This is a more intense version. We will dry it in the sun to speed up the drying process.
> *Student 10:* It is softer and easier to shape. We reduced the setting time because the starch is drying. The first model has hardened, cracked, and crumbled as it dried.
> …
> *Student 11*: It is difficult to stand. How are we going to make the dome?
> *Student 12*: We will do it with dough. We should either fill it in, or we should lay a foundation there like a solid stick.
> *Student 11*: Be it triangle or hexagon. After all, we are not just going to make it just from concrete. It can stand on something thick and flat.
> *Student 10*: The column we know. Then we make a dome.

The above dialogues show that the students in group D adapted their model according to the new condition of reducing setting time. They suggested a solution to the problem through a process of synthesis, with the application of all information acquired during the problem investigation, also using design, development, and communication. The conversation above also indicates the engineering sub-competency that elementary gifted students foresaw the consequences of actions based on their experiences and knowledge. Thus, all these indicators stated above could be accepted as exemplary evidence of *evaluating and judgment*.

Reflection and elaboration. When elementary gifted students' reflections on the design process were examined, they chose the ideal regolith and binder solution considering the interdependence, interactions, and relative importance of factors *(reflection)*. For example, the reflections of group C for their second model is shown in Figure 4.46.



C'nin en iyi olduğunu düşündük. Çünkü yoğunluk, rüzgar direnci ve maliyet bakımından en ideali olduğu sonucuna vardık. Kendi tarifimizde 4:3 tarifini kullandık ama fazladan un ve tutkal ekledik. Fazla un eklediğimizde regolitin yoğunluğu arttı. Tutkal eklemek yoğunluk ve rüzgar direncini fazladan olarak arttırdı. Regolitin fazla ama bağlayıcı maddesinde regolite göre orantılı olmasının gerektiğini bulduk.

Figure 4.46. The reflections of group C for their second model

As presented in Figure 4.46, the members of group C stated that their choices were the most optimal in terms of density, wind resistance, and cost, among other factors. They claimed that they used the 4:3 recipe as a reference point for their recipes, but that they supplemented it with flour and glue. Then, they asserted that increasing the amount of flour in the regolith increases its density, while increasing the amount of glue in the regolith increases its density and wind resistance. They reached the conclusion that the regolith should be in excess and that the binder should be in proportion to the regolith. Taking all of the aforementioned reflections of group C, they critically checked and reflected on solutions, reviewed parts of the process, and reflected on other ways to solve the problem *(reflection)*. Correspondingly, they reflected on the real problem and used mathematical knowledge to solve the problem *(reflection)*. Beyond that, the members of group C provided the details in the explanation of the ideas by applying proportional reasoning *(elaboration)*. Hence, their reflections of group C exhibited *exemplary* evidence for *elaboration* since they demonstrated knowledge of the technical breadth and depth of the process by presenting many details and experiences.

In addition, the structure was constructed by group D using the building material that they had developed, as shown in Figure 4.47

Figure 4.47. The second model of group D

The members of group D tested its flexibility by applying pressure. They commented that "*It became more fluid since we put the oobleck substance into it. It does not crumble that much when compared to the first model. We can also increase our amount of building material by keeping the ratio of everything the same".* As stated in the sub-competency of assumption, they pointed out that they should have enough material to construct the settlement on Mars because the spacecraft may need to be sent again, resulting in a higher cost. The above dialogue between the members of group D shows evidence that they reflected on the real problem and assumptions they made by using mathematical knowledge *(reflection).* In addition to this, they provided more details in the explanation of ideas by presenting the technical breadth and depth of the process and showed *exemplary* evidence of *elaboration*. More specifically, their ideas were explicitly explained in detail by integrating their science, mathematics, and engineering knowledge and experiences. Consequently, these findings provide *exemplary* evidence for groups C and D regarding *reflection*, in that they identified strengths and weaknesses in their own thinking, as well as different perspectives on the situation, when the students compared their final models to the initial model of the situation.

Finally, this part of the section presents the findings from groups E and F about the organizing competency and its related sub-competencies when they are engaged in the *Dr. Ahmet's Will* activity. The organizing competency of elementary gifted students engaged in this activity was investigated in terms of the sub-modeling competencies: *evaluating and judgment*, *reflection* and *elaboration*. The extent to which elementary gifted students in groups displayed the organizing modeling competency and its sub-modeling competencies when they are engaged in the activity is shown in Figure 4.48. According to the group modeling competency observation guide proposed by de Villiers (2018), the sub-modeling competencies of the groups were classified as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)*, and 3 *(exemplary)*.

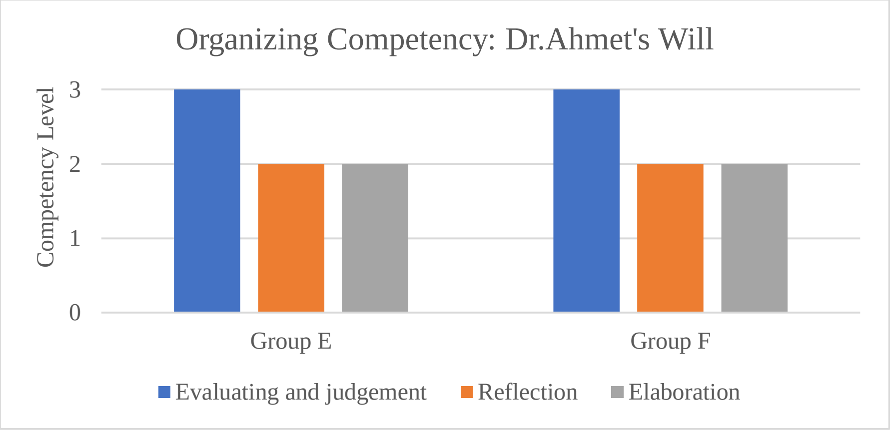
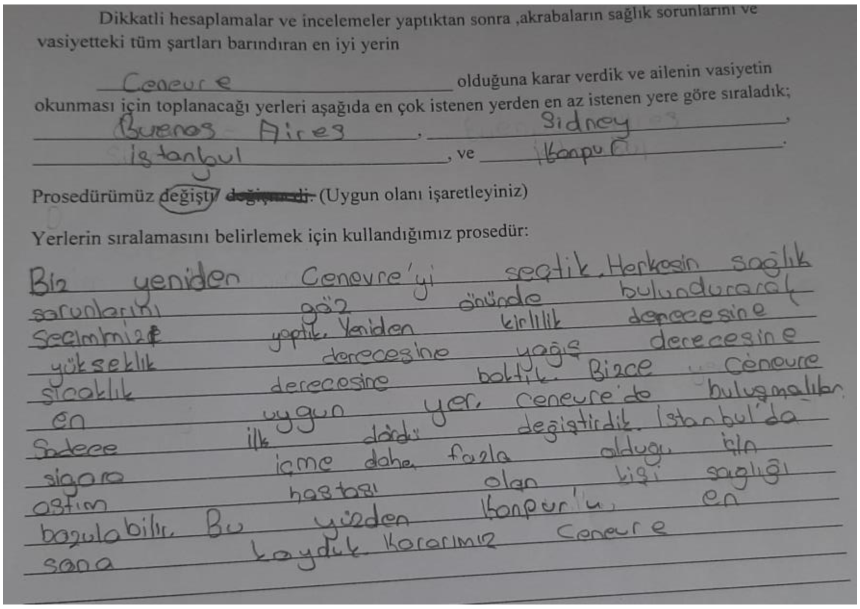


Figure 4.48. Organizing Competency: Dr. Ahmet's Will

As is shown in Figure 4.48, the findings of the current study revealed that the elementary gifted students in groups E and F generally displayed proficient evidence of the modeling sub-competencies of organizing when engaged in the *Dr. Ahmet's Will* activity. When the sub-competency of evaluating and judgment was considered, both groups showed *exemplary* evidence of the *evaluating and judgment* sub-modeling competency. As a result, the following part provides examples of this competency related to the sub-categories indicated above.

Evaluating and judgment. The findings of this study revealed that elementary gifted students viewed the *Dr. Ahmet's Will* problem in a different form for the current situation *(evaluating and judgment)*. That is to say, they validated their first strategy for the situation related to illness *(evaluating and judgement)*. As stated in the competency of the adjusting section of this chapter, the groups E and F analyzed, formulated, interpreted and examined the first model and created their model for the new situation *(evaluating and judgment)*. In addition, the analysis of the data revealed that they found their models to be quite similar. They expressed that both groups used almost the same methods to determine the best location for the relatives. An example regarding the *evaluating* and engineering *judgment* of group F is shown in Figure 4.49.



Dikkatli hesaplamalar ve incelemeler yaptıktan sonra ,akrabaların sağlık sorunlarını ve vasiyetteki tüm şartları barındıran en iyi yerin

Cenevre olduğuna karar verdik ve ailenin vasiyetin okunması için toplanacağı yerleri aşağıda en çok istenen yerden en az istenen yere göre sıraladık;

Buenos Aires, Sidney,

İstanbul, ve Kanpur.

Prosedürümüz [✓ değişti]/ ~~değişmedi~~. (Uygun olanı işaretleyiniz)

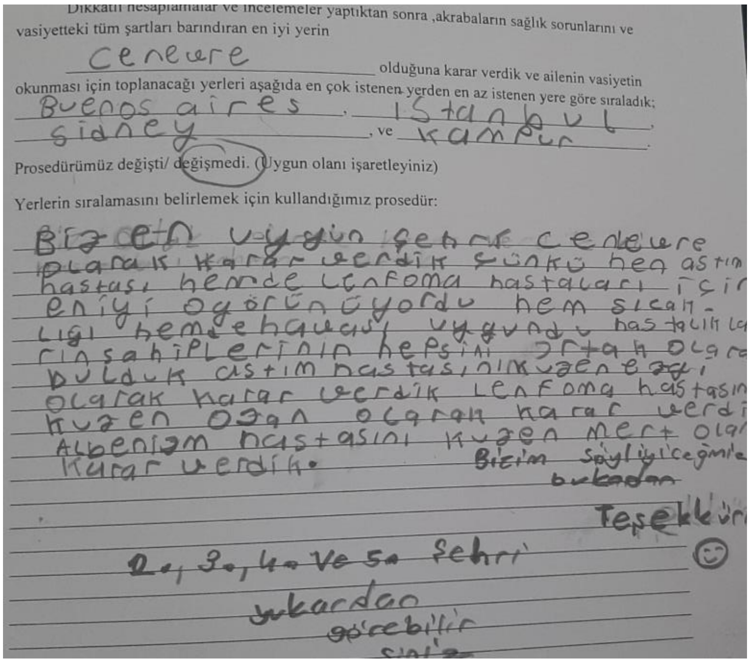
Yerlerin sıralamasını belirlemek için kullandığımız prosedür:

Biz yeniden Cenevre'yi seçtik. Herkesin sağlık
sorunlarını göz önünde bulundurarak
seçimimizi yaptık. Yeniden kirlilik derecesine
yükseklik derecesine yağış derecesine
sıcaklık derecesine baktık. Bizce Cenevre
en uygun yer. Cenevre'de buluşmalılar.
Sadece ilk dördü değiştirdik. İstanbul'da
sigara içme daha fazla olduğu için
astım hastası olan kişi sağlığı
bozulabilir. Bu yüzden Kanpur'u en
sona koyduk. Kararımız Cenevre

Figure 4.49. The second letter of group F

As presented in Figure 4.49, the members of group F validated their first choice by utilizing all knowledge gathered during the problem investigation such as illnesses, pollution level, rainfall, and the degree of temperature and altitude, and to develop the whole proposed solution *(evaluating and judgment).* In addition, they foresaw the consequences of their actions by considering that the smoking rate may affect the patients suffering from asthma *(evaluating and judgment).* The statements in the above sections show *exemplary* instances regarding *evaluating and judgment* through the provision of comprehensive, insightful analysis, synthesis, and evaluation, and the establishment of clear connections to real-world circumstances and previous information.

Reflection and elaboration. The analysis of the data indicated that elementary gifted students, when they are engaged in the *Dr. Ahmet's Will* activity, recognized and addressed social, cultural, and environmental consequences that are reasonably predictable, and thus demonstrated an indicator of the engineering sub-competency of *reflection*. They reflected on the real problem and used mathematical knowledge to solve the problem, especially interpretation of the data. In addition, both groups provided substantial details explaining the ideas and depth of the process as described in the preceding section and the second letters, and showed *proficient* evidence of the *elaboration* sub-competency. Although both groups critically checked and reviewed parts of the process, they did not reflect on the methods they used in their letters for the second model. For instance, group F did not reflect on the decisions about their letter, as shown in Figure 4.50.



Dikkatli hesaplamalar ve incelemeler yaptıktan sonra ,akrabaların sağlık sorunlarını ve vasiyetteki tüm şartları barındıran en iyi yerin Cenevre olduğuna karar verdik ve ailenin vasiyetin okunması için toplanacağı yerleri aşağıda en çok istenen yerden en az istenen yere göre sıraladık;

Buenos aires, İstanbul,
sidney, ve Kampur.

Prosedürümüz değişti/ [✓ değişmedi]. (Uygun olanı işaretleyiniz)

Yerlerin sıralamasını belirlemek için kullandığımız prosedür:

Biz en uygun şehri Cenevre olarak karar verdik çünkü hem astım hastası hemde lenfoma hastaları için en iyi görünüyordu hem sıcaklığı hemde havası uygundu hastalık ların sahiplerinin hepsini ortak olara bulduk astım hastasının kuzen ezgi olarak karar verdik lenfoma hastasın kuzen ozan olarak karar verdi Albenizm hastasını kuzen mert olar karar verdik.

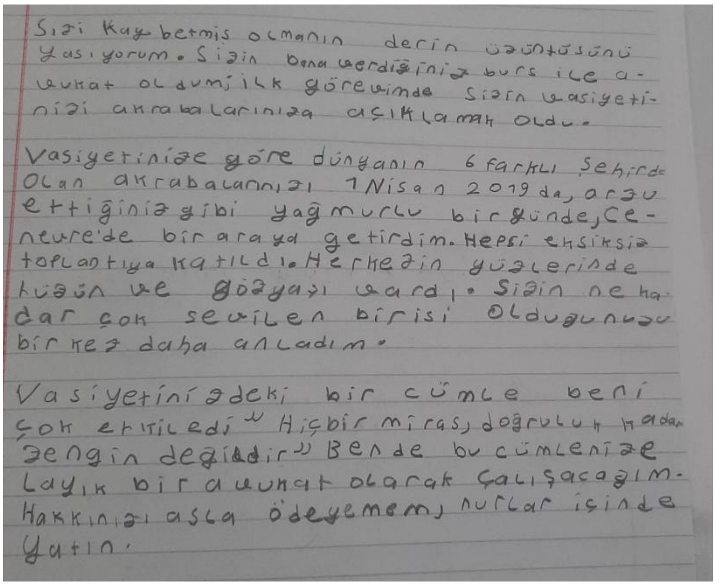
Bizim söyleyiceğimiz bukadar

Teşekkür

2, 3, 4 ve 5 şehri yukardan görebilir siniz

Figure 4.50. The second letter of group E

In the letter as shown in Figure 4.50, the members of group E gave information about assumptions they made related to diseases and some criteria they applied to determine the best location. This can be seen as evidence that they drew on experience and knowledge *(elaboration)*. Although the above dialogues of group E as mentioned in the previous section critically checked and revised part of the process, they did not reflect on the process in writing the letter. This showed evidence that they did not communicate with stakeholders in an effective way. However, student 14 preferred to write a letter to Dr. Ahmet individually, unlike group E. The student reported taking responsibility for determining the best location for the relatives as seen in Figure 4.51.



Sizi kaybetmis olmanın derin üzüntüsünü yasıyorum. Sizin bana verdiğiniz burs ile avukat oldum; ilk görevimde sizin vasiyetinizi akrabalarınıza açıklamak oldu.

Vasiyetinize göre dünyanın 6 farklı şehirde olan akrabalarınızı 7 Nisan 2019 da, arzu ettiğiniz gibi yağmurlu bir günde, Cenevre'de bir araya getirdim. Hepsi eksiksiz toplantıya katıldı. Herkezin yüzlerinde hüzün ve gözyaşı vardı. Sizin ne kadar çok sevilen birisi olduğunuzu bir kez daha anladım.

Vasiyetinizdeki bir cümle beni çok etkiledi "Hiçbir miras, doğruluk kadar zengin değildir" Bende bu cümlenize layık bir avukat olarak çalışacağım. Hakkınızı asla ödeyemem, nurlar içinde yatın.

Figure 4.51. The reflection of student 15

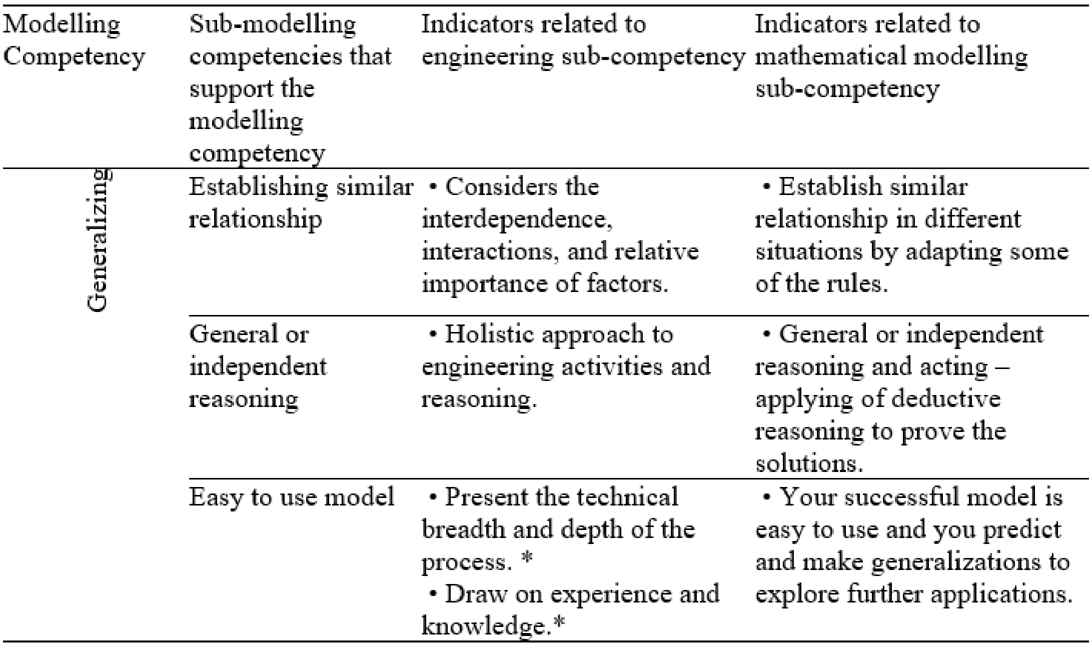
As presented in Figure 4.51, student 15 stated that one became a lawyer with the scholarship received from Dr. Ahmet and that one’s first duty was to fulfill his will. Hence, the student showed a high level of imagination in problem solving, taking conditions and constraints into account. Together these findings provide evidence that the elementary gifted students did not reflect on all the processes in the letters that they wrote. As a result, analyzing the entire process holistically became critical for examining elementary gifted students’ experiences when they are engaged in engineering-based MEAs. Accordingly, both groups exhibited *proficient* sub-competency in terms of *reflection* when their overall design processes are considered. Although they identified the strengths and weaknesses in their thinking and alternative perspectives about the problem when comparing the first situation, they did not evaluate them in the context of alternative perspectives.

To summarize, the organizing competency of elementary gifted students when they engaged in engineering-based MEAs were categorized under the sub modeling competencies of *evaluating and judgment*, *reflection* and *elaboration*. In addition to the group modeling competency observation guide proposed by de Villiers (2018), *elaboration* of sub-modeling competency emerged from the data of the current study. The findings of this study demonstrated that elementary gifted students in groups exhibited *exemplary* instances of engagement with engineering-based MEAs in that they provided an in-depth description along with the best idea that they selected. In addition, they generally displayed *exemplary* evidence of the competency to reflect on their own thought process, considering relevant principles that can influence the solution. As stated before, their judgment was rich in content and presented clear analysis, synthesis, and connections to real-life situations or previous content. On the other hand, holistic analysis of the entire process became crucial for evaluating the experiences of elementary gifted students engaging in engineering-based MEAs since they did not present the details of their explanations in writing.

4.7. Modeling Competency 7: Generalizing

The final modeling competency based on the classification of cognitive modeling competencies suggested by de Villiers (2008), the findings regarding the generalizing competency are stated in this section of the chapter. In this regard, the generalizing competency was investigated with respect to the sub-modeling competencies of *establishing a similar relationship*, *general or independent reasoning* and an easy *to use model*. In addition, the extent to which elementary gifted students in groups displayed the generalizing modeling competency and its sub-modeling competencies when they engaged in *Bridge Construction*, *Mars Lunarcrete*, and *Dr. Ahmet's Will* activities are presented in this section. In each activity, examples of two different groups are provided to demonstrate the sub-modeling competencies of the students in the study. Specifically, the generalizing competency of the elementary gifted students was analyzed through its sub-modeling competencies and their indicators related to engineering and mathematical modeling sub-competency as shown in Table 4.7. In this table, new codes that emerged within the scope of the current research are highlighted with "*".

Table 4.7. Classification of generalizing competency framework



Modelling Competency	Sub-modelling competencies that support the modelling competency	Indicators related to engineering sub-competency	Indicators related to mathematical modelling sub-competency
Generalizing	Establishing similar relationship	• Considers the interdependence, interactions, and relative importance of factors.	• Establish similar relationship in different situations by adapting some of the rules.
	General or independent reasoning	• Holistic approach to engineering activities and reasoning.	• General or independent reasoning and acting – applying of deductive reasoning to prove the solutions.
	Easy to use model	• Present the technical breadth and depth of the process. * • Draw on experience and knowledge.*	• Your successful model is easy to use and you predict and make generalizations to explore further applications.

As presented in Table 4.7, the analysis of this study revealed that the sub-competencies of generalizing: *establishing a similar relationship, general or independent reasoning* and *an easy to use model* proposed by de Villiers (2018) emerged as sub-codes. Thus, the indicators of generalizing sub-competencies in terms of engineering and mathematical modeling sub-competencies when elementary gifted students are engaged in engineering-based MEAs are indicated in Table 4.7.

As the first example of engineering-based model eliciting activities, the findings for the *Bridge Construction* activity related to the generalizing competency of group A and B were examined through the sub-competencies of *establishing a similar relationship, general or independent reasoning and an easy to use model.* In addition, the extent to which elementary gifted students displayed the generalizing modeling competency and related sub-modeling competencies when engaging in the *Bridge Construction* activity is presented in Figure 4.52. Based on the group modeling competency observation guide proposed by de Villiers (2018), the sub-modeling competencies of the groups were classified as 0 *(unsatisfactory),* 1 *(emergent/developing),* 2 *(proficient)* and 3 *(exemplary).*

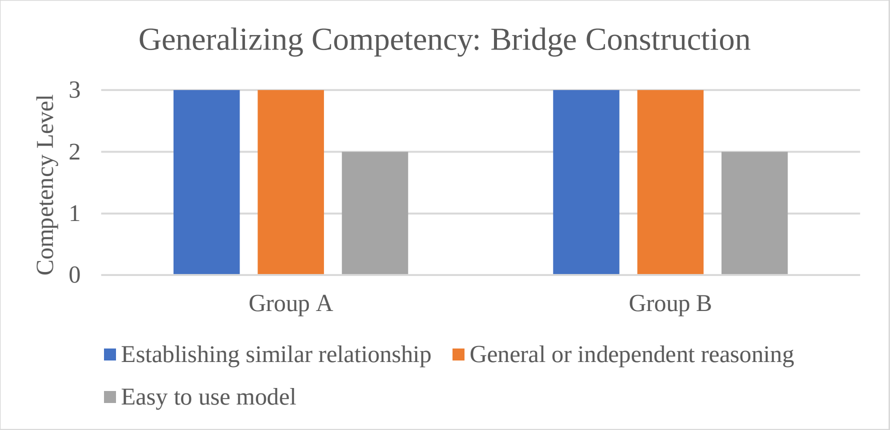


Figure 4.52. Generalizing Competency: Bridge Construction

As seen in Figure 4.52, when elementary gifted students in groups A and B engaged in the *Bridge Construction* activity, they generally exhibited *exemplary* evidence of the modeling sub-competencies of generalizing. From the graph above, it can be asserted that both groups displayed *proficient* evidence of the *easy to use model* sub-modeling competency. In order to explain and expand on the components of the generalizing competency, sample extracts which are representative of each sub-competency are provided below.

Establishing a similar relationship, *general or independent reasoning*, and *an easy to use model*. The reflections of groups A and B showed that the bridges they constructed can be used for other conditions in the same way or if some changes can be made, it can be expanded *(establishing a similar relationship)*. The below conversation shows evidence for their reasoning to prove the final prototype *(general or independent reasoning)*:

> *Student 5*: For example, why did you fix the arch with a rope? I think this is not solid enough. It could be stronger if you fix it with either wire or tongue stick like this.
> *Student 1*: We did not use the tongue sticks anymore because it could cost more, the wire could be too costly, so we used rope in the prototype. However, different things such as iron can be used while constructing the real bridge.

As deduced from the dialogue above, they considered the interdependence, interactions, and relative importance of factors by *establishing a similar relationship*.

In addition, as in the below dialogue, the students investigated the importance of the factors considered for the first and second prototype of group B to generalize the result. For the first bridge prototype:

> *Student 5*: I think these arrows and two signboards were a bit unnecessary because this bridge is in the village. I do not think that many visitors and tourists will come to the village. So, you could not make it. Instead, you could make the arch stronger or you could add one or two more abutments.
> *Student 1*: We made three abutments because we also have fixed it with extra holding mechanisms. We did what you said because we thought it would be a cute village

For the second prototype:

> *Student 1:* Can we use this elsewhere? Could you make normal small bridges instead?
> *Student 5:* We can, but we chose this one that is low cost, useful, very steady, and logical.
> *Student 1*: Well, does it not go with the current?
> *Student 5*: You squeeze this bridge between two lands

As can be understood from the dialogue above, their connection to a real life application and other disciplines such as engineering and science was accurate and realistic as well as indicative of their consideration of the interdependence, interactions, and relative importance of factors. Hence, this could be accepted as *exemplary* evidence regarding *establishing a similar relationship* for both groups. Accordingly, they exhibited *exemplary general or independent reasoning* sub-modeling competency since they applied deductive reasoning to prove their solutions in the context of specific situations. Consequently, their model can be transferred to other similar situations, but needed minor simplifications. Thus, both groups showed a *proficient* level of the *easy to use model* sub-modeling competency.

As a second example, the generalizing competency of group C and group D related to the *Mars Lunarcrete* activity was investigated regarding the sub-competencies of *establishing a similar relationship, general or independent reasoning and easy to use model*. Moreover, the distribution of each sub-competency regarding the extent to which elementary gifted students in groups C and D, when they are engaged in the *Mars Lunarcrete* activity, displayed the generalizing modeling competency is represented in Figure 4.53. Based on the group modeling competency observation guide proposed by de Villiers (2018), the level of sub-modeling competencies for the groups was classified as 0 *(unsatisfactory),* 1 *(emergent/developing),* 2 *(proficient)* and 3 *(exemplary).*

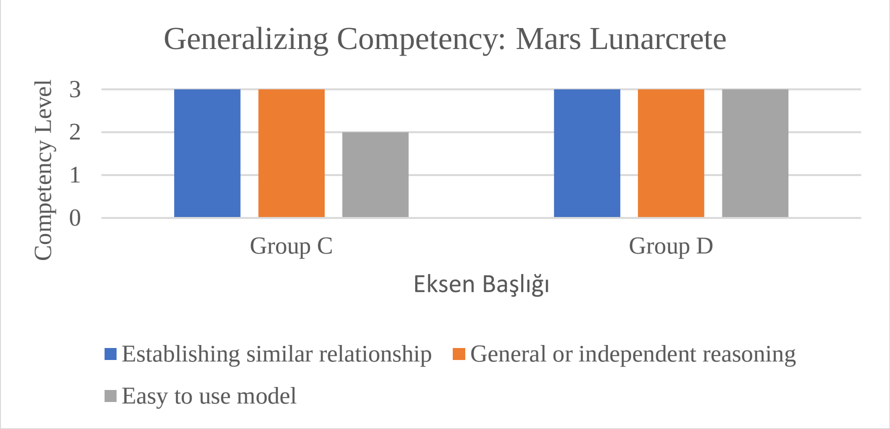


Figure 4.53. Generalizing Competency: Mars Lunarcrete

As shown in Figure 4.53, when elementary gifted students in groups C and D engaged in the *Mars Lunarcrete* activity, they generally exhibited exemplary generalizing sub-competencies. For *the easy to use model* sub-modeling competency, group C displayed a proficient level of sub-modeling competency as presented in the Figure 4.53. The following part provides examples of this competency related to the sub-categories indicated above.

Establishing a similar relationship, general or independent reasoning, and *an easy to use model*. When elementary gifted students' reflections were examined, they *established a similar relationship* in different situations by adapting some of the rules. Below is a dialogue demonstrative of the holistic approach they embodied to engineering activities and reasoning *(general or independent reasoning)*:

> *Student 12:* Conditions on the moon may be similar to those of Mars.
> *Student 11:* You have to look at the pressure, but the difference between the air temperatures is similar.
> *Student 12:* I guess it can be used for that.
> *Student 10:* Can it be used for other planets as well?
> *Student 12:* Why not if the conditions are similar.
> *Student 11:* Of course, it has to be evaluated.
> …
> *Student 10:* Also, wrapping it with insulation may be good for protection. For example, if we wrap it with an aluminum jacket in the first aid kit I mentioned,

> it will be heat resistant. Thus, it can be used in places with different air temperatures.
> …
> *Student 12:* An inclined structure can be used when building in a windy location. This reduces the effect of the wind.

As revealed in this conversation, elementary gifted students in group D applied deductive reasoning to prove their models and identified a generalization easily when presented with a specific situation and so showed *exemplary* indicators of the *general or independent reasoning* sub-modeling competency. In addition, they adapted their model easily in another related situation and the predictions were accurate. This could be accepted as *exemplary* evidence regarding the *easy to use model* sub-modeling competency. Although group C did not create a totally working model for the situation, they made some generalizations based on their experiences and exhibited optimism regarding exploration of further applications *(easy to use model).* Thus, they exhibited a *proficient* level of the *easy to use model* sub-modeling competency. For instance:

> *Student 8:* If the spacecraft were to leave Earth, would it be closer to Mars or the Moon? It can also affect the situation.
> *Student 9:* The moon is closer.
> *Student 8:* Would it not be more affordable?

As can be deduced from the dialogue above, the members of group C connected the solution process with concepts from mathematics and science. In this way, they applied deductive reasoning to compare the conditions of similar situations and prioritized the economic requirement in another related situation *(general or independent reasoning).* This could be presented as exemplary evidence for group C regarding the *general or independent reasoning* sub-modeling competency through the ability to identify a generalization in a specific situation. Together these findings provide evidence regarding the engineering competency in that both groups considered the interdependence, interactions, and relative important factors to make generalizations. In addition, their connection to real-life applications and other disciplines was accurate and realistic and so they showed *exemplary* evidence regarding the *establishing similar relationship* sub-modeling competency.

As the final example of engineering-based model eliciting activities, the findings of the *Dr. Ahmet's Will* activity related to the generalizing competency of groups E and F were investigated under the sub-competencies of *establishing similar relationship,* *general or independent reasoning and easy to use model*. In addition, the extent to which the groups exhibited the generalizing modeling competency and its sub-competencies when engaging in the *Dr. Ahmet's Will* activity is given in Figure 4.54. According to the group modeling competence observation guide proposed by de Villiers (2018), the sub-modeling competencies of the groups for generalizing competency were classified as 0 *(unsatisfactory)*, 1 *(emergent/developing)*, 2 *(proficient)*, and 3 *(exemplary)* proposed by de Villiers (2018).

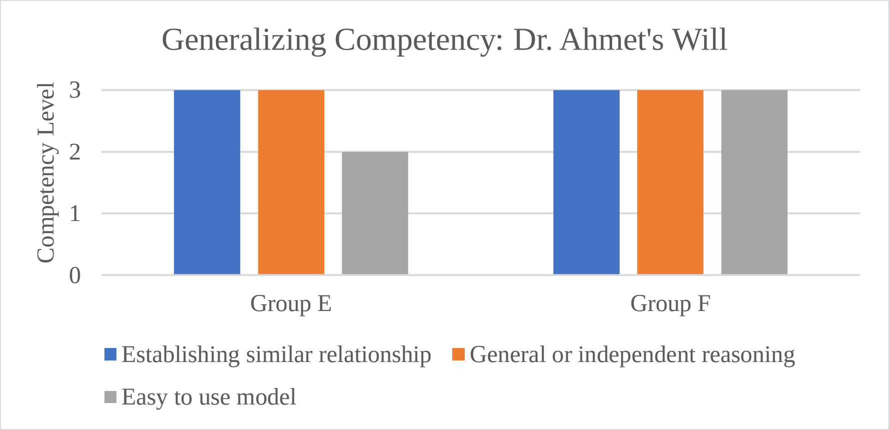


Figure 4.54. Generalizing Competency: Dr. Ahmet's Will

As illustrated in Figure 4.54., the findings of this study indicated that elementary gifted students in groups E and F exhibited mostly *exemplary* instances of the modeling sub-competencies of generalizing when they engaged in the *Dr. Ahmet's Will* activity. The above figure also shows that group E displayed a proficient level of the *easy to use model* sub-modeling competency engaging in the activity. The sample instances of this competency related to the above-mentioned sub-categories are provided below.

Establishing a similar relationship, *general or independent reasoning*, and *an easy to use model*. When elementary gifted students' reflections about the generalizability of their models were examined, both groups claimed that their models can be used in different situations by adapting some of the rules *(establishing a similar relationship)*. The following extract shows that group E's model can easily be adapted in another related situation *(easy to use model)*:

> *Student 14:* Our strategy is just right for this problem.
> *Student 13:* We can adapt and use our strategy in other locations. For instance, when traveling, considerations such as temperature and weather can be made.

> *Student 15:* Travel time and cost are also important factors to travel somewhere.
> *Student 14:* When preparing the brochure, we first gave information about Geneva, then wrote the famous things there, and put a city map for tourists to go to Geneva easily.
> *Student 15:* You follow a similar method to ours when creating brochures for any location. There can be a map of the place, attractions to visit, important information, and special foods.
> *Student 13:* For example, if we had prepared the brochure for another place, the basic things would not have changed. This was also beneficial for me, for example, I have a lot of courses and I can plan them easily. We considered many things to develop a strategy.

As seen in the dialogue above, the model of group C could be transferred to other similar situations and predictions can be made from their model, but needed minor simplifications. Thus, this could be accepted as evidence that they showed a *proficient* level of the *easy to use model* sub-modeling competency. This conversation indicates that the elementary gifted students assessed not just the generalizability of their methods, but also the generalizability of the techniques used to create their brochures. In addition, student 13 emphasized that designing a strategy considering many factors can also be applied to their personal life *(establishing a similar relationship)*. Hence, they applied deductive reasoning to prove the solution of factors to consider when traveling anywhere *(general or independent reasoning)*. On the other hand, the reflections of the members of group F indicate that their model is successful and easy to use to explore further applications *(easy to use model)*. For instance:

> *Student 16:* We considered all the ideas and chose the best one.
> *Student 19:* We considered many factors and planned. We decided that we could not jeopardize someone's safety in our plan.
> *Student 17:* Our strategy is strong. It can be adapted to different situations. We do not need several modifications.
> *Student 18:* For instance, in the Olympic context discussed previously, it is critical that the location be available to everyone. Additionally, if a security-related or risky circumstance exists, it should be considered. Anyone attending the Olympics is at risk of developing health concerns. We also examined the attendance of both young and old persons. As a result, we can easily apply our strategy when selecting a location.

The above extract indicates that the elementary gifted students *established a similar relationship* with the previous experience. The members of group F also used an example to show their model easily adapted to real-life applications and provided *exemplary* evidence regarding the *easy to use model* sub-modeling competency. As an engineering sub-competency, they used a holistic approach that you can apply your strategy in a wide variety of situations without much modification if you develop a sound plan considering many factors. Hence, both groups displayed *exemplary* instances regarding *general or independent reasoning*. In addition, both groups considered the interdependence, interactions, and relative importance of factors, reflecting the engineering competency whether their model is adaptable or not. Taking all the aforementioned items into account, both groups displayed *exemplary* evidence of the *establishing similar relations*hip sub-modeling competency since their connection to real-life applications was accurate and realistic.

In conclusion, the findings from elementary gifted students when they are engaged in engineering based model eliciting activities were analyzed in terms of *establishing a similar relationship* in different situations by adapting some of the rules, *general or independent reasoning* and *the easy to use model* with allowing predictions sub-competencies of generalizing. The analysis of the data revealed that elementary gifted students exhibited *exemplary* modeling competency in generalizing competency considering interdependence, interactions and relative importance of factors when they engaged in some of the engineering-based model eliciting activities. In addition, they applied deductive reasoning to prove the solutions in *exemplary* competency of *general or independent reasoning*. Hence, the revision was made in the group modeling competency observation guide proposed by Villiers (2018). The stated explanations above were added to the relevant dimensions in accordance with the findings of the current study.

4.8. Summary of the Findings

The preceding seven sections of this chapter provide the findings regarding the emerging categories of cognitive modeling competencies identified among elementary gifted students when they are engaged in engineering-based model eliciting activities. The modeling process and constructed models of the students working in groups were further investigated in this study. Based on the data from three engineering based-model eliciting activities, seven cognitive modeling competencies and multiple categories for each cognitive modeling competency were presented. To put it more explicitly, each cognitive modeling competency was analyzed through its sub-modeling competencies and their indicators related to engineering and mathematical modeling sub-competencies. In the first section, the internalizing competency was classified with regard to sub-modeling competencies: *understanding the problem, collecting relevant information*, *simplifying the situation*, *ethical consideration*, and *flexibility and novelty*. In addition to the group modeling competency observation guide proposed by Villiers (2018), *ethical consideration*, and *flexibility and novelty* sub-modeling competencies emerged from the data of the current study. In the second section, the interpreting competency was investigated in terms of sub-modeling competencies: *assumptions, determining particularities*, and *establishing conditions and constraints.* In the third section, the structuring competency was presented regarding sub-modeling competencies: *innovative planning and design*, and *constructing relations.* In the fourth section, the symbolizing competency was categorized under the sub-modeling competencies: *choosing appropriate symbols, using the symbols, approaching problems methodically*, and *applying interdisciplinary knowledge.* Moreover, the group modeling competency observation guide given by de Villiers (2018) has been updated to include the new sub-competency of *applying interdisciplinary knowledge* as a result of this study. In the fifth section, the adjusting competency was examined through the sub-modeling competencies: *refining and testing*, *explaining*, *deriving an elegant solution*, *adaptability and transferability*, as well as *creative approach*. The data from the current study revealed the sub-modeling competencies of *adaptability and transferability* and *creative approach*. In the sixth section, the organizing competency was investigated in terms of the modeling sub-competencies of *evaluating and judgment, reflection,* and *elaboration*. The findings of this study led to the emergence of an *elaboration* sub-modeling competency in addition to the group modeling observation guide proposed by de Villiers (2018). In the last section, the generalizing competency was investigated with regard to sub-modeling competencies: *establishing a similar relationship*, *general or independent reasoning*, and *the easy to use model.* Each section also provided an investigation of the extent to which elementary gifted students in groups displayed such cognitive modeling competencies when they engaged in engineering-based MEAs, namely, “Bridge Construction”, “Mars Lunarcrete”, and “Dr. Ahmet’s Will”. Based on the group modeling competency observation guide proposed by de Villiers (2018), the findings of this study demonstrated that elementary gifted students in groups generally displayed exemplary instances when they engaged in engineering-based model eliciting activities. However, some groups exhibited proficient indicators of sub-modeling competencies: *flexibility and novelty, innovative planning and design, constructing relations, using the symbols, creative approach, reflection, elaboration*, and *easy to use model*. In this regard, the key findings of this study will be discussed in the following chapter through a critique of the existing body of literature.

CHAPTER 5

CONCLUSION, DISCUSSION, AND IMPLICATIONS

The purpose of the current study is to investigate the cognitive modeling competencies of elementary gifted students when they are engaged in engineering-based model eliciting activities. In accordance with this purpose, this chapter presents the conclusion and discussion of the research findings obtained from this study, and the key points mentioned in the findings section are discussed and criticized vis-a-vis references to prior research in the literature. Along with the research questions of this study, the first section of this chapter is divided into seven parts, each discussing the findings regarding the internalizing, interpreting, structuring, symbolizing, adjusting, organizing, and generalizing competencies of elementary gifted students. Subsequently, implications for educational practices in the second section are addressed. In the last section of this chapter, the limitations and recommendations of the study are mentioned.

5.1. Conclusion and Discussion of Findings

In line with the aim of this study, this chapter discusses the findings of this study over seven main sections and their related subsections, each of which focuses on different cognitive modeling competencies. In this regard, the findings of three engineering-based MEAs are presented based on the adapted version of the group modeling competency observation guide proposed by de Villiers (2018), as stated in the findings chapter. The seven main sections each represent one of the following competencies: internalizing, interpreting, structuring, symbolizing, adjusting, organizing, and generalizing. Each section examines the extent to which elementary gifted students in groups displayed such cognitive modeling competencies when they are engaged in engineering-based MEAs, namely, *Bridge Construction*, *Mars Lunarcrete*, and *Dr. Ahmet’s Will*.

5.1.1. Internalizing

This part of the section discusses the findings of the first modeling competency, internalizing. The data analysis of this study revealed that in addition to the sub-codes suggested by de Villiers (2018) - the internalizing sub-competencies of *understanding the problem*, *collecting relevant information*, and *simplifying the situation* - the following sub-codes also emerged based on the findings of the current study: *ethical consideration* and *flexibility and novelty*. In other words, the internalizing competency was investigated under the five sub-modeling competencies of understanding the problem, collecting relevant information, simplifying the situation, ethical consideration, and flexibility and novelty.

With respect to the findings related to the *understanding the problem* sub-modeling competency, elementary gifted students in all groups exhibited *exemplary* modeling of the sub-competency of internalizing. That is to say, all of them clearly defined and summarized the main issues and explicitly explained why they are problems. It was revealed that *understanding the problem* was neither a simple nor straightforward competency for elementary gifted students. To put it more precisely, contrary to other studies (Kaygısız, 2021; Şahin, 2014) that claim elementary students do not spend the necessary amount of time on the modeling process to completely understand the problem, the elementary gifted students in this study did spend a significant amount of time on it. This finding may be explained by the fact that the complexity of the real-world situation in engineering-based MEAs required an initial in-depth investigation. In other words, it may have taken the student groups some time to attempt to understand the ill-structured and complex modeling problems, ones involving numerous variables that they had not previously encountered. However, all the groups managed their time efficiently through the adoption of group roles and sharing of tasks – collaborative behaviors they are accustomed to from group work on the activities conducted in BILSEM. In this way, this conclusion may have been nurtured by the fact that engineering-based MEAs encouraged collaboration and team work. In addition, Blum and Leiß (2007) reported students' difficulty in moving from a complex problem situation to a real-world problem statement. If this step in the modeling process is completed more rapidly and superficially, the depth of thought in the next steps reduces (Biccard, 2010; Hıdıroğlu, 2012). In line with this view, the high level display of the *understanding the problem* sub-modeling competency could be important in revealing other modeling competencies, in that this sub-modeling competency constitutes a prerequisite competency for the other modeling competencies, namely, interpreting, structuring, symbolizing, adjusting, organizing, and generalizing.

Despite their ultimately exemplary demonstration of this sub-modeling competency of internalizing, elementary gifted students in group E initially had difficulty *understanding the problem* in the *Dr. Ahmet's Will* activity. Subsequent to the long group discussion, the group members explained and summarized the main issues in the problem situation. This finding is consistent with studies finding that in-group discussions are effective for displaying modeling competencies (Maaß, 2007; Mousoulides, 2007). A possible explanation of this finding might be related to the nature of the given activity. In parallel to Kaygısız (2021) and Mousoulides (2007), the fact that the students in this group were confronted with many quantitative and qualitative data at the same time and their perception that they were expected to provide a rapid response may have led to initial confusion.

Despite the fact that the students were able to understand the core problematic issues in most of the engineering-based MEAs in this study, this understanding was insufficient for them to create comprehensive solutions. They needed to first deepen their comprehension with further data. For example, elementary gifted students in group B used information from not only the problem text but also the video related to the Bridge Construction activity to clarify the problem details. Similarly, the students in group D tried to understand the real-life situation beyond the data in order to more meaningfully grasp the problem in the Mars Lunarcrete activity. Thus, they referred to previous experiences about the condition of Mars to make sense of the problem. These findings are consistent with the argument emphasized by Biccard and Wessels (2011), according to which understanding a real-life situation should be defined in relation to the context of the problem and the student's previous experience. By utilizing informal information, students can have a better understanding of the problem situation (English & Watters, 2005). Moreover, Lesh (2007) asserted that the inclusion of real-life situations that may be of interest to students in MEAs makes it easier to understand the problem. Hankeln (2020) explored the cultural influences on the modeling processes of students, revealing that context played a significant role in the solution process beyond just motivating them to solve the problem. In this regard, addressing everyday problems that may attract the interest of elementary gifted students may have also contributed to the high level of the *understanding the problem* sub-modeling competency observed in the context of Turkey. Hoh (2008) provides another potential explanation for this finding in gifted students' strong desire to learn more about their own interests.

Similar to the first sub-modeling competency discussed above, elementary gifted students in all groups demonstrated *exemplary* indicators of the second sub-modeling competency of internalizing, *collecting relevant information*. Specifically, they uncovered hidden or implied information not readily apparent and used all relevant information for the problem situation when they were engaged in engineering-based MEAs. Contrary to other studies (Blum, 2015; Maaß, 2007; Şahin, 2014), elementary gifted students in all groups sought further important and appropriate information that was not provided apparently in problem situations and data sets. Collecting further relevant information related to type of bridge, materials, and material costs beyond the information obtained from the data set when the students were engaged in the *Bridge Construction* activity can be an example of this type of enhancement.

Moreover, elementary gifted students scrutinized carefully relevant information about the problem by questioning, brainstorming, and clarifying. For instance, the students in group C uncovered information not readily apparent such as the relationship between density and wind resistance in the *Mars Lunarcrete* activity. In a similar vein, in the *Dr. Ahmet's Will* activity, both groups considered not only the given data but also implied information related to the characteristics of each relative. Hence, elementary gifted students' exemplary demonstration of this sub-competency of internalizing may be due to the structure of the activities (Aydın-Güç, 2016; Blum, 2015; Ng, 2018; Tekin-Dede, 2015). Specifically, the complex and less-restricted nature of engineering-based MEAs may have led the students in this study sample to search for more information to solve the problems. Another view in support of the high-level modeling sub-competency is that it provides opportunities for gifted students to acquire information from other sources such as discussions with their peers or from approved online sources to support their creative process (Lee et al., 2021). In the current study, the student-driven learning environment enabled elementary gifted students to collect relevant information from these sources, and the researcher provided opportunities for research as well as flexible guidance.

Another sub-modeling competency of internalizing that elementary gifted students in all groups displayed *exemplary* indicators of *simplifying the situation* by using multiple representations to explain and simplify the problem. The body of research supports the notion that simplifying the situation is a crucial sub-competency for progressing forward in the modeling process (Biccard, 2010; Maaβ, 2006; Mousoulides, 2007; Şahin, 2019). That means, in order for students to reveal other competencies at the desired level, they must demonstrate the sub-competency of *simplifying the situation* to a significant degree. In this regard, it is reasonable to expect that elementary gifted students in this study could exhibit a high level of other modeling competencies.

Importantly, simplifying requires revealing the essential features of the problem (Blum & Borromeo-Ferri, 2009). In line with this view, the findings of this study indicated that all six groups simplified the real-world situation by connecting essential concepts. However, even in the same activity, the groups did not determine the same factors affecting the context. For instance, in the Bridge Construction activity, group A used not just the given problem text but also the video, whereas group B used previous real-life experience to evaluate the factors affecting the context offered in the activity. Indeed, the cognitively demanding nature of the process of simplifying and developing the idealized form of the problem was highlighted by Blum and Leiß (2007), and prior studies have demonstrated that students struggle with simplifying the situation (Biccard, 2010; Kaygısız, 2021; Mousoulides, 2007; Şahin, 2019; Tekin-Dede, 2015). On the contrary, the high level performance in *simplifying the situation* in the present study may be due to gifted students' desire to engage in cognitively challenging processes (Sayı & Yurtseven, 2021). Mann and colleagues (2011) argued that engineering-based activities offer the potential for varied levels of sophistication, breadth, and depth of understanding, hence providing properly challenging tasks for gifted students. Thus, elementary gifted students in this study could have the opportunity to demonstrate their competencies in their areas of strength.

Furthermore, this sub-modeling competency lends itself to engineering tasks, which by their nature require detailed focus on all sub-components of both problem and solution. All groups in the present study documented functional solution requirements and made proposals to clients of different engineering-based MEAs such as peasants, corporations, and lawyers. This finding is similar to a previous study that showed elementary students consider several stakeholder views (Siverling et al., 2019). The findings of the current study also indicated that elementary gifted students established different design aspect standards based on the components of the engineering-based MEAs. For instance, they designed the bridges to ensure safety, durability and serviceability, and the building material to ensure strength, heat resistance, economy, and determined the best place to ensure weather conditions, time, and cost. In line with the studies on engineering design activities (English et al., 2017; Lyon & Magana, 2019), elementary gifted students in this study not only take into account a number of aspects and the connections between them, but they also assign priorities to a number of factors and choose some above others.

Different from the aforementioned sub-competencies of internalizing, *ethical consideration* emerged from the data analysis of the current study. All groups displayed exemplary instances of the *ethical consideration* sub-modeling competency. To put it more precisely, all groups clearly identified the central ethical problem, the interested and affected parties, and also how engineering affects people and places. This finding is consistent with Mousoulides and English (2011), who found that MEAs provide opportunities for students to attend to ethical considerations.

Moreover, the engineering design challenge offered the appropriate context for addressing socio-scientific concerns and ethical considerations (Katehi et al., 2009). In today’s society, the ability to think both locally and globally for students’ own benefit and the welfare of their communities is increasingly important. In this regard, the *ethical consideration* sub-modeling competency is also considered an essential aptitude for 21st-century skills and engineering habits of mind (Katehi et al., 2009). In line with these views, concern for others is accepted as an indicator of giftedness in the 21st-century (Chowkase, 2022). Thus, it was evident that elementary gifted students in this study demonstrate the 21st-century indicator of giftedness when they are engaged in engineering-based MEAs. Relatedly, high ethical standards are determined as an attribute of successful engineers. This attribute matches the characteristics of gifted students, who have a strong sense of justice and intellectual honesty (Mann et al., 2011). Hence, the high level of *ethical consideration* of gifted students could be explained by their high self-awareness, and high expectations of self and others (Clark, 2008; David et al., 2011; Sak, 2010). The finding of this study also supports the finding of Sak (2014), who claimed that the role of gifted students is important in bringing solution suggestions in areas where society requires assistance.

Another sub-competency of internalizing that emerged from the findings of the present study is *flexibility and novelty*. Unlike the other sub-modeling competencies of internalizing, groups in different engineering based MEAs showed different degrees of *flexibility and novelty*. Both groups in the *Dr. Ahmet Will* activity and group C in the *Mars Lunarcrete* activity showed proficient instances of this sub-modeling competency. All six groups in this study offered the data a great deal of flexibility. Importantly, the groups who showed exemplary display of flexibility and novelty also came up with original ideas to develop new and unique solutions, whereas the group who showed a proficient level of this competency did not. Similar to the *ethical consideration* sub-modeling competency mentioned previously as the newly emerging category, the available literature advocates not only MEAs (Chamberlin & Moon, 2005; Lu & Kaiser, 2021; Şengil-Akar, 2017; Taşkın, 2016) but also engineering-design based activities (Mann et al., 2011; Şen, 2018) to elicit from students' *flexibility and novelty* as components of creativity. For instance, in the *Bridge Construction* activity, neither group found the materials sufficient to construct a bridge. They offered a great deal of flexibility with the given data related to the type of bridges, materials, and material cost. Consequently, they added new variables to existing problems and made associations among variables. In addition, the idea of bringing household materials from the house and determining their cost via an internet search could be accepted as novel ideas since they are extraordinary for the scope of the problem. However, the finding is contrary to previous studies, which have suggested that immediate access to materials accelerated the construction process (Portsmore et al., 2012; Welch & Lim, 2000). Because of the open-ended nature of the activities involved in engineering-based MEAs, they can be considered a creatively directed activity since they require and encourage mental flexibility and offer opportunity for the production of novel ideas similar to the study of Lu and Kaiser (2021). On the other hand, research studies highlight the importance of the context of the model eliciting activity in displaying the creativity of gifted students (Şengil-Akar, 2017; Taşkın, 2016).

The findings of this study thus support some aspects of previous studies. That is to say, the context of the activities influences the occurrence of flexibility but the incidence of novelty is influenced by the group dynamic. Due to the rapid pace of technological advancement, an engineer must possess a variety of skills, including originality, ethics, adaptability, and a desire to continue their education throughout their careers (NAE & NRC, 2009). Hence, the current study supports the view that engineering-based MEAs could be an effective tool for elementary gifted students to display their special characteristics, which overlap with successful engineers (Mann et al., 2011).

5.1.2. Interpreting

The second part of the section discusses the findings of the interpreting modeling competency with respect to the sub-modeling competencies of *assumptions, determining particularities*, and *establishing conditions and constraints*. Further, the extent to which elementary gifted students in different groups displayed the interpreting modeling competency and its sub-modeling competencies when engaged in three different engineering-based MEAs are investigated in this section. Although a new sub-modeling competency has not emerged in the interpreting modeling competency, as did occur in the previous section, the exemplary level belonging to the sub-modeling competencies of *determining particularities* and *establishing conditions and constraints* has been expanded according to the data of this study.

In accordance with the findings of this study, all six groups exhibited an *exemplary* level of the *assumptions* sub-modeling competency. That is to say, elementary gifted students in groups demonstrated innovative and insightful assumptions and showed clear and coherent consideration for the consequences of the assumptions. The crucial role of making assumptions in constructing the model and the inherent difficulty of this sub-modeling competency have been mentioned in many studies (Blum & Borromeo-Ferri, 2009; Chan et al., 2012; Maaß, 2006; Mousoulides, 2007; Şahin, 2014, Tekin-Dede, 2015). Such difficulties have resulted in the construction of simple models (Maaß, 2006; Şahin, 2014). In the present study, where the opposite was observed, the elementary gifted students' high level of making assumptions may imply the construction of more sophisticated models. Accordingly, all of the groups that participated in this study made complicated assumptions based on real-life observations throughout the design process and group reports. Although the literature supports the idea that long-term studies are required to reliably display a high level of the assumption sub-modeling competency (Biccard, 2010; Chan et al., 2012; Kaygısız, 2021; Şahin, 2014), the findings of the current study indicate that elementary gifted students showed this sub-competence in engineering-based MEAs within a five-week period. Gifted students' ability to absorb different amounts of information in a short time (Clark, 2008) could explain this finding.

In addition to using their real-life experiences, the students in this study especially considered practical, economic, and social factors for making assumptions about the problem situations. In the *Dr. Ahmet's Will* activity, both groups assumed that old people could not withstand long journeys and also that their ability to make the journey would depend on their relatives' ability to finance it for them. Another example, this one from the *Bridge Construction* activity, is that both groups made *assumptions* regarding the possibility of the bridge collapsing because of the failure of different components and produced an alternative support mechanism for that condition, thus demonstrating evidence of their ability to manage risk as an engineering sub-competency. In accordance with the present findings, previous studies (Householder & Hailey, 2012; Katehi et al., 2019) have proposed that engineering design activities should offer students not only the technical but also the social dimension of engineering. Hence, these findings of the current study revealed that engineering-based MEAs encourage the consideration of social, economic, safety, and other impacts of engineering design decisions for elementary gifted students. This might be due to gifted students' sensitivity to human concerns (Renzulli et al., 2006; Sak, 2010). Their sensitivity towards the elderly, those with less money, and the bridge that was about to collapse might stimulate the formation of a sense of belonging with those people, the emotional need to help, and the cultivation of the motivation to do something about such situations. Thus, the participants of this study showed the component of concern for others, which is one of the indicators of giftedness in the 21st-century (Chowkase, 2022).

Turning to another sub-competency of interpreting, all groups in this study demonstrated exemplary instances of the *determining particularities* sub-modeling competency. Specifically, elementary gifted students recognized important quantities and variables in the problem and how they related to the problem considering practical, economic, social, environmental, quality assurance, and safety factors. The findings of the current study revealed that during engineering-based MEAs that require the construction of concrete models, the students determined the particularities for the construction of the bridges and settlement on Mars by drawing. On the other hand, in the *Dr. Ahmet's Will* activity, the students arrived at an understanding of the important quantities and variables that can influence the problem situation via group discussion. However, elementary gifted students' engagement in all engineering-based MEAs shared a common feature in that they determined the particularities of the task based on collaboration with one another. This finding is in line with those of other studies such as one by Lesh and colleagues (2003), who advocated that the ability to successfully work in groups on MEAs could be attributed to communication, social interaction, and shared responsibility among group members.

With respect to the analysis of the sketches, all four groups used basic and two-dimensional sketches. This finding is consistent with the literature, which holds that the level of sophistication in initial design is generally low (Crismond & Adams, 2012; English, 2019). Interestingly, however, the findings of the current study support the idea that young students do not fully use their sketch as a vehicle to develop and communicate their design plans (Crismond & Adams, 2012; Welch et al., 2000). For this reason, students were asked to give a justification for their sketches and initial planning. Similar approaches have been employed in the literature to influence students' engagement in engineering design discussion and the quality of their final solutions (Guzey & Aranda, 2017). The students' preliminary plan indicated that they interpreted the client's requirements and identified the accepted criteria for the work product. For instance, in the *Mars Lunarcrete* activity, strength, durability, and affordance were determined to be the acceptance criteria for both groups. Hence, the study's elementary gifted students' preferences in determining task particularities vary depending on the context of the problem. This finding may be explained by trade-off issues (Dasgupta, 2019; English, 2019) such as choosing a more expansive bridge due to stability concerns. In other words, the students in the present study optimized the design solution while reasoning about the definition of an optimal model and trade-off, considering the effect of multiple design parameters, such as cost and final pressure. Furthermore, gifted students' sensitivity to human concerns (Chowkase, 2022; Davis et al, 2011, Renzulli, 2015) might have motivated them to mostly focus on factors related to social or community concerns. Considering this special group's interest in global and environmental issues, it may be reasonable to expect them to offer more substantial ideas and opinions in qualitative data.

All groups provided an exemplary display of the last sub-modeling competency of interpreting, *establishing conditions and constraints*. That is to say, elementary gifted students established clear conditions and constraints in terms of efficient utilization and interaction of people, materials, equipment, means and funding, as well as explanations for such conditions and constraints. The findings of this study showed that elementary gifted students also applied previous experience when establishing conditions and constraints similar to the findings from previous studies (Mousoulides, 2010; Şahin, 2017). Such studies claim that students demonstrate the indicatives of this sub-competency despite no previous experience on modeling (Kaygısız, 2021; Şahin, 2017). While students are expected to exhibit this proficiency at an optimal level, it has been stated that students have difficulty in establishing conditions and constraints in MEAs with three or more variables (Leong & Tan, 2015; Tekin & Dede, 2015). On the contrary, the highest level of this sub-competency in the present study indicated that elementary gifted students identified conditions and constraints with many variables. For instance, in the *Dr. Ahmet's Will* activity, both groups manipulated many variables for the problem situation in terms of flight costs, travel time, temperature, altitude, snowfall, and pollution levels, as well as social factors. It is likely that the students who attend BILSEMs get high scores on the general ability test, and as a result, differ from their peers in terms of their ability to deal with many variables. Van Tassel-Baska and Brown (2007) provide a potential explanation for this finding, namely students' desire to participate in complex and challenging activities with their intellectual peers.

5.1.3. Structuring

The third modeling competency, structuring, is discussed here as part of the sub-modeling competencies of *innovative planning and design*, as well as *constructing* *relations* based on the group modeling observation guide proposed by de Villiers (2018). However, the revision related to the exemplary level definition of the sub-modeling competencies was made in that guide based on the findings of the current study. In this regard, whereas the explanation of using *innovative planning and design* to set up situational models was added to the *innovative planning and design* sub-modeling competency, the explanation of *considering the interdependence, interactions, and relative importance of factors* was added to the *constructing relations* sub-modeling competency.

The findings from the *innovative planning and design* sub-competency of structuring indicate that groups in different engineering-based MEAs demonstrated varying levels of competence in this sub-competency. Both groups in the *Dr. Ahmet's Will* activity demonstrated proficient instances of the sub-modeling competency, while the other groups in the *Bridge Construction* and *Mars Lunarcrete* activities provided an exemplary display of the *innovative planning and design* sub-competency. To put it another way, the other groups used innovative planning and design to set up situational models with multiple representations to explain the problem as well as to complete an accurate model generated by groups E and F. A possible explanation for this difference might be that the *Dr. Ahmet's Will* activity requires students to set up the procedure rather than prototypes. Hence, the likelihood of unexpected situations occurring was low as elementary gifted students in groups E and F discussed the problem situation in detail before creating their situational model.

Throughout the activities, it was observed that the students in the present study attempted to come up with innovative ideas for their designs. This might be due to the fact that engineering-based MEAs offered students an enriched and flexible learning environment. During this process, students were able to freely discuss their opinions and thoughts, while simultaneously transferring the same into tangible products, offering an opportunity to assess their demonstration of the structuring competency. This finding is consistent with prior studies (Sen et al., 2021) indicating that engineering-based activities support the creative skills of gifted students.

In particular, the findings of the current study revealed that elementary gifted students applied the iterative nature of the engineering design process by differentiating their initial plan to create a realistic representation of the original situation. For instance, both groups in the *Bridge Construction* activity changed the measurement of their bridge's abutments while structuring their models. Another example of students' demonstration of the structuring competency comes from the *Mars Lunarcrete* activity, in which gifted students did not completely depend on the given data, but made inferences based on the given recipes. Similarly, both groups in the *Dr. Ahmet's Will* activity used not only the data that was provided but also incorporated the characteristics of the relatives. These findings contradict the finding (English & Mousoulides, 2011; Şahin, 2019) that students in MEAs did not go beyond the provided data. As suggested by Lee and colleagues (2021), a possible explanation might be that gifted students' access to resources is necessary for creative production. An adaptive environment in the planning phase of the engineering-based MEAs could provide access to the necessary materials for investigations demanding creative production. In addition, the main study of the present research was implemented in five sections: planning, constructing, testing, re-constructing, and reflection, in line with the findings obtained from the pilot study. Hence, the time given to students to plan and access resources prior to the implementation phase might have contributed to this finding.

Furthermore, it has been argued that the use of iterations in engineering-based learning environments is an effective method of developing a deeper understanding of concepts related to science, technology, engineering, and mathematics (Park et al., 2018). Considering that, in the *Bridge Construction* activity, the height of the suspension bridge is 10 m above the stream in the real situation, the change of the abutments measurements of the bridge can be given as an example of Group A members' deep understanding of mathematics. In addition, group D in the *Mars Lunarcrete* activity used a comparison of ratios and inferred that although using a 4:3 ratio required more materials, the density was also higher and would offer better protection than the 2:1 ratio. This is a clear demonstration of their deep understanding of science, mathematics, and engineering. Consistent with the findings of this study, previous research has demonstrated that students develop their own challenges to solve throughout a study, as well as the goals they intend to accomplish, rather than relying on the predetermined challenges (English, 2019). The possible explanation aligns with a previous research finding (Van Tassel-Baska, 2003), which concluded that gifted students appreciate complex and challenging activities. Hence, engineering-based MEAs may provide a suitable environment for the exhibition of elementary gifted students' innovation skills. This is supported by previous research (Şen, 2018), which has shown that engineering-based activities enable gifted students to demonstrate their innovative, creative, and imaginative skills through collaboration with their peers.

When creating a model for the activities, all groups in the present study focused primarily on the client's needs, similar to the findings in other studies conducted with elementary students (Watkins, Spencer, & Hammer, 2014). Quite to the contrary, however, high school students in engineering design experiences generally consider the technical aspects of the design (Berland et al., 2013). Only one of the six groups in the present study highlighted design aesthetics, but all six groups discussed the functionality of their design solutions. This is in contrast to previous studies in which students initially focused on design aesthetics rather than function (Barnett, 2005; Fortus et al., 2004). Furthermore, all groups approach the problem with the belief that they were able to solve the design problem using the techniques that they developed. Even when the sample of the present study experienced problems such as those relating to the nature and amount of material or data, they generated a solution to overcome them. An explanation for this may be found in the characteristics of gifted students, namely that they are persistent, goal-oriented, and deeply interested in the issues they care about (Hoh, 2008). It can be concluded that the engineering-based MEAs in this study might capture the attention of elementary gifted students based on their commitment to the task. Moreover, all groups approached the problem situation by considering the possibility of improvement, optimistically learning from their mistakes, and persevering in difficult situations throughout the engineering design process. This finding is consistent with the findings of Şen's study (2018), which investigated the skills exhibited by gifted students at BILSEM when they were engaged in engineering-design based activities. She found that such activities, which include challenging problem situations, positively affect gifted students' problem solving skills and also contribute to their cooperation. Thus, development of engineering and design-oriented solutions to real-world problems might be helpful in revealing their problem-solving skills.

Regarding the *constructing relations* sub-competency of structuring, all groups provided an exemplary display of the sub-modeling competency. To put it more explicitly, elementary gifted students in groups created a general rule or formula or strategy, model or prototype for solving problems considering the interdependence, interactions, and relative importance of factors. In other words, the findings of this study indicated that all groups identified and constructed relations between key variables when engaged in engineering-based MEAs. However, the key variables for each activity vary depending on the context of the activity. This is consistent with research (English & Fox, 2005) that claimed distinct groups focus on distinct relationships among crucial factors even within the same activity. While group D established relations between key variables such as density, wind resistance, and cost, group C established relations between the key variables, the nature of the material, and pressure and temperature differences. In this study, all groups distinguished between the key and other variables by categorizing the variables according to their relative importance.

While prioritizing the aforementioned variables, a particularly interesting finding of this study revealed that elementary gifted students ascribe greater significance to considerations of the social, safety, and environmental implications of their decisions. This finding is in parallel with studies (Mass, 2006; Tekin-Dede, 2015) that support the idea that students associated the variables affecting the problem situation with real life experiences during the MEAs. In engineering-based MEAs where cost was a factor, the groups applied this category, which aligns with prior observations from students' design discussions (Guzey & Aranda, 2017; Siverling et al., 2019). Contrary to the findings from the studies on MEAs conducted with elementary students (English & Watters, 2005; Kaygısız, 2021; Şahin, 2019), the findings of the current study revealed that elementary gifted students used logical strategies and provided justifications for their decisions when determining the key variables in this study, rather than employing intuitive and informal strategies. Due to gifted students' ability to deduce the logical interrelations between concepts beyond their peers and concern for others (Sak, 2010), engineering-based MEAs may serve as a differentiation tool in bringing solution suggestions to areas where society requires assistance.

Moreover, the findings of the current study revealed that elementary gifted students considered the interdependence, interactions, and relative importance of various factors. In the *Bridge Construction* activity, the development of alternative support mechanisms in the case of the collapse of bridge abutments can be given as an interesting example of such. Both groups in the activity identified links in a complex environment and articulated unexpected outcomes that can occur. Although this engineering sub-competency was not specified in the literature (Guzey et al., 2016; Wheeler et al., 2019), the students in this study exhibited exemplary evidence of this sub-competency. This striking finding may be explained by the common attributes of gifted students and successful engineers, namely their ability to comprehend the broad picture, recognize patterns, and connect different topics (Mann et al., 2011). Given the opportunity to put their understanding of science and mathematics to practice in the real-world in this study, the students generated a solution-oriented model for a particular situation. In this process, they behaved like engineers, trying to create realistic and useful products by evaluating not only technical but many other factors as well.

In addition, elementary gifted students displayed their competency in dealing with such a challenging task by addressing a variety of problem components at the same time (e.g. costs, time, temperature differences, material consumption, engineering procedures, and stability and strength challenges). The findings of this study are thus consistent with those of previous studies, even though other studies have emphasized that this is a challenging situation for elementary students to handle (English et al., 2017; Guzey & Jung, 2021). This finding might be explained by gifted students' preference to engage in complex and challenging activities (Sayı & Yurtseven, 2021). Providing adequately challenging opportunities that are aligned with the unique skills of gifted students encourages flexible thinking among these students (Lee et al., 2021). Thus, engineering-based MEAs offered the participants the opportunity to engage in conceptual investigation and encouraged them to consider more complex ideas.

However, similar to the findings of the research (English, Hudson & Dawes, 2013), cost and material limitations constrained their structuring of the model. Correspondingly, the findings of this study indicated that these elementary gifted students maintained a balance between the effectiveness of the solution process and the cost/time involved. Contrary to this implication, researchers (Biccard & Wessells, 2011; Şahin, 2019; Tekin-Dede, 2015) have argued that the groups that developed discussions on modeling through the consideration of the quantities in the data table tried to reach a solution through the data that they could relate mathematically, and had difficulty associating data sets from other disciplines in interdisciplinary activities (English & Watters, 2005). However, in this study, groups used not only the quantitative data but also qualitative data, which was not provided in the data sets. Beyond that, they also considered the social, safety, and environmental implications of their solutions in addition to the interdependence and interactions of the same. As an example, in the *Dr. Ahmet's Will* activity, the students incorporated the characteristics of the relatives into the data that they considered. Previous research supports this idea that informed designers are individuals who engage in reflective design thinking (Crismond & Adams, 2012). Şengil-Akar's (2017) research on MEAs in her study of gifted students attending BİLSEM provides additional support from the Turkish context. She concluded that progressive thinking is a factor in gifted students' ability to generate innovative models since the creative thinking skills of the students emerged throughout the process. Overall, the iterative nature of both the engineering design process and mathematical modeling might enable elementary gifted students to reveal their unique thought processes through the use of engineering-based MEAs, which require the knowledge and skills of various disciplines.

5.1.4. Symbolizing

The findings related to symbolizing, the fourth modeling competency, and its related sub-competencies are discussed in this part of the section. The study data revealed that in addition to the symbolizing sub-competencies, suggested by de Villiers (2018), of *choosing appropriate symbols*, *using the symbols*, and *approaching the problems methodically*, the new sub-competency of *applying interdisciplinary knowledge* emerged based on the findings of the current study.

The findings indicate that all groups displayed an exemplary level of the *choosing appropriate symbols sub-modeling competency*. That means, elementary gifted students chose the mathematical, scientific, or engineering tools that would lead to an elegant solution. The findings further indicate that the students not only chose appropriate mathematical tools but also scientific and engineering tools because of the interdisciplinary nature of engineering-based MEAs. This finding is different from that of other research (English, 2007; Şahin, 2019), which claims that elementary students generally choose verbal expressions over mathematical symbols when they are involved in MEAs. In the present study, however, all six groups chose the appropriate symbols throughout all phases of the design process. For example, in the planning phase of the *Bridge Construction* activity, both groups gave information about the height and weight of the bridge sketches by using units that correspond to the situational condition. In the structuring phase of the *Mars Lunarcrete* activity, both groups chose the appropriate materials, demonstrating consideration of the nature of the material necessary to engineer their structure. Moreover, students in both activities decided to procure extra materials in order to further strengthen their bridge or structure. The students made effective decisions where the materials at their disposal were insufficient to solve the project problem. Clark (2008) suggests this may be explained by gifted students' accelerated and flexible thought processes. They might develop their own ways of thinking about problems and ideas that are unique to them. Such a dynamic was observed in the current study.

On the other hand, not all groups exhibited an exemplary level of the *using the symbols sub-modeling competency*. One of the groups in the *Bridge Construction* and *Dr. Ahmet's Will* activities displayed a proficient level of the *using the symbols* sub-modeling competency. They explained and described the symbols used in their model accurately. The groups, however, who displayed an exemplary level of the *using the symbols* sub-modeling competency present possible alternative methods for working with the problem. An explanation for this may be related to the fact that students are generally accustomed to obtaining the immediate answer in regular classrooms and may not typically need to generate alternative solutions to problems (de Villiers, 2018; Şahin, 2019). The data analysis performed for this study showed that the students used mathematical symbols to set up the mathematical model. The symbols that they used differed according to the context of the activity, an observation similar to the findings of another study (Siverlig et al., 2019). For example, in the *Bridge Construction* activity, both groups generally used symbols related to length measurement, but in the *Mars Lunarcrete* activity, the symbols the students generally used related to liquid and weight measurement, and in the *Dr. Ahmet's Will* activity, they mostly related to time measurement. It could be noted that some mathematical symbols are used in all engineering-based MEAs, such as those used for comparing and contrasting.

In addition, all groups showed engineering competency by displaying mastery of the established methods, procedures, and techniques in the practice area as components of the symbolizing competency. A sample instance can be given from the *Dr. Ahmet's Will* activity, where students in group F converted temperatures from Fahrenheit to Celcius, which they then used in their daily lives in order to resolve conflict among group members. Moreover, all groups discovered relations and regularities by comparing many variables, similar to observations made in a number of other studies (English, 2007; Guzey et al., 2006; Mousoulides, 2011). However, the findings of this study do not support the claim that elementary students experience difficulty in dealing with more than two variables at the same time (Kaygısız, 2021) or variables related to multiple disciplines (Mousoulides, 2011). It was observed that the sample in the present study generally looked for deeper and more complex interactions than those offered by the activities themselves, which were seen as more obvious. Another unexpected finding of this study is that students in group C discussed the type of drawing they used and the concept of perspective drawing when they planned the structures in the *Mars Lunarcrete* activity. The acknowledged high visual-spatial ability of gifted students (David & Rimm, 2004) may be at play here. Another reason for discussing the type of drawing in the group might be that student 9 is talented in visual arts. Since this student had received a specialized education in the subject of visual arts, he may have discussed the concept of 3D drawing with his group members. This talent plays a significant role in determining whether or not a student is suitable for skilled labor such as engineering, yet it is widely ignored in gifted education (Andersen, 2014). In a similar vein, the findings of this study support the notion that engineering-based activities may provide gifted students with opportunities to apply their visual-spatial ability in practice. Hence, the combination of gifted students with diverse specific skills may also nurture the various aspects of the groups as stated.

Exemplary instances of another sub-competency of symbolizing, *approaching problems methodically*, were displayed by all groups. That is to say, all aspects of their reasoning were completely accurate and they translated the structure of the situation into a satisfactory solution. Ludwig and Xo (2010) emphasized the importance of appropriately applying mathematical reasoning to each step of the modeling process to overcome the cognitive difficulties encountered in the process, and this is in fact what seems to have occurred to the students of the present study. The design processes of all groups involved a step-by-step consideration of facts and evidence, as well as the logical conclusions that could be drawn from this consideration of facts and evidence. It is thus unsurprising that this led to all aspects of their reasoning resulting in satisfactory solutions. Such a finding speaks to the idea that students are able to generate shared knowledge when they participate in an activity that prompts them to reflect on the reasoning and procedures used by other group members in the collaborative process (Kaiser & Sriraman, 2006; Lesh & Zawojewski, 2007; OECD, 2013).

Furthermore, the findings of this study showed that elementary gifted students transferred the real world situation into mathematical language to solve the problem. During the *Mars Lunarcrete* activity, the students in group D sketched the garage with a forward sloping door, explaining that it would be more resistant to wind. The elementary gifted students were also observed to have applied an acceptable level of understanding and technological knowledge to execute engineering decisions. For instance, both groups in the *Dr. Ahmet's Will* activity applied knowledge of how rainfall, pollution level, and temperature differences affect travel in science and the design process in order to make arrangements that fulfill all requirements as efficiently and economically as possible in engineering. A possible explanation for these might be due to gifted students' ability to transfer ideas easily (Vogelearr & Resing, 2018). It has been observed that the context of the engineering-based MEAs related to real-life could be effective in transferring ideas into a variety of diverse situations. Consistent with the research (Şen, 2018), the use of engineering-based activities as enrichment tools enables gifted children to transfer their knowledge of science, technology, engineering, and mathematics disciplines, as well as other fields, to real-world problems.

A particularly interesting finding is the newly emerging sub-competency of symbolizing, *applying interdisciplinary knowledge*. In-depth investigations undertaken by all groups, characterized by the application of interdisciplinary knowledge to solve problems, reflected an exemplary level of this sub-modeling competency. Students from all six groups discussed existing technologies, different components of engineering, unit-based and non-unit-based science, and mathematics content in their attempts to ultimately develop and justify solutions to their engineering problems. The interdisciplinary nature of MEAs, especially engineering-based MEAs (Diefes-Dux et al., 2004; Lesh et al., 2000; Lyon & Magana, 2021), may account for this finding. However, the students' high level of performance in this sub-competency may also be attributable to gifted students' talent in recognizing uncommon connections between fields or concepts (Wellisch & Brown, 2013). It was observed in the present study that elementary gifted students integrated science and mathematics into the engineering design problems via a consideration of the context of the study, similar to past research (English et al., 2016; Guzey, Moore, & Roehrig, 2010; Siverling et al., 2019). Contrary to what was observed in previous research on bridge construction MEAs (English, 2016; Guzey et al., 2010), the members of group B used the nature of the material to construct a new bridge. They indicated knowledge of material use and force (science), measurement, geometry, cost effectiveness (mathematics), and design process and strategies to construct stronger and more stable bridges (engineering). In fact, the members of group B reflected on how they apply science, technology, engineering, and mathematics knowledge to the nature of spaghetti in science, adjusting the measurement of cardboard and spaghettis, placing beads in equal intervals in mathematics, and the whole design process in engineering. Consistent with the findings from previous research, they also explained how they use geometric shapes and engineering principles to improve the strength and stability of their bridges and structures (English & King, 2018). Furthermore, students in the present study were found to employ their knowledge of the processes and procedures underpinning science to support their decisions in accordance with mathematical knowledge. In a similar vein, Şen (2018) proposed that engineering-based activities might provide an authentic learning environment to integrate gifted students' previous knowledge and academic knowledge of science and mathematics. This may well apply to the participants of the current study. Since these students were chosen based on a general intelligence test, the fact that they are typically proficient in science and mathematics may have facilitated their ability to establish relationships in these fields.

Another interesting finding from the present study concerns the students' ability to reflect on the scientific knowledge and reasoning behind their design decisions. An interesting example can be given from group D in the *Mars Lunarcrete* activity. One of the group members added new pieces of scientific knowledge based on their previous experience, integrating an aluminum jacket inside the first aid kits to be used for thermal insulation based on previous experience with such a jacket. Different from previous studies in which students initially concentrated on design appearance and aesthetics rather than functionality and practicality (Barnett, 2005), the gifted students in the present study investigated the functionality and practicality of their designs. The findings of this study are, however, in line with the view that students consider design benefits and trade-offs when engaged in engineering-based activities (English et al., 2016; Lyon, 2021).

A large body of literature supports the idea that engineering-based problems can be complicated and require students to use information from a range of disciplines, which is similar to the work done by working engineers in their professional lives (English, 2016; Guzey et al., 2016; Siverling et al., 2019). Hence, the development of an in-depth understanding of science, mathematics, and technology could be promoted by providing gifted students with opportunities to engage in engineering-based MEAs that are of interest to them and that motivate them. Due to the interdisciplinary nature of such activities (Maass et al., 2019), they have the potential to bring to light gifted students' strengths in competencies that are similar to those of successful engineers.

5.1.5. Adjusting

In accordance with the aim of the current study, the findings from the fifth modeling competency, adjusting, are discussed in this part of the section. In addition to the adjusting sub-competencies of *refining and testing*, *explaining*, and *deriving an elegant solution* proposed by de Villiers (2018), the analysis of the present study revealed that *adaptability and transferability*, as well as *creative approach* also emerged as sub-modeling competencies. In this regard, the extent to which elementary gifted students in varied groups display the adjusting modeling competency and related sub-modeling competencies when engaged in various engineering-based MEAs is investigated through the use of specific examples.

First of all, the findings of this study showed that all groups displayed an exemplary measure of the *refining and testing* sub-modeling competency of adjusting. In other words, all groups related the underlying structure of the problems to other similar problems. While some groups refined parts of the model, other groups went through the entire modeling process again if the solutions they developed did not fit the new situation. For instance, in the *Dr. Ahmet's Will* activity, neither of the groups involved changed their first and last preferences for the new situation, but changed other options. On the other hand, in the *Bridge Construction* activity, group B decided to construct a totally new bridge for a new situation using new material. Consequently, all groups in the present study dealt with the changing circumstances by totally or partially refining their models. This was consistent with the findings of English and King (2015), who asserted that refining and testing the model has been crucial in fostering a deeper knowledge of the concepts behind the problem as well as in generating a better final product.

In fact, the iterative nature of engineering design activities encourages students to consider the possibility of improvement, to remain optimistic, to learn from failure, and to continue under difficult conditions (Wheeler et al., 2019). In a similar vein, in the current study, it was observed that elementary gifted students realized that failure was a natural part of the process, and that unsuccessful attempts were an opportunity to improve the design and try again. As many gifted students demonstrate perfectionism and perseverance (Clark, 2008; Sak, 2010), engineering-based MEAs provide them the flexibility to take risks, test an idea, and then make it better. This might have enabled the sample in this study to recognize that an imperfect design does not indicate failure, but rather an opportunity to learn and revise what they have created. Overall, the findings of this study confirm previous research stating that the emphasis on design iteration and improvement of an initial product are a cornerstone of engineering-based activities (English, King & Smeed, 2017; Huffman, 2015; Tank et al., 2019) for elementary gifted students engaged in engineering-based MEAs.

Moving on to a new sub-modeling competency, the data revealed that all groups in different engineering-based MEAs exhibited an exemplary level of *explaining*. To put it plainly, the students gave in-depth explanations of their reasoning when adjusting their model. The findings indicate that elementary gifted students not only recognized quantities and variables that can influence the problem situation but also surprisingly identified interested and affected parties and their expectations. Specifically, they also identified environmental impacts of the engineering activity during participation in the MEAs. For instance, group A in the *Bridge Construction* activity decided to build a shed to prevent getting waters on rainy days. They thus made their bridge more durable for peasants by ensuring they would be less affected by rainfall.

In particular, the findings indicate that all student groups in the present study communicated with stakeholders verbally or in writing. Critically, all groups reflected their experiences constructing the first model into the second situation with its working and non-working aspects. Engaging in engineering-based MEAs, elementary gifted students participated in group discussions, and it was observed that these types of settings were effective for revealing the reasoning skills of the elementary gifted students in this study. Similarly, Şen (2018) indicated that the discussion environment and the appropriate guidance of this discussion are effective in revealing the reasoning and problem-solving skills of gifted students. In the present study, the researcher actively encouraged students to freely share their opinions and ideas while engaged in engineering-based MEAs as a group. They were given the opportunity to express their opinions in the form of questions and subsequent answers geared to promote mental activity. To assist elementary gifted students to become more intellectually active, *why and how* questions were posed at the end of each session throughout the process to elicit their in-depth explanation. Such an implementation is supported by previous research (Şen et al., 2021), which has shown that a supportive classroom environment provides many opportunities for gifted students to express ideas, pose questions, and develop arguments together in a collaborative setting.

Despite the fact that the specific engineering design challenges vary in each activity, all of the student groups supported some of their design ideas and decisions in an effort to discover the best potential solutions to the engineering problems they faced. These findings confirm previous research (Guzey & Aranda, 2017; Siverling et al., 2021) and demonstrate that the participants in this study used evidence-based reasoning to make decisions about engineering design solutions - behavior which is comparable to, but likely less sophisticated than, the evidence-based decisions made by engineers in the profession. This finding may be explained by the idea that the traits of successful engineers are often exhibited by gifted students (Mann et al., 2011). It was observed that elementary gifted students' reasoning skills are effective in adjusting their design and model in engineering-based MEAs. The critical role that their explanations play was evident in the selection of the material to be used in the creation of models, the suitability of the design for the intended purpose, and the adjustment of the finished model. Importantly, a similar set of skills is employed by engineers. Some examples of such skills include the determination of the functionality of their design and the potential effects on the natural setting (Katehi et al., 2009).

Continuing with another sub-competency of adjusting, all six groups demonstrated exemplary instances of the *deriving an elegant solution* sub-modeling competency. That is, they employed elegant approaches that lead to solutions. However, the diverse groups exhibited exemplary evidence of the sub-modeling competency in a variety of approaches. While some of the groups sought the most elegant solution with the least amount of materials and efforts, other groups did so through the use of appropriate methods and materials. Correspondingly, the students proposed measures to mitigate the negative effects of the engineering activity and to maximize the desired solution. These findings are in line with the view that students iteratively refined the design solutions while reasoning about the definition of an optimal model and the trade-offs (Dasgupta, 2019; English & King, 2015; Mentzer et al., 2014). One plausible explanation for why the sample in this study demonstrates a high level of the *deriving an elegant solution* sub-modeling competency is that they expressed the functional solution requirement and gained client acceptance when they were internalizing the problem. Another possible explanation is that engineering-based MEAs produce an environment that is similar to the one that elementary gifted students experience when they work as engineers in the field (Mann et al., 2011).

The newly emerging sub-competency of adjusting, *adaptability and transferability,* is one of the most striking findings of the current study. All groups displayed exemplary evidence of this sub-modeling competency by easily adapting to the new condition and transferring their previous knowledge inside or outside of school settings. These are vital skills for the 21st-century, and activities that promote adaptability and transferability are suggested not only for regular students (Bertrand & Namukasa, 2020) but also gifted students (Renzulli, 2020). This finding might be related to the cognitive characteristics of gifted students, which include their ability to transfer knowledge across disciplines and thus creatively solve the problems in a different context, both within and outside of the classroom (Sen et al., 2021; Şengil-Akar, 2017; Taberi 2014; Vogelaar & Resing, 2018). For instance, in the *Mars Lunarcrete* activity, both groups created their second model determining the minimum amount of substances in the recipes different from the process of creating the first model. Hence, they made new connections between pieces of knowledge by using their experiences when constructing their first model and adapted their solution easily. This might imply that the second iteration is constructed on the experiences that elementary gifted students obtained from the first iteration of the design cycle.

Furthermore, the analysis of this study revealed that elementary gifted students display *transferability and adaptability* when adjusting the solution of the problem. This finding may be attributable to the notion that students perform better in modeling when they are provided with a substantial amount of time (Biccard, 2010; Mousoulides, 2007; Şahin, 2019; Tekin-Dede, 2015). The implementation of this study as a summer school study at BILSEM allowed more time to be allocated for the solutions of engineering-based MEAs than would have been during the regular semester. Hence, one of the most common challenges in such implementations, the time constraints encountered by gifted students while exhibiting their creativity, was eliminated in this study (Lee et al., 2021). Another possible explanation for this finding might be found in the display of a high level of modeling competencies in the first iteration. That is to say, elementary gifted students' exemplary performance in constructing their first model may lead to the demonstration of their unique characteristics (Pativisian, 2006) of adaptability and transferability. Specifically, it was observed that engineering-based MEAs provide elementary gifted students opportunities to use their previous knowledge by adapting it to meet the original conditions of the problem and think more flexibly.

Another striking finding of the present study is the emergence of a new sub-modeling competency of adjusting, *creative approach*. Groups within various engineering-based MEAs displayed different levels of competence in this sub-competency. While one of the groups in each engineering-based MEA showed an exemplary display of the *creative approach* sub-modeling competency, the other group showed a proficient display of the same. Those at the exemplary level took a creative approach to solving the problems, explaining the underlying reasons. In accordance with the present findings, previous studies (Morris et al., 2019; Siegle, 2014; Sen et al., 2021;) have demonstrated that authentic engineering-based MEA experiences gave elementary gifted students autonomy in their decision-making process and strategies and also the opportunity to show their creativity. The idea of constructing the floating bridge could be accepted as an exemplary instance of the creative approach since they came up with a new, unique, unexpected, and useful idea that was also appropriate and adaptive in terms of the existing constraints of the *Bridge Construction* problem. Another interesting example could be the decision of group D in the *Mars Lunarcrete* activity to use an oobleck substance made with cornstarch and water as a binder. These findings support arguments from previous research (Chamberlin & Moon, 2005; Lu & Kaiser, 2021; Mann et al., 2011; Wessels, 2014) claiming that the open-ended and complex nature of engineering-based MEAs make them creativity-oriented activities, since they require and encourage mental flexibility and provide opportunities for the generation of creative ideas.

Parallel to this argument (Lu & Kaiser, 2021), one of the most striking findings of the present study supports the idea that creativity should be incorporated into the continuous expansion of the modeling competencies framework, with various characteristics of creativity embedded within the process phases for elementary gifted students. A possible explanation for this interesting finding may be related to the relationship between giftedness and creativity (Chowkase, 2022; Guillford, 1967; Renzulli, 2005; Sternberg, 2005). Considering that the participants of this study, students who attended BILSEM, were defined by individual intelligence tests and group intelligence tests, it is a reasonable expectation that they would exhibit the creativity behaviors associated with the concept of giftedness. This is supported by previous research, which has shown that BILSEM students demonstrated different dimensions of creativity when engaged in MEAs (Şengil-Akar, 2017). The varying degrees of the *creative approach* sub-modeling competency found among the study's six groups may derive from the distinct dynamics of each group. During group work, the students demonstrated their individual differences; nonetheless, it was observed that they were able to arrive at a consensus by convincing each other and reasoning together. The findings of the present study suggest that the process of creativity is one in which elementary gifted students influence each other. Accordingly, ideas evolve by going in a variety of different directions, and the students move forward by cooperating with one another in the case of the problems. Therefore, the models that the students put together at the end of this study are the result of a common mind and a social process. Consequently, engineering-based MEAs could serve as an enrichment tool to meet the needs of gifted students and show their creativity.

5.1.6. Organizing

This part of the section discusses the findings for organizing, the sixth modeling competency. Data analysis revealed that in addition to the organizing sub-competencies of *evaluating and judgment* and *reflection* suggested by de Villiers (2018), *elaboration* emerged as a new sub-competency. Across the board, the findings of this study revealed that all six groups exhibited an exemplary degree of the *evaluating and judgment* sub-modeling competency. In other words, they offered comprehensive, insightful analysis, synthesis and evaluation as well as clear connections made to real-life situations or previous content. There is widespread consensus that the key to successful modeling is the competency to generate answers that are both more precise and general through iterative reflection and validation procedures (Blum, 1991). The findings of the current study support this idea, that all groups displayed the engineering sub-competency and all of the information gathered was used to generate a comprehensive solution through a process of synthesis that included design, development, and communication during the investigation of the problem. Consistent with other emerging research (Chan et al., 2012), elementary gifted students validated their models by evaluating many solution choices in order to make decisions and accomplish the goal they had set out to attain.

Through within-group and whole class discussion, the evidence in the findings section clearly demonstrates that student groups take a structured and systematic approach to developing their models. In line with English (2003), these developments occurred without guidance from the instructor or researcher, and involve students forming, defining, explaining, validating, checking, and communicating their own ideas. Further, an important aspect of these developments is that the social interactions occur spontaneously. This is likely related to working collaboratively in groups (Lesh & Doerr, 2003; Tangney et al, 2001), as the students evaluate and judge their views with their and other group members. This finding further supports the idea that students' social interaction in the modeling process is critical in demonstrating their organizing modeling competency (Biccard, 2010; Chan et al., 2012; English & Watters, 2004; Maaβ, 2006). On the other hand, discussion sessions during the engineering design processes present suitable learning environments to reveal gifted students' reasoning skills (Brown & Joerg, 2018). Hence, the iterative nature of the engineering design process to repeatedly express, test, and revise the current state of gifted students' thinking might contribute to the high degree of this sub-modeling competency on display. Another possible explanation for the finding might be the amount of evaluation time that students would most likely need in order to engage in model validation (Maaß, 2006).

Extending the discussion of the *evaluating and judgment sub-modeling competency*, one of the interesting findings is that elementary gifted students evaluated a situation in the absence of full evidence. To put it more explicitly, they demonstrated an ability to analyze a situation without having access to all of the evidence. This finding contradicts with the literature that elementary students (Kaygısız, 2021; Şahin, 2019) and even pre-service teachers (Aydın-Güç 2015; Gürel, 2018) do not need to evaluate and judge their models. For instance, group C in the *Mars Lunarcrete* activity did not create a thorough working model but did make an inference based on their experiences. This may be an indication of the critical thinking skills of elementary gifted students (Şen et al., 2021). Even if most gifted students already have critical thinking skills due to the nature of their giftedness (Reis & Renzulli, 2009), the studies emphasize that differentiation strategies should be used to promote these skills (Ozdemir, 2016; Van Tassel-Baska, 2010). This suggests that as engineering-based MEAs involving analysis, evaluation, and reconstruction require and foster critical thinking, they are most likely to help those in the present study demonstrate their critical thinking skills. In the present study, the elementary gifted students made connections between the steps while applying the procedures and deduced how the process was expected to be completed by analyzing the overall working process. When they were engaged in engineering-based MEAs, they employed an exploratory approach while simultaneously analyzing the suitability of the design and generating a realistic model by revealing comprehensive ideas. Hence, the students in the present study were not only able to investigate a variety of options, but also, when given the opportunity to reflect on their own thought processes, they engaged in critical intensive analysis of different points of view in order to develop alternative solutions. The possible explanation of this finding might be that engineering-based MEAs provide enrichment in the process since elementary gifted students took on the role of active investigators, applying skills such as analyzing, synthesizing, and evaluating information as well as exploring new ways of thinking (Taber, 2014). By means of these activities, these students could actively engage in critical thinking, complex problem solving and making inferences in the context of the problems.

Secondly, the data analysis indicated that the groups, except in the *Dr. Ahmet's Will* activity, displayed an exemplary degree of the *reflection* sub-modeling competency. On the other hand, both groups in that activity exhibited a proficient level of reflection. To put it more explicitly, all groups identified strengths and weaknesses in their own thinking, but recognized alternate perspectives about the problem when comparing the other perspectives. In addition, the groups who showed exemplary instances evaluated them in the context of alternate points of view. The students in different groups made critical checks on their solutions and models while engaging in engineering-based MEAs in the whole process. According to the findings of the current study, all groups reflected their own experiences and thoughts. Using a variety of tools for reflection, elementary gifted students constructed their models. Examples of such reflection tools include drawings, letters, and the changing roles that students took on throughout the MEAs; the values, attitudes, or feelings that can contribute to higher levels of engagement; and the problem-solving strategies that are productive throughout the various stages of the engineering design process. These tools may provide a direction for the development of self-reflection (Hamilton et al., 2008). Hence, the findings of this study revealed that elementary gifted students take on the role of decision makers by engaging in self-reflection and group discussion as they work through the design process. Furthermore, this finding supports the argument that students should be encouraged to engage in constructive design discussions so that they may make and justify effective decisions (Guzey & Aranda, 2017).

However, there are many studies that reach the conclusion that students do not need to check and reflect on solutions or other ways of solving the problem (Biccard, 2010; Blum & Ferri, 2009; Kaygısız, 2021; Maaß, 2006; Şahin, 2019). Nonetheless, the students in the present study not only reflected on their solutions but also considered the social, cultural and environmental effects of their solution as well as the interdependence, interactions, and relative importance of such factors. On the other hand, analysis of written reflections revealed varying degrees of reflectivity, with high levels of reflection being rare. Thus, the findings of this study confirm the notion that using only written reflections is not adequate for elementary gifted students, and that other methods should be used to support their reflection. A possible explanation for this might be related to gifted students' ability of verbalization (Ogurlu, 2010). Since gifted students think quickly and put their thoughts into practice (Clark, 2008), verbalization skills should be taken into account when evaluating the thinking process and revealing how it is realized. In addition, research studies have emphasized the importance of reflecting on the solutions found within groups and in whole class discussions (Biccard, 2010; Chan et al., 2012; Doerr & English, 2003; Lesh & Doerr, 2003). The findings of the current study support the idea that interaction with group members and other groups enabled the groups to check their assumptions, recognize different applications of mathematical concepts, recognize weaknesses and strengths in their models, recognize better and more useful models and how they were formed, and acquire different ways of thinking.

Similar to the reflection sub-modeling competency, while two groups in the *Dr. Ahmet's Will* activity showed a proficient degree of the *elaboration* sub-modeling competency, the other four groups showed an exemplary level of the same. The groups that demonstrate a proficient measure of the sub-modeling competency presented the technical breadth and depth of the process by providing substantial details in the explanation of their ideas. The other groups that demonstrated an exemplary degree of the sub-modeling competency, on the other hand, provide even more details in the explanation of their ideas. This unexpected finding indicates that elementary gifted students display the components of creativity as a sub-modeling competency when they are engaged in engineering-based MEAs similar to studies focused on gifted students' creativity when involved in MEAs (Şengil-Akar, 2017; Wessels, 2014). Though definitions have been provided for these components, which are also cited as creativity indicators, there are no specific descriptions of how these indicators might be observed in students or what behaviors reveal these indicators (Guilford, 1966; Şengil-Akar, 2017; Taşkın, 2016; Wessels, 2014). Consequently, it may be stated that this condition provides a significant obstacle to the objective evaluation of creativity (Chamberlin & Moon, 2005). Considering the lack of a fully structured program in the education-teaching process carried out in BILSEMs and the presence of an activity-based teaching process, elementary gifted students' engagement in engineering-based MEAs for five weeks might provide insights into how they elaborate on their ideas through the process.

Based on the findings of this study, engineering-based MEAs could serve as differentiation tools within the classroom, as a form of enrichment beyond classroom learning, or as the site for learning experiences that are both engaging and challenging for elementary gifted students. To put it more precisely, engaging with such challenging tasks would offer participants the opportunity to demonstrate varying degrees of sophistication and breadth and depth of understanding, not to mention competencies in their areas of strength. This study also indicates that elementary gifted students demonstrate various levels of creativity when they are engaged in engineering-based MEAs. Accordingly, such activities could be used as a diagnostic tool both to identify children who are talented or creative in the fields of science, mathematics, engineering, or the arts, and also to identify different dimensions of their creativity such as originality and flexibility, as other studies have done (Chamberlin & Moon, 2005; Şengil-Akar, 2017). When elementary gifted students are engaged in engineering-based MEAs as a group, students with varying levels of creativity could promote collective creativity and nurture their creative thinking skills.

Although determining the degree of creativity and of its components among elementary gifted students was not the aim of this study, the students engaging in engineering-based MEAs did exhibit such components as sub-modeling competencies. Thus, the findings of the current study present empirical evidence for the study of Lu and Kaiser (2021), who proposed a framework for the modeling competencies incorporating creativity. The findings of this study indicate that the elaboration of ideas, as suggested by the researchers, occurs during the validation of the solutions phase. The high-level elaboration sub-modeling competency may have emerged when the students were permitted to work in-depth on their own ideas and shape them without being evaluated within and between groups. In such environments, the students could be encouraged to think flexibly and self-reflection could be required. In the *Bridge Construction* activity, for instance, the dialogue between two students from different groups reveals that their own self-reflection provides details in the explanation of ideas on proportional reasoning by, in this case, establishing a connection between the length of the finger and the double-sided road. The discussions among students on their engineering design process and final models were also key contributors to the findings as evident in their displays of applied science, mathematics, and engineering knowledge as well as real-life experiences. Hence, it may have provided technical depth to the process as opportunities were provided for students to use their ideas in a wide range of materials and under different circumstances. Since elaboration can only be examined for appropriate problems (Klavir & Gorodetsky, 2011), an unexpected finding of this study shows that engineering-based MEAs may contribute to reveal the elaboration component of creativity for elementary gifted students, contrary to the research (Lu & Kaiser, 2021). Consistent with the research (Biccard, 2010; Lesh & Zawojewski, 2007; Sen et al., 2021; Şengil-Akar, 2017), group presentations at the conclusion of the engineering design process and explanations of how elementary gifted students created their models in the process may contribute to students' ability to work in groups, allow them to share their ideas, explain, justify, listen to and respect the ideas of individuals with differing viewpoints, demonstrate analytical and creative thinking skills by exposing them to multiple perspectives, and display students' ability to think flexibly by considering multiple perspectives. Since the level of elaboration was related to the amount of detail in the idea (Guilford, 1967), the various levels of elaboration in groups might be explained by their strategies to employ client acceptance and a specialized brainstorming approach. This means that the more an idea is studied, the more it is processed, and the more it is expanded by drawing on a variety of associations, the more detail they may provide at the end of the process.

5.1.7. Generalizing

Based on the classification of cognitive modeling competencies proposed by de Villiers (2018), this part of the section discusses the findings of the generalizing competency, the final modeling competency. It is important to note that based on the findings of the present study, an adjustment was made to that guide's description of the exemplary level of sub-modeling competencies. In this regard, the generalizing competency was investigated with respect to the sub-modeling competencies of *establishing a similar relationship*, demonstrating *general or independent reasoning* and *creating an easy to use model*. Moreover, the extent to which elementary gifted students in groups displayed the generalizing modeling competency and its sub-modeling competencies when they are engaged in three different engineering-based MEAs were addressed in this section.

Regarding *establishing a similar relationship*, all six groups exhibited exemplary indicators of this sub-competency. On this basis, their connection to a real-life application or other disciplines was accurate and realistic. Beyond that, data analysis revealed that elementary gifted students considered the interdependence, interactions, and relative important factors different from the *exemplary* indicators of *establishing a similar relationship* sub-competency proposed by de Villiers (2018). This finding is in surprising conflict with the findings from studies that students of varying grade levels (Chan et al., 2012; Kaygısız, 2021; Şahin, 2019) and even pre-service teachers (Aydın-Güç, 2015; Çakmak-Gürel, 2018) could not exhibit their generalization competence and its related sub-modeling competencies at the desired level. In the literature, the context of MEAs and students' modeling experiences are mentioned as the explanation for why students are unable to display generalizing competencies at an adequate level (Biccard, 2010; Chan et al., 2012; Kaygısız, 2021). In contrast, however, to previous research, the findings of the present study propose that generalizing competency may not be related to modeling experience. In particular, in this study, elementary gifted students generally exhibited an exemplary degree of the generalizing competency and its related sub-competencies, despite not having modeling experience. This could be explained by the exceptional ability of gifted students to communicate their ideas, establish relationships, and generalize (Gardner, 2011; Şengil-Akar, 2017). It was observed that elementary gifted students in this study provided instances from real life or scientific knowledge in order to be convincing in their explanations, generalizations, and arguments. They also engaged in investigations and discussions regarding the generalizations and arguments that they developed. This finding supports the previous research (Sak, 2014), stating that gifted students act on the basis of facts and evidence throughout the problem-solving process and that rationality serves as the foundation for their views during the decision-making process. Another possible reason for this finding could be that this sub-modeling competency of *establishing a similar relationship* relies greatly on the quality of the situation model. Hence, it was reasonable to expect a high level of generalizing ability because the sample of the current study generally showed a high level of competency in structuring and organizing information. It can also be explained by the fact that they are competent in the previous steps of modeling and have acquired modeling experience by the end of the process.

Continuing on to another sub-competency of generalizing, all six groups exhibited an exemplary degree of the *demonstrating general or independent reasoning* sub-modeling competency. Specifically, they demonstrated the ability to identify a generalization easily when presented with a specific situation and also applied deductive reasoning to prove the solution. This sub-modeling competency is an essential component of not only mathematical competencies (Niss & Hojgaard, 2021) but also modeling (Geiger et al., 2021; Lehrer & Schable, 2000; Lesh & Doerr, 2003). In addition, evidence-based reasoning is a crucial element of engineering-design based problems with an iterative nature (Siverling et al., 2021). In line with these views, the findings of the present study revealed that elementary gifted students demonstrate evidence of engineering as well as scientific and mathematical reasoning. Due to their high reasoning ability, it is common for elementary gifted students to demonstrate these special characteristics when engaged in MEAs (Taşkın, 2016), especially engineering-based MEAs (Sen et al, 2021).

Finally, groups in different engineering-based MEAs demonstrated varying levels of competence in the *creating an easy to use model* sub-modeling competency of generalizing. Except group D in the *Mars Lunarcrete* activity and group F in the *Dr. Ahmet's Will* activity, the other groups displayed a proficient level of this sub-modeling competency. While the models of group D and F could easily be adapted in other situations and their predictions were accurate, the models of other groups needed minor simplifications to use as a model for other situations. In contrast to previous studies (Biccard, 2010; Kaygısız, 2021; Tekin-Dede, 2015), the findings of the present study indicate that elementary gifted students exhibit *proficient* or *exemplary* displays of *creating an easy to use model*. A possible explanation might be that the fact of students being given sufficient time to complete the engineering-based MEAs produced more positive findings in terms of their modeling competency (Kaygısız, 2021; Tekin-Dede, 2015). Another possible explanation is that their exemplary modeling competencies were demonstrated at previous stages similar to the idea of

Aydın-Güç (2015). Thus, the findings from the current study support the idea that the quality of the situation model has an impact on the generalizability of the model (Biccard, 2010). That is to say, the stronger the models are, the easier it could be for the groups to construct generalizable models.

In addition to their applicability to the specific circumstance in which the model was developed, the changed and improved models could be applicable to other situations and other products. This idea is similar to the assumption that specific representations of the students' model may have limited application, but that the conceptual structure that underlies the representation may be more generalizable (Mousoulides, 2007). For instance, group C in the *Mars Lunarcrete* activity did not come up with a model that was completely functional for the situation, yet they made some generalizations based on their experiences. The most likely reason for this is that elementary gifted students recognize uncommon connections between concepts (Wellisch & Brown, 2013) and transfer their knowledge easily to novel situations (Vogelaar & Resing, 2018).

Although the core characteristics were the same for all three engineering-based MEAs in this study, the findings of the study indicate that certain aspects of the activities, particularly those related to design-related activities, promoted different types of sub-modeling competencies. For all forms of design challenges in the present study, elementary gifted students discussed the design as a whole, the types of materials they were to use, the design's functionality, and whether the client would accept the solution or not. The findings suggest that the less-restricted nature of engineering-based MEAs enables gifted elementary students to incorporate their imagination into the design process through the flexibility in the given data. When their intended output was a product, the participant students discussed the structure of their design, and if they were expected to produce a prototype, they also discussed the implementation of their design. In contrast, students focused on the steps involved in the process rather than the structure when working on process-oriented design such as in the *Dr. Ahmet's Will* activity. These findings of the current study suggest that different types of design problems within engineering-based MEAs inspire students to examine various engineering components.

Taking all of the aforementioned into account, integrating the engineering design process into MEAs is using engineering-based MEAs as a tool to enable elementary gifted students to demonstrate their creativity and innovative thinking in order to find solutions to real-world problems. This they do by incorporating engineering design process and mathematical modeling builds on their existing mathematics and science knowledge. In addition, engineering-based MEAs encourage students to engage in groups, in which they learn to communicate and work together to find solutions to challenging situations. To clarify this, the iterations that groups go through when revising a model using design-test-redesign cycle could produce new cognitive structures and understandings in group members more effectively than a single iteration could. Hence, the findings of the present study present empirical evidence using engineering-based MEAs as a reflection tool (Hamilton et al., 2008) to nurture problem-solving personalities for elementary gifted students.

Moreover, another important finding of the current study is related to the parallelism between the engineering design process and mathematical modeling process for elementary gifted students. The iterative nature of both processes, which enable the students to elicit learning of the content or process, has been emphasized in the literature (Groshong, 2018; Mann et al., 2011; Zawojewski et al., 2008). This relationship has been ignored, however, when designing modeling experiences and engaging in such processes (English, 2017), particularly with elementary (Dorie et al., 2014; Portsmore et al., 2012) and gifted students (Mann et al., 2011). Hence, the present study gives researchers insight into elementary gifted students by examining the terminology and similarities of both processes from an integrated perspective. To be more specific, this study proposes that the iterative nature of these processes enables elementary gifted students to reveal their unique thought processes through the use of engineering-based MEAs, which often require the knowledge and skills of various disciplines.

Last but not least, one of the most striking findings of the current study is an adaptation and enrichment of a modeling competency observation guide and classification of cognitive modeling competencies for elementary gifted students in the Turkish context, developed on the basis of the study (de Villiers, 2018). Since the present study followed a holistic approach and elementary gifted students were expected to demonstrate seven modeling competencies by maintaining the process with instructions while being uninformed of the engineering design process and modeling competencies. Even in the current study, elementary gifted students display some new sub-modeling competencies (ethical consideration, flexibility and novelty, applying interdisciplinary knowledge, adaptability and transferability, creative approach and elaboration) when they are engaged in engineering-based MEAs. That is to say, the mentioned sub-modeling competencies were added to the framework proposed by de Villiers (2018). Beyond that, some indicators related to the sub-modeling competencies in the classification of cognitive modeling competencies were expanded based on the findings of the present study. Therefore, the present study attempted to fill this gap in knowledge in the field of mathematics and engineering education, and it did so by merging the engineering competencies that can co-develop with mathematical modeling competencies through engineering-based MEAs. Consequently, this study provides insight into the investigation of elementary gifted students' modeling competencies, and clues for researchers regarding how to construct activities and environments to make elementary gifted students' thinking visible to others.

The accessible literature has indicated that the older the students are, the more successfully they complete the modeling process (Henning & Keune, 2007; Ludwig & Xu, 2010) and that students struggle with the competence of making an assumption (Yıldırım, 2019), as well as the competencies of verification and justification (Kabar & İnan, 2018; Kaiser, 2007; Maaß, 2006; Tekin-Dede & Yılmaz, 2015). However, the findings from the present study suggest that engineering-based MEAs enable elementary gifted students to demonstrate modeling competencies at a generally exemplary level since these activities are appropriate for their unique characteristics and that they satisfy their needs. In this regard, this study provides contributions to the literature in mathematics education as well as science, engineering and gifted education by determining the modeling competencies of elementary gifted students and which competencies are revealed at specific level. Despite the fact that it was not the primary goal of the current study, the results of this study may be useful to future researchers in terms of how to develop activities to promote students' modeling competencies.

5.2. Implications for Educational Practices

The findings of this study have three main implications for educational practices. These are discussed with regards to students, teachers and educational resource designers for BILSEM. First of all, the findings of this study reveal that engineering-based MEAs could be implemented as effective tools to differentiate the instruction for gifted students. The complex, interdisciplinary, and flexible nature of engineering-based MEAs could make them a significant tool to overcome the problems that gifted students generally experience such as boredom, lack of challenge, and motivation (Mann et al., 2011). In this study, gifted students took decisive action to develop their models, demonstrating task commitment throughout the process even if confronted with unexpected situations. This could be evidence that engagement in engineering-based MEAs is interesting for elementary gifted students. The findings of the current study further reveal that engineering-based MEAs enable elementary gifted students to engage in a collaborative learning environment in order to construct and strengthen their models. In this regard, the students could be given opportunities to externalize their views by combining their perspectives with those of their group members, and thus reflecting on their own and others' perspectives. The fact that students struggle to handle multiple data at once during the process and that they built on each other's ideas may indicate that these activities are challenging for them.

In addition, the exemplary indicators related to new sub-modeling competencies such as applying interdisciplinary knowledge, elaboration, adaptability and transferability, and creative approach may provide evidence that engineering-based MEAs require higher-order thinking for elementary gifted students. Considering all these instances, the findings of the present study showed that engineering-based MEAs could be used as a differentiated tool that presents the initially assumed characteristics such as being interesting, challenging, and requiring higher-order thinking (Ozdemir, 2016). To put it another way, differentiation provides students with a variety of educational opportunities while engaged in the activities as well as interpreting, processing, or creating the information that they have obtained. Hence, the findings of the present study imply that engineering-based MEAs could be effective for all giftedness fields, not only mathematics but also field-specific giftedness.

Beyond eliciting students' interdisciplinary knowledge to solve problems, the findings of this study indicate that engineering-based MEAs enable elementary gifted students to also display modeling competencies, which bear a substantial overlap with 21st-century skills such as flexibility, problem solving, creativity, and innovation, as well as social skills. In addition to this, it can be inferred that the students demonstrated their collaboration, communication, and critical thinking skills in the instances involving engineering-based MEAs. Thus, such activities could be used as challenging and authentic tasks to integrate engineering and incorporate 21st-century challenges into the classroom.

Another implication of this study concerns teachers of elementary gifted students. The relevant literature indicates that teachers of gifted students in both heterogeneous and homogeneous classrooms need appropriate differentiation strategies to address the unique needs of gifted students (Bildiren & Citil, 2021; Reis et al., 2004). In particular, the structure of the activity and the use of effective teaching strategies have crucial importance when providing education to gifted students (Diezmann & Watters, 2000). Taking into consideration the lack of a fully structured teaching program framework at BILSEMs (Çetin & Doğan, 2018), as well as the lack of special teaching programs followed in regular schools to meet the needs of gifted students, the current study provides sample engineering-based MEAs implemented and revised in accordance with the views of gifted students and their teachers. Hence, teachers could apply such activities by adapting their classroom or could develop similar activities by considering the characteristics of engineering-based MEAs and the instructional needs of gifted students.

The findings of this study bear implications regarding possible practical considerations in differentiated instruction. Due to the time pressure in regular heterogeneous classrooms to keep up with the curriculum (Dedebaş, 2017), mathematics application courses could be used to apply activities such as those above in heterogeneous groups including gifted students. Accordingly, teachers could plan activities for gifted students that provide challenges, flexibility, and extension. The findings of this study support the idea that engineering-based MEAs, with their less constrained and more complicated nature, constitute a good example of activities that can be used by teachers to meet the needs of gifted students. Furthermore, the findings of this study reveal that engineering-based MEAs generally enable elementary gifted students to display their modeling competency to a considerable extent. Hence, teachers could examine how the students demonstrate their competencies as they progress through the modeling cycle. In addition, rather than presenting students with ordinary and routine problems, teachers could provide engineering-based MEAs in order to elicit gifted students' autonomy, creativity, higher-order thinking, and reasoning skills.

Moreover, the engineering design process-based learning model could be an effective teaching strategy in that it organizes learning around the engineering design process for gifted students (Hann & Shim, 2019; Mann & Mann, 2016). The findings of this study imply that elementary gifted students were competent in deriving new learning opportunities from their failures, participating actively and willingly in the process, and collaborating with their group members. Overall, the teacher could use engineering-based MEAs and the engineering design process to provide educational opportunities for gifted students to display their existing capacity at the highest level.

The final implication of the present study concerns educational resource designers for BILSEM. Considering the problems related to developing and applying activities to satisfy the needs of gifted students in BILSEM described previously (Bildiren & Çitil, 2021; Çetin & Doğan, 2018), the findings of the current study imply that engineering-based MEAs provide educational opportunities to overcome such problems. In keeping with the necessity of maximizing gifted students' potential in the learning environments offered to them in BILSEM (MoNE, 2012), the findings of this study present empirical evidence regarding the implementation of engineering-based MEAs for elementary gifted students who attended individual talent recognition programs. The MEAs' specific benefits in such a context are their ability to elicit students' creativity and their areas of strength in specific fields.

In the individual talent recognition programs, the activities that promote gifted students' creativity are developed and implemented in order to help them realize their unique skills and potential (MoNE, 2012). Correspondingly, the findings of this study implied that engineering-based MEAs reveal students' creativity. Consistent with the literature (Chamberlin et al., 2013; Chamberlin & Moon, 2005), engineering-based MEAs could be used as a tool to identify creativity in different areas. The findings also revealed that elementary gifted students in groups generally showed components of creativity as sub-competencies such as *flexibility and novelty, creative approach,* and *elaboration.*

Alongside creativity, in order to identify areas where gifted students are talented and to prepare them for further talent programs subsequent to the individual talent recognition program, the students could be involved in interdisciplinary activities instead of activities specific to each field such as science, mathematics, and design. Hence, the areas that they can work on in-depth in the successive programs in BILSEMs could be determined through the exploration of ideas. The findings of this study indicate that elementary gifted students generally display exemplary instances of sub-competencies in *applying interdisciplinary knowledge* and *adaptability and transferability* when they are engaged in engineering-based MEAs. Since such activities give the opportunity to work together with more than one discipline, similar to the literature (Mann et al., 2011; Sen, 2019; Şengil-Akar, 2017), special talents could be noticed and directed towards the areas to be selected for the successive program. In all of these aspects, the current study implies that engineering-based MEAs could be implemented in BILSEMs as appropriate activities to determine their special skills and reveal their creativity in different areas.

To summarize, in light of the findings of the current study, there are a variety of educational implications valid for students, teachers, and educational resource designers for BILSEM. Additionally, this study addresses several limitations and makes some recommendations for future research in the relevant subject. The following section discusses the limitations of this study.

5.3. Limitations and Recommendations of the Study

There are some limitations which may affect the findings of this study. The following provides details on each of these limitations, the different approaches that have been proposed to address them as well as recommendations for further research in light of the previously described findings and relevant literature.

One of the limitations might involve the engineering-based model eliciting activities used in the study. The problem situations in such activities were mostly concerned with civil engineering, architectural engineering, and data mining engineering. Hence, future research might integrate the various fields of engineering into the problem situation. Although these activities meet the criteria of this study and the participants have never seen them before, the possibility may exist that the results are biased by the elementary gifted students' previous knowledge. Moreover, there was only one engineering-based model eliciting activity that was presented to each group throughout the five weeks of the study. Therefore, the specific activity may have influenced their performances if they were not comfortable or did not specialize in the content areas that the activity covered. To overcome this limitation, the special needs of elementary gifted students who attended individual talent programs at BILSEMs were identified through discussions with BILSEM teachers. Then, the characteristics of differentiated activities that address the special needs of elementary gifted students were recognized based on the relevant literature. Accordingly, engineering-based MEAs were considered as appropriate activities due to their interdisciplinary, open-ended, complex and authentic nature to meet the needs of gifted students. In addition, it was determined that elementary gifted students are interested in the events that are taking place in their environment and in the globe, as well as the problems on the agenda (Taber, 2014). Considering their interests, specialists in mathematics and science education, engineering, and science and mathematics teachers at BILSEM chose the context of the problems. For example, it was believed that the *Send your name to Mars* project, which was on the agenda at the time, *Mars Lunarcrete* activity would stimulate the students' attention. Besides, the context of *Bridge Construction* and *Dr. Ahmet's Will* activities include regional and global concerns that students may experience frequently in their daily lives. Hence, the researcher attempted to minimize these limitations that might have occurred and affected the findings as best as possible before data collection.

Considering that students' modeling competencies vary depending on MEAs (Mousoulides, 2007; Şahin, 2019), future studies might focus on different types of engineering-based MEAs and examine how gifted students' modeling competencies differ in these activities. Moreover, further research may examine more than one activity over a longer period of time, as each group participated in only one engineering-based MEA in a five-week period of the current study. Thus, the studies might be conducted to determine the effects of different engineering-based MEAs on the same group of students or to determine whether specific modeling competencies develop over time. In addition, future studies might examine the effects of such activities on the creativity of gifted students.

A second limitation might be the method of data collection. This study is based on the assumption that students verbalize their thought processes when solving problems by the think aloud method. They may not feel the pressure of a time constraint. However, the students might not have reported all of their thoughts because they needed to spend extra effort applying the think aloud technique. A further issue in data collection, interview bias, might be a concern in this study. In order to minimize the effects of this threat, the researcher conducted an introductory meeting with the participants in which the researcher tried to build trust by introducing herself and the purpose of the study. To conquer these data collection-based limitations, triangulation was used in order to improve the credibility of the findings across the interviews. Member checking was also used to validate the obtained data.

Another limitation may relate to the small number of participants. The participants of the study were selected from students attending the Science and Art Center. Inclusion was based on a sample of gifted students selected by teachers and then achieving IQ tests with scores of 130+. Due to the uniqueness in design and methodology, purposive sampling was used to select participants that meet the selection criteria and provide more detail on their processes when they are engaged in engineering-based MEAs. However, generalization of the study findings into other settings was not the aim of this study. The findings of the study and its implications are important for understanding elementary gifted students' engagement in engineering-based MEAs at BILSEM and provide information for other researchers to conduct future investigations in this particular area. In line with this, the following presents the recommendations for future research studies.

Although engineering-based activities (Guzey et al., 2016) and MEAs (Lesh & Doerr, 2003) are suggested for all students - both regular and gifted - in the literature, the current study used engineering-based MEAs for only one specific group of students, the gifted demographic. The participants of this study were limited to the students who were enrolled in the individual talent recognition program at BILSEM and who were determined to be gifted based on BILSEM’s acceptance criteria in the years they were nominated to BILSEM. Although there have been some modifications to the acceptance criteria of BILSEM, engineering-based MEAs could still be used for gifted students at BILSEM since the identification of giftedness is mainly based on the fact that gifted students demonstrate particular characteristics than their peers. In future studies, the research could be replicated with gifted students who determined to be gifted based on the current BILSEM’s acceptance criteria and who attended successive programs at BILSEM. Moreover, long-term research could be conducted on the development of modeling competencies of gifted students.

In addition, it can be suggested that further studies examine the experiences of regular students at different grade levels when they are engaged in engineering-based MEAs. Furthermore, research with gifted students in homogeneous and heterogeneous classrooms could be carried out and their findings compared to those of their peers. Comparable research studies can also be conducted in different nations, with the results of these studies being used to explore cultural variations. Thus, the extent to which gifted students or regular students display modeling competencies when they are engaged in engineering-based MEAs could be discussed in a variety of contexts.

Last but not least, the present study revised the framework proposed by de Villiers (2018) to determine the modeling competencies of elementary gifted students by merging the related mathematical modeling and engineering sub-competencies based on the findings of this study. As a possible extension of this research, the framework might be tested in various settings with different groups of students. In addition, researchers might investigate whether the new codes revealed in this study have been reported in other investigations involving other student groups. For instance, future research might determine whether other studies reveal components of creativity and ethical consideration. In summary, further research is needed to validate the current group modeling competency observation guide that emerged from the data of this study with multiple reference groups with varying levels of experience.

REFERENCES

Akaygun, S., & Aslan-Tutak, F. (2016). STEM images revealing stem conceptions of pre-service chemistry and mathematics teachers. *International Journal of Education in Mathematics, Science and Technology*, *4*(1), 56-71.

Akgül, G. (2021). Teachers' metaphors and views about gifted students and their education. *Gifted Education International*, *37*(3), 273-289.

Akgündüz, D. , Aydeniz, M., Çakmakçı, G., Çavaş, B., Çorlu, M. S., Öner, T., & Özdemir, S. (2015). *STEM eğitimi Türkiye raporu*. İstanbul: Scala Basım.

Andersen, L. (2014). Visual–spatial ability: Important in STEM, ignored in gifted education. *Roeper Review*, *36*(2), 114-121.

Anney, V. N. (2014). Ensuring the quality of the findings of qualitative research: Looking at trustworthiness criteria. *Journal of Emerging Trends in Educational Research and Policy Studies*, *5*(2), 272-281.

Assouline, S. G., & Lupkowski-Shoplik, A. (2021*). Developing math talent: A comprehensive guide to math education for gifted students in elementary and middle school.* Routledge.

Ataman A. , (1998). Üstün Zekâlılar ve Üstün Yetenekliler, Anadolu Üniversitesi *Açıköğretim Fakültesi Yayınları*, 183-185.

Ataman, A. (2012). Üstün yetenekli çocuk kimdir. Geleceğin mimarları üstün yetenekliler sempozyumu, 27, 4-15.

Aydin Güç, F. (2015). *Matematiksel modelleme yeterliklerinin geliştirilmesine yönelik tasarlanan öğrenme ortamlarında öğretmen adaylarının matematiksel modelleme yeterliklerinin değerlendirilmesi* [Unpublished doctoral dissertation]. Karadeniz Teknik University, Trabzon.

Aygun, B. (2010). *The needs analysis of the mathematics program for the gifted students attending lower secondary education schools* [Unpublished master's thesis]. Hacettepe University, Ankara.

Aztekin, S. , & Şener, Z. T. (2015). The Content Analysis of Mathematical Modelling Studies in Turkey: A Meta-synthesis Study. *Education & Science/Egitim ve Bilim*, *40*(178).

Baker, C. K., & Galanti, T. M. (2017). Integrating STEM in elementary classrooms using model-eliciting activities: responsive professional development for mathematics coaches and teachers. *International Journal of STEM Education*, *4*(1), 1-15.

Barbosa, J. C. (2006). Mathematical modelling in classroom: A socio-critical and discursive perspective. *ZDM*, *38*(3), 293-301.

Barnett, M. (2005). Engaging inner city students in learning through designing remote operated vehicles. *Journal of Science Education and Technology*, *14*(1), 87– 100.

Bayazıt, İ., & Koçyiğit, N. (2017). Üstün Zekâlı Ve Normal Zekâlı Öğrencilerin Rutin Olmayan Problemler Konusundaki Başarılarının Karşılaştırmalı Olarak İncelenmesi. *Abant İzzet Baysal Üniversitesi Eğitim Fakültesi Dergisi*, *17*(3), 1172-1200.

Berland, L. K., Martin, T. H., Ko, P., Peacock, S. B., Rudolph, J. J., & Golubski, C. (2013). Student learning in challenge-based engineering curricula. *Journal of Pre-College Engineering Education Research*, *3*(1), 53– 64.

Bertrand, M. G., & Namukasa, I. K. (2020). STEAM education: student learning and transferable skills. *Journal of Research in Innovative Teaching & Learning*.

Betts, G. T., & Neihart, M. (1988). Profiles of the gifted and talented. *Gifted child quarterly*, *32*(2), 248-253.

Biccard, P. (2010). *An investigation into the development of mathematical modelling competencies of grade 7 learners* [Unpublished doctoral dissertation]. University of Stellenbosch.

Biccard, P., & Wessels, D. C. (2011). Documenting the development of modelling competencies of grade 7 mathematics students. *Trends in teaching and learning of mathematical modelling*, 375-383.

Bildiren, A., & Çitil, M. (2022). Educating Gifted Children in Turkey: A Retrospective Analysis and the Current State. *International Journal of Educational Reform*, 10567879211050345.

Blomhøj, M. (2004). Mathematical modelling: a theory for practice. In *International perspectives on learning and teaching mathematics* (pp. 145-159). National Center for Mathematics Education.

Blomhøj, M. (2009). Different perspectives in research on the teaching and learning mathematical modelling. *Mathematical applications and modelling in the teaching and learning of mathematics*, *1*.

Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching mathematics and its applications*, *22*(3), 123-139.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. *Trends in teaching and learning of mathematical modelling*, 15-30.

Blum, W., & Kaiser, G. (1997). Vergleichende empirische Untersuchungen zu mathematischen anwendungsfähigkeiten von englischen und deutschen Lernenden [Comparative empirical studies at mathematical application skills of English and German learners]. Unpublished manuscript, German Research Foundation, Bonn, Germany.

Blum, W., & Leiβ, D. (2005). *"Filling Up "-the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks*. Paper presented at the Cerme 4–Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education.

Blum, W., & Leiβ, D. (2007). How do students and teachers deal with modelling problems? C. Haines, P. Galbraith, W. Blum, y S. Khan. *Mathematical Modelling: Education, Engineering and Economics*, 222-231.

Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects—State, trends and issues in mathematics instruction. *Educational studies in mathematics*, *22*(1), 37-68.

Bogdan, R. C. , & Biklen, S. K. (2007). *Qualitative research for education: A introduction to theory and methods* (5th ed.). Boston: Allyn and Bacon.

Borromeo-Ferri, R. B. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM*, *38*(2), 86-95.

Borland, J. H. (2009). Myth 2: The gifted constitute 3% to 5% of the population. Moreover, giftedness equals high IQ, which is a stable measure of aptitude: Spinal tap psychometrics in gifted education. *Gifted Child Quarterly*, *53*(4), 236-238.

Buckingham Shum, S., & Crick, R. D. (2016). Learning Analytics for 21st Century Competencies. *Journal of Learning Analytics, 3*(2), 6-21.

Bukova-Güzel, E., Dede, A. T., Hıdıroğlu, Ç. N., Ünver, S. K., & Çelik, A. Ö. (2016). Matematik eğitiminde matematiksel modelleme: araştırmacılar, eğitimciler ve öğrenciler için. *Pegem Atıf İndeksi*, 001-146.

Bursic, K. M. , & Atman, C. J. (1997). Information gathering: A critical step for quality in the design process. *Quality Management Journal*, *4*(4), 60-75.

Capobianco, B. M., & Tyrie, N. (2009). Problem solving by design. *Science and Children*, *47*(2), 38.

Caprile, M., Palmén, R., Sanz, P., & Dente, G. (2015). Encouraging STEM studies labour market situation and comparison of practices targeted at young people in different member states. *Policy Department A, 12*.

Carroll, D. R. (1997). Bridge engineering for the elementary grades. *Journal of Engineering Education*, *86*(3), 221-226.

CCSSI (2014). Common Core State Standards for Mathematics. 12 Ekim 2019 tarihinde erişildi. http://www.corestandards.org/Math/

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2021). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics*, 1-32.

Chamberlin, S. A. , Chamberlin, M. T. (2001). *On-time arrival*. Unpublished manuscript.

Chamberlin, S. A., & Coxbill, E. (2012). Using model-eliciting activities to introduce upper elementary students to statistical reasoning and mathematical modeling.

Chamberlin, S. A., & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *Journal of Secondary Gifted Education*, *17*(1), 37-47.

Chan, C. M. E., Ng, K. E. D., Widjaja, W., & Seto, C. (2012). Assessment of primary 5 students' mathematical modelling competencies. *Journal of Science and Mathematics Education in Southeast Asia*, *35*(2), 146–178.

Chick, H., & Stacey, K. (2013). Teachers of mathematics as problem-solving applied mathematicians. *Canadian Journal of Science, Mathematics and Technology Education*, *13*(2), 121-136.

Chowkase, A. A. (2022). Three C's conception of giftedness: A call for paradigm shift. *Gifted Education International*, 02614294211064703.

Christiaans, H. H. C. M. , & Dorst, K. H. (1992). Cognitive models in industrial design engineering: a protocol study. *Design theory and methodology*, *42*(1), 131-140.

Cirillo, M., Pelesko, J. A., Felton-Koestler, M. D., & Rubel, L. (2016). Perspectives on modeling in school mathematics. In C. R. Hirsch & A. Roth McDuffie, (Eds.), *Annual perspectives in mathematics education: Mathematical modeling and modeling mathematics* (pp. 3-16). Reston, VA: National Council of Teachers of Mathematics.

Clark, B (2008). *Growing Up Gifted*. New Jersey: Pearson Education.

Clinkenbeard, P. R. (2007). Economic arguments for gifted education. *Gifted Children*, *2*(1), 3.

Cobb, P. (2002). Modelling, symbolizing and tool use in statistical data analysis. In: Gravemeijer, K., Lehrer, R., van Oers, B., & Verschaffel, L. (Eds.), *Symbolizing, modelling and tool use in mathematics education*, (pp. 171–195). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Cole, J., Linsenmeier, R., McKenna, A., & Glucksberg, M. (2010, June). Investigating engineering students' mathematical modeling abilities in capstone design. In *2010 Annual Conference & Exposition* (pp. 15-812).

Cooper, E. E. (2000). Spatial-temporal intelligence: Original thinking processes of gifted inventors. *Journal for the Education of the Gifted*, *24*(2), 170-193.

Coxbill, E., Chamberlin, S. A., & Weatherford, J. (2013). Using model-eliciting activities as a tool to identify and develop mathematically creative students. *Journal for the Education of the Gifted*, *36*(2), 176-197.

CPALMS. (2017a). Uncle Henry's Dilemma Model Eliciting Activity. Accessed on 18 March 2017.Retrieved from https://www.cpalms.org/PreviewResourceLesson/Preview/31681

CPALMS. (2017b). Areas Habitation Corporation and the Search for Lunarcrete Model Eliciting Activity. Accessed on 20 March 2017.Retrieved from https://www.cpalms.org/PreviewResourceLesson/Preview/79355

Crammond, B. (2004, Fall). Can we, should we, need we agree on a definition of giftedness? *Roeper Review*, *27*(1), 15-16.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five traditions.* Thousand Oaks, CA: Sage Publications.

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Upper Saddle River, NJ: Pearson Education.

Creswell, J. W. (2014). *Research design: Qualitative, quantitative and mixed methods approaches* (4th ed.). Thousand Oaks, CA: Sage Publications.

Crismond, D. P., & Adams, R. S. (2012). The informed design teaching & learning matrix. *Journal of Engineering Education-Washington*, *101*(4), 738.

Crouch, R., & Haines*, C. (2004). Mathematical modelling: Transitions between the real world and the mathematical model. *International Journal of Mathematical Education in Science and Technology*, *35*(2), 197-206.

Crowe, S. , Cresswell, K. , Robertson, A. , Huby, G. , Avery, A. , & Sheikh, A. (2011). The case study approach. *BMC medical research methodology*, *11*(1), 1-9.

Cunningham, C. M. & Hester, K. (2007). *Engineering is elementary: An engineering and technology curriculum for children*. In Proceedings of the 2007 American

Society for Engineering Education Annual Conference & Exposition, Honolulu, Hawaii: American Society for Engineering Education.

Cutts, N. E., & Moseley, N. (2004). Üstün zekâlı ve yetenekli çocukların eğitimi. [*Education of gifted and talented students], çev: İsmail Ersevim, İstanbul: Özgür Yayınları*.

Çetin, A., & Doğan, A. (2018). Bilim ve sanat merkezlerinde görev yapan matematik öğretmenlerinin karşılaştıkları sorunlar. *Ankara Üniversitesi Eğitim Bilimleri Fakültesi Özel Eğitim Dergisi*, *19*(4), 615-641.

Dailey, D. (2017). Using engineering design challenges to engage elementary students with gifts and talents across multiple content areas. *Gifted Child Today*, *40*(3), 137-143.

Dasgupta, C. (2019). Improvable models as scaffolds for promoting productive disciplinary engagement in an engineering design activity. *Journal of Engineering Education*, *108*(3), 394-417.

Davis, G. A., & Rimm, S. B. (2004). Characteristics of gifted students. *Education of the gifted and talented*, *5*, 32-53.

Davis, G. A., Rimm, S. B., & Siegle, D. (2014). Leadership, affective learning, and character education. *Education of the Gifted and Talented*, 273-290.

De Corte, E. (2013). Giftedness considered from the perspective of research on learning and instruction. *High Ability Studies*, *24*(1), 3-19.

Dedebaş, E. (2017). *An Investigation of fifth grade students' behaviors and difficulties through multiple implementation of model eliciting activities* [Unpublished master's thesis]. Middle East Technical University, Ankara.

Denzin, N.K. , & Lincoln, Y.S. (2005). Introduction: The discipline and practice of qualitative research. In N.K. Denzin and Y.S. Lincoln (Eds.) *The Sage handbook of qualitative research* (3rd ed., pp.1-32). Thousand Oaks, CA: Sage.

De Villiers, L. (2018). *The concurrent development of mathematical modelling and engineering technician competencies of first-year engineering technician students* [Unpublished doctoral dissertation]. Stellenbosch University, Stellenbosch.

Diefes-Dux, H.A., Hjalmarson, M. A., Miller, T.K., & Lesh, R. (2008). Chapter 2: Model eliciting activities for engineering education. In J. S. Zawojewski, H. A. Diefes-Dux, & K. J. Bowman (Eds.), *Models and modeling in engineering education: Designing experiences for all students* (pp. 17-35).

Diefes-Dux, H. A., Moore, T., Zawojewski, J., Imbrie, P. K., & Follman, D. (2004, October). A framework for posing open-ended engineering problems: Model-eliciting activities. In *34th Annual Frontiers in Education, 2004. FIE 2004.* (pp. F1A-3). IEEE.

Diezmann, C. M., & Watters, J. J. (2000). Catering for mathematically gifted elementary students: Learning from challenging tasks. *Gifted Child Today*, *23*(4), 14-52.

Dimitriadis, C. (2011). Developing mathematical ability in primary school through a 'pull-out'programme: a case study. *Education 3-13*, *39*(5), 467-482.

Doerr, H. M., & Lesh, R. (2011). Models and modelling perspectives on teaching and learning mathematics in the twenty-first century. *Trends in teaching and learning of mathematical modelling*, 247-268.

Doğan, M. F., Gürbüz, R., Erdem, Z. Ç., & Şahin, S. (2019). Using mathematical modeling for integrating STEM disciplines: A theoretical framework. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *10*(3), 628-653.

Dorie, B. L., Cardella, M., & Svarovsky, G. N. (2014). Capturing the design thinking of young children interacting with a parent.

Eisenhart, M. A., & Finkel, E. (1998). *Women's science: Learning and succeeding from the margins*. University of Chicago Press.

English, L. D. (2003). Reconciling theory, research, and practice: A models and modelling perspective. *Educational Studies in Mathematics*, *54*(2), 225-248.

English, L. D. (2007). Interdisciplinary modelling in the primary mathematics curriculum. *Mathematics: Essential Research, Essential Practice Volume 1*, 275-284.

English, L. D. (2007b). Children's strategies for solving two and three dimensional combinatorial problems. *Journal for Research in Mathematics Education*, *24,* 255-273

English, L. D. (2013). Complex modelling in the primary and middle school years: An interdisciplinary approach. In *Teaching mathematical modelling: Connecting to research and practice* (pp. 491-505). Springer, Dordrecht.

English, L. D. (2017). Advancing elementary and middle school STEM education. *International Journal of Science and Mathematics Education*, *15*(1), 5-24.

English, L. D. (2019). Learning while designing in a fourth-grade integrated STEM problem. *International Journal of Technology and Design Education*, *29*(5), 1011-1032.

English, L. D., Hudson, P. B., & Dawes, L. (2012). Engineering design processes in seventh-grade classrooms: bridging the engineering education gap. *European Journal of Engineering Education*, *37*(5), 436-447.

English, L., Hudson, P., & Dawes, L. (2013). Engineering-based problem solving in the middle school: Design and construction with simple machines. *Journal of Pre-College Engineering Education Research*, *3*(2), 1-13.

English, L. D., & King, D. (2019). STEM integration in sixth grade: Desligning and constructing paper bridges. *International Journal of Science and Mathematics Education*, *17*(5), 863-884.

English, L. D., & Mousoulides, N. G. (2011). Engineering-based modelling experiences in the elementary and middle classroom. In *Models and modeling* (pp. 173-194). Springer, Dordrecht.

English, L. D., King, D. & Smeed, J. (2017). Advancing integrated STEM learning through engineering design: Sixth-grade students' design and construction of earthquake resistant buildings. *The Journal of Educational Research*, *110*(3), 255-271.

Erbaş, A. K., Kertil, M., Çetinkaya, B., Çakıroğlu, E., Alacacı, C., & Baş, S. (2014). Matematik eğitiminde matematiksel modelleme: Temel kavramlar ve farklı yaklaşımlar. *Kuram ve Uygulamada Eğitim Bilimleri*, *14*(4), 1-21.

Erduran, S. (2020). Science Education in the Era of a Pandemic. *Science & Education*, *29*(2), 233-235.

Ericson, K. A. , & Simon, H. A. (1984). *Protocol Analysis: Verbal reports as data*. A Bredford Book.

Fortus, D., Dershimer, R. C., Krajcik, J., Marx, R. W., & Mamlok-Naaman, R. (2004). Design-based science and student learning. *Journal of Research in Science Teaching*, *41*(10), 1081– 1110.

Frankel, J. R., & Wallen, N. E. (2006). *How to Design and Evaluate Research in Education*.(6. Baskı) McGraw Hill.

Freudenthal, H. (2006). *Revisiting mathematics education: China lectures*. Dordrecht: Kluwer Academic Publishers.

Gainsburg, J. (2006). The mathematical modeling of structural engineers. *Mathematical Thinking and Learning*, *8*(1), 3-36.

Gainsburg, J. (2013). Learning to model in engineering. *Mathematical Thinking and Learning*, *15*(4), 259-290.

Galbraith, P. L., Henn, H. W. , & Niss, M. (Eds.). (2007). *Modelling and applications in mathematics education: The 14th ICMI study* (Vol. 10). Springer Science & Business Media.

Galbraith, P. L., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM*, *38*(2), 143-162.

Gardner, H. (1983). Frames of mind: The theory of multiple intelligences. New York, NY: Basic Books. Gardner, H. (1993). *Frames of mind: The theory of multiple intelligences* (10th anniversary ed.). New York, NY: Basic Books.

Gardner, H. (1999). *The disciplined mind: What all students should understand*. New York, NY: Simon & Schuster.

Gardner, H. (2011). Multiple intelligences: Reflections after thirty years. *Washington, DC: National Association of Gifted Children Parent and Community Network*.

Gagné, F. (1985). Giftedness and talent: Reeexamingig a reexamination of the definition. *Gifted Child Quarterly*, *29*(3), 103-112.

Gagné, F. (2004). Transforming gifts into talents: The DMGT as a developmental theory. *High ability studies*, *15*(2), 119-147.

Gagné, F. (2005). From gifts to talents: The DMGT as a developmental model. In Sternberg, R. J., Davidson, J. E. (Eds.), *Conceptions of giftedness* (2nd ed., pp. 98-120), New York, NY: Cambridge University Press.

Gagné, F. (2010). Motivation within the DMGT 2.0 framework. *High ability studies*, *21*(2), 81-99.

Gagné, F. (2015). Academic talent development programs: A best practices model. *Asia Pacific Education Review*, *16*(2), 281-295.

Galbraith, P. (2012). Models of modelling: Genres, purposes or perspectives. *Journal of Mathematical Modelling and application*, *1*(5), 3-16.

Geiger, V., Galbraith, P., Niss, M., & Delzoppo, C. (2022). Developing a task design and implementation framework for fostering mathematical modelling competencies. *Educational Studies in Mathematics*, *109*(2), 313-336.

Gordin, P. C. (2006). *An instrumental case study of the phenomenon of collaboration in the process of improving community college developmental reading and writing instruction* [Unpublished doctoral dissertation]. University of South Florida, Tampa, FL.

Gravemeijer, K. (2002). Preamble: From models to modeling. In *Symbolizing, modeling and tool use in mathematics education* (pp. 7-22). Springer, Dordrecht.

Gravemeijer, K., & Stephan, M. (2002). Emergent models as an instructional design heuristic. In K. Gravemeijer, R. Lehrer, B. Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 145-169). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Greenes, C. (1981). Identifying the gifted student in mathematics. In J., A. Hatch (Ed.). *Doing qualitative research in education settings*. Albany: State University of New York Press

Groshong, K. A. (2018). *Defining mathematical modeling for K-12 education* [Unpublished doctoral dissertation]. The Ohio State University.

Gross, M.U.M., MacLeod, B., Drummond, D., Merrick, C. (2001). *Gifted Students in primary schools: Differentiating the curriculum*. Sydney: GERRIC, University of New South Wales.

Grünewald, S. (2012, July). Acquirement of modelling competencies–first results of an empirical comparison of the effectiveness of a holistic respectively an atomistic approach to the development of (metacognitive) modelling competencies of students. In *12th International Congress on Mathematical Education* (Vol. 8, p. 15).

Guilford, J. P. (1950). Creativity. *American Psychologist, 5*(9), 444–454.

Guilford, J. P. (1967). Creativity: Yesterday, today and tomorrow. *The Journal of Creative Behavior*, *1*(1), 3-14.

Guzey, S., & Aranda, M. (2017). Student participation in engineering practices and discourse: An exploratory case study. *Journal of Engineering Education*, *106*(4), 585-606.

Guzey, S. S., Moore, T. J., & Harwell, M. (2016). Building up STEM: An analysis of teacher-developed engineering design-based STEM integration curricular materials. *Journal of Pre-College Engineering Education Research*, *6*(1), 11– 29.

Guzey, S. S., Moore, T. J., & Roehrig, G. H. (2010). Curriculum development for STEM integration: bridge design on the white earth reservation. *Handbook of curriculum development*, 347-366.

Gül, M.D. (2021). *Designing a multimodal pedagogical model in gifted education: impacts on representational competence of teachers of gifted* [Unpublished doctoral dissertation]. Yıldız Technical University, Istanbul.

Hallahan, D., Kauffman, J., & Pullen, P. (2009). *Exceptional learners: An introduction to special education*. Boston: Pearson Education, Inc.

Hamilton, E., Lesh, R., Lester, F., & Brilleslyper, M. (2008). Model-Eliciting Activities (MEAs) as a bridge between engineering education research and mathematics education research. *Advances in Engineering Education*, *1* (2), 1–25

Han, H. J., & Shim, K. C. (2019). Development of an engineering design process-based teaching and learning model for scientifically gifted students at the Science Education Institute for the Gifted in South Korea. *Asia-Pacific Science Education*, *5*(1), 1-18.

Hankeln, C. (2020). Mathematical modeling in Germany and France: a comparison of students' modeling processes. *Educational Studies in Mathematics*, *103*(2), 209-229.

Harrison, C. (2004). Giftedness in early childhood: The search for complexity and connection. *Roeper Review*, *26*, 78-84.

Henning, H., & Keune, M. (2007). Levels of modelling competencies. In *Modelling and applications in mathematics education* (pp. 225-232). Springer, Boston, MA.

Heuser, B. L., Wang, K., & Shahid, S. (2017). Global Dimensions of Gifted and Talented Education: The Influence of National Perceptions on Policies and Practices. *Global Education Review*, *4*(1), 4-21.

Hıdıroğlu, Ç. N. (2012). *Teknoloji destekli ortamda matematiksel modelleme problemlerinin çözüm süreçlerinin analiz edilmesi: Yaklaşım ve düşünme süreçleri üzerine bir açıklama* [Unpublished doctoral dissertation]. Dokuz Eylul University, Izmir.

Hıdıroğlu, Ç. N., & Bukova Güzel, E. (2016). Transitions between Cognitive and Metacognitive Activities in Mathematical Modelling Process within a Technology Enhanced Environment. *Necatibey Faculty of Education Electronic Journal of Science & Mathematics Education*, *10*(1).

Hıdıroğlu Ç. N., Dede, A. T., Kula S. & Güzel, E. B. (2014). Matematiksel modelleme süreci çerçevesinde öğrencilerin kuyruklu yildiz problemine ilişkin çözümleri. *Mehmet Akif Ersoy Üniversitesi Eğitim Fakültesi Dergisi*, *1*(31), 1-17.

Hoh, P.S. (2008). Cognitive characteristics of the gifted. In J. A. Plucker & C. M. Callahan, *Criticalissues and practices in gifted education: What the research says* (1st ed., pp. 57–83). PrufrockPress.

Householder, D. L., & Hailey, C. E. (Eds.). (2012). *Incorporating engineering design challenges into STEM courses*. Retrieved from http://ncete.org/flash/pdfs/NCETECaucusReport.pdf.

Huffman, T. J. (2015). *The effects of a model-eliciting activity on high school student design performance* [Unpublished doctoral dissertation]. Purdue University.

Işık, A.., & Mercan, E. (2015). Ortaokul Matematik Öğretmenlerinin Model Ve Modelleme Hakkındaki Görüşlerinin İncelenmesi. *Kastamonu Eğitim Dergisi*, *23*(4), 1835-1850.

İnan, M. (2018). *7. sınıf öğrencilerinin matematiksel modelleme süreçlerinin incelenmesi* [Unpublished master's thesis]. Tokat Gaziosmanpasa University.

Jensen, A. R. (1980). Chronometric analysis of intelligence. *Journal of Social and Biological Structures, 3*(2), 103-122.

Jensen, T. H. (2007). Assessing mathematical modelling competency. In C. Haines, P. L. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modeling (ICTMA 12): Education, Engineering and Economics* (pp. 141-148). Chichester, UK: Horwood Publishing.

Johnson, D. T. (2000). *Teaching mathematics to gifted students in a mixed-ability classroom.* Reston, VA: ERIC Clearinghouse on Disabilities and Gifted Education.

Julie, C., & Mudaly, V. (2007). Mathematical modelling of social issues in school mathematics in South Africa. In *Modelling and applications in mathematics education* (pp. 503-510). Springer, Boston, MA.

Kabar, M. G. D., & İnan, M. (2018). Investigating Seventh Grade Students' Mathematization Processes and Mathematical Models: The Lawnmower Problem. *Turkish Journal of Computer and Mathematics Education (TURCOMAT), 9*(2), 339-366.

Kaiser, G. (2007). Modelling and modelling competencies in school. *Mathematical modelling (ICTMA 12): Education, engineering and economics,* 110-119.

Kaiser, G., Blomhøj, M., & Sriraman, B. (2006). Towards a didactical theory for mathematical modelling. *Zentralblatt für Didaktik der Mathematik*, *38*(2), 82-85.

Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *ZDM*, *38*(3), 302-310.

Kaiser, G., & Stender, P. (2013). Complex modelling problems in co-operative, self-directed learning environments. In *Teaching mathematical modelling: Connecting to research and practice* (pp. 277-293). Springer, Dordrecht.

Kang, O., & Noh, J. (2012). *Teaching mathematical modelling in school mathematics*. Proceedings of the 12th International Congress on Mathematical Education (pp. 145–157). Seoul: COEX.

Kaplan Sayı, A., & Yurtseven, N. (2021). How do gifted students learn? Their learning styles and dispositions towards learning. *Education 3-13*, 1-15.

Kapur, J. N. (1982). The art of teaching the art of mathematical modelling. *International Journal of Mathematical Education in Science and Technology*, *13*(2), 185-192.

Karabey, B. ve Yürümezoğlu, K. (2015). Yaratıcılık ve üstün yetenekliliğin zeka kuramları açısından değerlendirilmesi. *Buca Eğitim Fakültesi Dergisi*, *40*, 86-106.

Katehi, L., Pearson, G., & Feder, M. (Eds.) (2009). *Engineering in K12 Education. Understanding the status and improving the prospects*. Committee on K-12 Engineering Education, National Academy of Engineering and National Research Council. Washington, DC: National Academies Press.

Kaygısız, İ. (2021). *İlkokul 4. sınıf öğrencilerinin matematiksel modelleme yeterliklerinin incelenmesi: bir öğretim deneyi*. [Unpublished doctoral dissertation].Anadolu Üniversitesi, Eskisehir.

Kertil, M., & Gurel, C. (2016). Mathematical modeling: A bridge to STEM education. *International Journal of Education in mathematics, science and Technology*, *4*(1), 44-55.

Kim, K. H. (2006). Can we trust creativity tests? A review of the Torrance Tests of Creative Thinking (TTCT). *Creativity Research Journal*, 18, 3-14.

King, D., & English, L. D. (2016). Engineering design in the primary school: Applying STEM concepts to build an optical instrument. *International Journal of Science Education*, *38*(18), 2762-2794.

Knott, A. (2014). *The Process of Mathematisation in Mathematical Modelling of Number Patterns in Secondary School Mathematics* [Unpublished master's thesis].Stellenbosch University

Kocayayla, C. (2019). *Ortaokul 7. sınıf öğrencilerine yönelik matematiksel modelleme etkinliklerinin geliştirilmesi ve öğrencilerin modelleme yeterliklerinin incelenmesi* [Unpublished master's thesis]. Bolu Abant İzzet Baysal University.

Koç, D. (2020). *An investigation on thesis and dissertations on mathematical modeling in turkey in the last two decades* [Unpublished master's thesis]. Middle East Technical University.

Krutetski, V. A. (1976). *The psychology of mathematical abilities in school children*. Chicago: The University of Chicago Press.

Kurnaz, A. (2014). Yirminci yılında bilim ve sanat merkezlerinin raporlar ve yönetici görüşlerine dayalı olarak değerlendirilmesi. *Üstün Yetenekliler Eğitimi ve Araştırmaları Dergisi (UYAD)*, *2*(1).

Küçükoğlu, M. (2014). *Üstün ve özel yetenekli öğrencilerin eğitiminde model olan Beyazıt Ford Otosan İlköğretim Okulu üzerine bir araştırma* [Unpublished master's thesis]. Istanbul Sabahattin Zaim University.

Lee, S. W., Baek, J. I., & Lee. J, G. (2013). The development and the effects of educational program applied on STEAM for the mathematical prodigy. *Education of Primary School Mathematics*, *16*(1), 35-55.

Lee, L. E., Meyer, M. S., & Crutchfield, K. (2021). Gifted classroom environments and the creative process: A systematic review. *Journal for the Education of the Gifted*, *44*(2), 107-148.

Lehrer, R., & Schauble, L. (2007). A developmental approach for supporting the epistemology of modeling. In *Modelling and applications in mathematics education* (pp. 153-160). Springer, Boston, MA.

Lesh, R. (2010). Tools, researchable issues & conjectures for investigating what it means to understand statistics (or other topics) meaningfully. *Journal of Mathematical Modelling and Application, 1*(2), 16-48.

Lesh, R., & Carmona, G. (2003). Piagetian conceptual systems and models for mathematizing everyday experiences. *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*, 71-96.

Lesh, R. A., & Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Routledge.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. R. (2000). Principles for developing thought-revealing activities for students and teachers. In *Research design in mathematics and science education* (pp. 591-646). Lawrence Erlbaum Associates, Inc.

Lesh, R., & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. *Mathematical thinking and learning*, *5*(2-3), 109-129.

Lesh, R., & Sriraman, B. (2005). Mathematics education as a design science. *ZDM*, *37*(6), 490-505.

Lesh, R., & Yoon, C. (2007). What is distinctive in (our views about) models & modelling perspectives on mathematics problem solving, learning, and teaching?. In *Modelling and applications in mathematics education* (pp. 161-170). Springer, Boston, MA.

Lesh, R., & Zawojewski, J. S. (2007). Problem solving and modeling. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763–804). Greenwich, CT: IAP.

Levent, F. & Bakioğlu, A. (2013). Üstün yeteneklilerin eğitiminde Türkiye için öneriler. *Üstün yetenekliler eğitimi ve araştırmaları dergisi* (uyad), 1(1), 31-44.

Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). *Problem solving in mathematics education*. Springer Nature.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic Inquiry.* Thousand Oaks, California: Sage Publications.

Lu, X., & Kaiser, G. (2022). Creativity in students' modelling competencies: conceptualisation and measurement. *Educational Studies in Mathematics, 109*(2), 287-311.

Ludwig, M., & Reit, X. R. (2012). A Cross-section Study about Modelling Task Solutions. In *ICME 12 Conference (S. 3376–3387), Seoul, Korea: ICME*.

Ludwig, M., & Xu, B. (2010). A comparative study of modelling competencies among Chinese and German students. *Journal für Mathematik-Didaktik, 31*(1), 77-97.

Lyon, J. A., & Magana, A. J. (2021). The use of engineering model-building activities to elicit computational thinking: A design-based research study. *Journal of Engineering Education, 110*(1), 184-206.

Maaß, K. (2006). What are modelling competencies?. *ZDM, 38*(2), 113-142.

Maaß, K., & Gurlitt, J. (2011). LEMA–Professional development of teachers in relation to mathematical modelling. *Trends in teaching and learning of mathematical modelling*, 629-639.

Maass, K., Geiger, V., Ariza, M. R., & Goos, M. (2019). The role of mathematics in interdisciplinary STEM education. *ZDM, 51*(6), 869-884.

Maiorca, C., & Stohlmann, M. (2014). The how and why of integrated STEM model-eliciting activities. In *Session conducted at the National Council of Teachers of Mathematics Annual Meeting). New Orleans: NCTM*.

Mammadov, S. (2012). The education of gifted K-8 students in Turkey: Policy analysis and program evaluation. *Unpublished Master's thesis). Boğaziçi University, Istanbul*

Mann, E. L., & Mann, R. L. (2021). Engineering design and gifted pedagogy. In *Engineering instruction for high-ability learners in K-8 classrooms* (pp. 33-44). Routledge.

Mann, E. L., Mann, R. L., Strutz, M. L., Duncan, D., & Yoon, S. Y. (2011). Integrating engineering into K-6 curriculum: Developing talent in the STEM disciplines. *Journal of Advanced Academics, 22*(4), 639-658.

Margolis, J., & Fisher, A. (2002). *Unlocking the clubhouse: Women in computing*. MIT press.

Marland, S. P. (1972). *Education of the gifted and talented*. Report to Congress. Washington, DC: U.S. Government Printing Office

Matthews, D. J., & Foster, J. F. (2005). Mystery to mastery: Shifting paradigms in gifted education. *Roeper Review*, *28*(2), 64-69.

Mentzer, N. (2014). High school student information access and engineering design performance. *Journal of Pre-College Engineering Education Research (J-PEER)*, *4*(1), 4.

Mentzer, N., Huffman, T., & Thayer, H. (2014). High school student modeling in the engineering design process. *International Journal of Technology and Design Education*, *24*(3), 293-316.

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation* (2nd ed.). San Francisco (CA): John Wiley & Sons.

Miedijensky, S. (2018). Learning environment for the gifted—What do outstanding teachers of the gifted think?. *Gifted Education International*, *34*(3), 222-244.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis; an expanded source book*. Sage Publications.

Mills, A. J., Durepos, G., & Wiebe, E. (Eds.). (2009). *Encyclopedia of case study research*. Sage Publications.

Ministry of Education. (2007). *Mathematics syllabus – Primary*. Singapore: Curriculum Planning and Developmental Division.

Milli Eğitim Bakanlığı [Ministry of National Education] (MoNE) (2012). Bilim Sanat Merkezi Yönergesi [Science and Art Center Directive] http://mevzuat.meb.gov.tr/html/2593_0.html

Milli Eğitim Bakanlığı [Ministry of National Education] (MoNE) (2018). Ortaokul matematik dersi (5, 6, 7, ve 8. Sınıflar) ögretim programı [Middle school mathematics curriculum grades 5 to 8]. Ankara, Turkey: Author.

Ministry of National Education [MoNE]. (2016). *STEM Education Report*. Ankara: General Directorate of Innovation and Education Technologies.

Moon, S. M., Kolloff, P., Robinson, A., Dixon, F. & Feldhusen, J. F. (2009). The purdue three stage model. In J. S. Renzulli et.al (Eds). Sytems & Models for *Developing Programs for the Gifted & Talented*, 2nd. Ed. (pp. 289- 321), Mansfield Center, C.T. Creative Learning Press.

Moore, T. J., Glancy, A. W., Tank, K. M., Kersten, J. A., Smith, K. A., & Stohlmann, M. S. (2014). A framework for quality K-12 engineering education: Research and development. *Journal of pre-college engineering education research (J-PEER)*, *4*(1), 2.

Moore, T., & Diefes-Dux, H. (2004, October). Developing model-eliciting activities for undergraduate students based on advanced engineering content. In *34th Annual Frontiers in Education, 2004. FIE 2004.* (pp. F1A-9). IEEE.

Moore, T. J., Miller, R. L., Lesh, R. A., Stohlmann, M. S., & Kim, Y. R. (2013). Modeling in engineering: The role of representational fluency in students' conceptual understanding. *Journal of Engineering Education*, *102*(1), 141-178.

Moore, T., & Richards, L. G. (2012). P-12 Engineering Education Research and Practice. *Advances in Engineering Education*, *3*(2), n2.

Morris, J., Slater, E., Fitzgerald, M. T., Lummis, G. W., & van Etten, E. (2021). Using local rural knowledge to enhance STEM learning for gifted and talented students in Australia. *Research in Science Education*, *51*(1), 61-79.

Mousoulides, N. (2007). *The Modeling Perspective in the Teaching and Learning of Mathematical Problem Solving*. Unpublished Doctoral Dissertation, University of Cyprus, Nicosia.

National Academy of Engineering & National Research Council. (2009). *Engineering in K-12 Education: Understanding the status and improving the prospects*. Washington: The National Academies Press.

National Association for Gifted Children. (2015). *Self-Assess Your P-12 Practice or Program Using the NAGC Gifted Programming Standards.*

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Niss, M. (2003, January). Mathematical competencies and the learning of mathematics: The Danish KOM project. In *3rd Mediterranean conference on mathematical education* (pp. 115-124).

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction to modelling and applications in mathematics education. In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education. The 14th ICMI Study* (pp. 3–32). New York, NY: Springer.

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, *102*(1), 9-28.

OECD (2013). *PISA 2012 results: what students know and can do (volume I): student performance in mathematics, reading and science*. OECD publishing.

OECD (2017*). PISA 2015 technical report.* Paris: OECD.

OECD (2019a). *PISA 2018 assessment and analytical framework*. Paris: OECD Publishing.

OECD (2019b). *PISA 2018 results volume I: What students know and can do*. Paris: OECD Publishing.

Oğurlu, Ü. (2010). Üstün zekali ve yeteneklilerde sosyal uyum ve arkadaşlik ilişkileri: Literatür taramasi. *Çukurova Üniversitesi Eğitim Fakültesi Dergisi*, *3*(39),90-99.

Ozdemir, D. A., & Işiksal Bostan, M. (2021). Mathematically gifted students' differentiated needs: what kind of support do they need?. *International Journal of Mathematical Education in Science and Technology*, *52*(1), 65-83.

Özdemir, D. (2016). Design and development of differentiated tasks for 5th and 6th grade mathematically gifted students. Unpublished Phd Dissertation, Middle East Technical University.

Özyaprak, M. (2016). Üstün Zekâli Ve Yetenekli Ögrenciler Için Matematik Müfredatinin Farklilastirilmasi [Differentiating Mathematics Curriculum for Gifted and Talented Students]. *HAYEF: Journal of Education*, *13*(2), 115-128.

Park, D. Y., Park, M. H., & Bates, A. B. (2018). Exploring young children's understanding about the concept of volume through engineering design in a STEM activity: A case study. *International Journal of Science and Mathematics Education*, *16*(2), 275-294.

Pativisan, S. (2006). *Mathematical problem solving processes of Thai gifted students*. Oregon State University.

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice*. Sage publications.

Peterson, J. S. (2009). Myth 17: Gifted and talented individuals do not have unique social and emotional needs. *Gifted Child Quarterly*, *53*(4), 280-282.

Piaget, J. (1964). Part I: Cognitive development in children: Piaget development and learning. *Journal of research in science teaching*, *2*(3), 176-186.

Plucker, J. A., & Dow, G. T. (2010). Attitude change as the precursor to creativity enhancement. *Nurturing creativity in the classroom*, 362-379.

Pollak, H. O. (1979). The interaction between mathematics and other school subjects. *New trends in mathematics teaching IV*, 232-248.

Portsmore, M., Watkins, J., & McCormick, M. (2012). *Planning, drawing and elementary school students in an integrated engineering design and literacy activity.* Paper presented at the 2nd P-12 Engineering and Design Education Research Summit, Washington, DC

Reis, S. M., Gubbins, E. J., Briggs, C. J., Schreiber, F. J., Richards, S., Jacobs, J. K., Eckert, R. D., & Renzulli, J. S. (2004). Reading instruction for talented readers: 30 case studies documenting few opportunities for continuous progress. *Gifted Child Quarterly*, *48*(4), 315–338.

Reis, S. M., & Renzulli, J. S. (2009). Myth 1: The gifted and talented constitute one single homogeneous group and giftedness is a way of being that stays in the person over time and experiences. *Gifted Child Quarterly*, *53*(4), 233-235.

Renzulli, J. S. (1977). The enrichment triad model: A plan for developing defensible programs for the gifted and talented. *Gifted Child Quarterly*, *21*(2), 227-233.

Renzulli, J. S. (1986). The three-ring conception of giftedness: A developmental model for creative productivity. In Sternberg, R .J., Davidson, J. E. (Eds.), *Conceptions of giftedness* (pp. 53-92). New York, NY: Cambridge University Press.

Renzulli, J. S. (2005). The three-ring definition of giftedness: A developmental model for promoting creative productivity. In Sternberg, R .J., Davidson, J. E. (Eds.), Conceptions of giftedness (2nd ed., pp. 246-280). New York, NY: Cambridge University Press.

Renzulli, J. S. (2012). Reexamining the role of gifted education and talent development for the 21st century. *Gifted Child Quarterly*, *56*(3), 150–159.

Renzulli, J. S. (2016). *The three-ring conception of giftedness: A developmental model for promoting creative productivity*. Prufrock Press Inc.

Renzulli, J. S. (2020). The Catch-A-Wave Theory of adaptability: Core competencies for developing gifted behaviors in the second machine age of technology. *International Journal for Talent Development and Creativity*, *8*(1-2), 79-95.

Renzulli, J. S. (2021). *The Three-Ring Conception of Giftedness: A Developmental Model for Promoting Creative Productivity 4* (pp. 55-90). Routledge.

Renzulli, J.S., Rizza, M., & Smith, L.H. (2002). *The learning styles inventory*. Mansfield Center, CT: Creative Learning Press.

Roman, H. T. (2012). *STEM—a powerful approach to real-world problem solving for gifted and talented students in middle and high school grades.* Gifted Education Press, Manassas, VA.

Rotigel, J. V., & Fello, S. (2004). Mathematically gifted students: How can we meet their needs?. *Gifted Child Today*, *27*(4), 46-51.

Runco, M. A., & Jaeger, G. J. (2012). The standard definition of creativity. *Creativity research journal*, *24*(1), 92-96.

Sak, U. (2010). Üstün zekâlılar: Özellikleri, tanılanmaları, eğitimleri. *Ankara: Maya Akademi*.

Sankar-DeLeeuw, N. (2004). Case studies of gifted kindergarten children: Profiles of promise. *Roeper Review*, *26*(4), 192-207.

Saranlı, A.G. ve Metin, N. (2012). Üstün yetenekli çocuklarda gözlenen sosyal-duygusal sorunlar. *Ankara Üniversitesi Eğitim Bilimleri Fakültesi Dergisi*, *45*(1), 139-163.

Sarı H. & Öğülmüş, K. (2014). Bilim ve sanat merkezlerinde (BİLSEM) karşılaşılan sorunların öğretmen ve öğrenci görüşleri açısından değerlendirilmesi, *Uluslararası Türk Eğitim Bilimleri Dergisi*, *2*(2) 254-265

Schiever, S. W., & Maker, C. J. (2003). New directions in enrichment and acceleration. *Handbook of gifted education*, *3*, 163-173.

Schukajlow, S., Leiss, D., Pekrun, R., Blum, W., Müller, M., & Messner, R. (2012). Teaching methods for modelling problems and students' task-specific enjoyment, value, interest and self-efficacy expectations. *Educational studies in mathematics*, *79*(2), 215-237.

Sen, C., Ay, Z. S., & Kiray, S. A. (2021). Computational thinking skills of gifted and talented students in integrated STEM activities based on the engineering design process: The case of robotics and 3D robot modeling. *Thinking Skills and Creativity*, *42*, 100931.

Sheffield, L. J. (1994). *The development of gifted and talented mathematics students and the National Council of Teachers of Mathematics Standards* (Research Monograph No. 9404). Storrs: National Research Center on the Gifted and Talented, University of Connecticut.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*, *22*(2), 63-75.

Siegle, D. (2014). Technology: Differentiating instruction by flipping the classroom. *Gifted Child Today*, *37*(1), 51-55.

Singer, F. M., Sheffield, L. J., Freiman, V., & Brandl, M. (2016). *Research on and activities for mathematically gifted students*. Springer Nature.

Siswono, T. Y. E. (2010). Levelıng students' creatıve thınkıng ın solvıng and posıng mathematıcal problem. *Journal on Mathematics Education*, *1*(1), 17-40.

Siverling, E. A., Suazo-Flores, E., Mathis, C. A., & Moore, T. J. (2019). Students' use of STEM content in design justifications during engineering design-based STEM integration. *School Science and Mathematics*, *119*(8), 457-474.

Springer, L., Stanne, M. E., & Donovan, S. S. (1999). Effects of small-group learning onundergraduates in science, mathematics, engineering, and technology: A meta-analysis. *Review of educational research*, *69*(1), 21-51.

Sriraman, B., & Lesh, R. A. (2006). Modeling conceptions revisited. *ZDM*, *38*(3), 247-254.

Stake, R. E. (1995). *The art of case study research.* Thousand Oaks, CA: Sage.

Stake, R. E. (2005). Qualitative case studies. In Denzin, N. K., Lincoln, Y. S. (Eds.), *The Sage handbook of qualitative research* (3rd ed., pp. 443-466). Thousand Oaks, CA: Sage.

Sternberg, R.J. (1986). *Intelligence applied: Understanding and increasing your intellectual skills*. San Diego: Harcourt Brace.

Sternberg, R. J. (2005). WICS: A model of positive educational leadership comprising wisdom, intelligence, and creativity synthesized. *Educational Psychology Review*, *17*(3), 191-262.

Sternberg, R. J. ve Zhang, L. F. (1995). What do we mean by giftedness? A pentagonal implicit theory. *Gifted Child Quarterly*, *39*(2), 88-94.

Şahin, F. (2015). Educational programs, services and support for gifted students in Turkey. *Eğitimde Kuram ve Uygulama*, *11*(4), 1207-1223.

Şahin, N. (2019). İlkokul 4. sınıf öğrencilerinin bilişsel modelleme yeterliklerinin belirlenmesi ve değerlendirilmesi, [Unpublished doctoral dissertation]. Ondokuz Mayıs University, Samsun.

Şahin, N., & Eraslan, A. (2017). Cognitive modeling competencies of third-year middle school students: The reading contest problem. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi*, *11*(2), 19-51

Şahin, H., & Zorlu, A. (2022). Türkiye'de özel yetenekli bireyleri tanılama süreci ve tanılamada kullanılan yöntemler. *Milli Eğitim Dergisi*, *51*(233), 863-885.

Şen, C. (2018). Mühendislik tasarımı odaklı bütünleşik STEM etkinliklerinde üstün zekâlı ve yetenekli öğrencilerin kullandığı beceriler [Unpublished doctoral dissertation]. Hacettepe University, Ankara.

Şengil-Akar, Ş. (2017). Üstün Yetenekli Öğrencilerin Matematiksel Yaratıcılıklarının Matematiksel Modelleme Sürecinde İncelenmesi. [Unpublished doctoral dissertation].Hacettepe University, Ankara.

Singapore Ministry of Education. (2007). Primary mathematics syllabuses. Ministry of Education,

Sternberg, R. J. (2018). Theories of intelligence. In S. I. Pfeiffer, E. Shaunessy-Dedrick, & M. Foley-Nicpon (Eds.), *APA handbook of giftedness and talent* (pp. 145–161). American Psychological Association.

Sternberg, R. J. (2019). Teaching and assessing gifted students in STEM disciplines through the augmented theory of successful intelligence. *High Ability Studies*, *30*(1-2), 103-126.

Subban, P. (2006). Differentiated instruction: A research basis. *International Education Journal*, *7*(7), 935–947.

Taber, K. S. (2016). Giftedness, intelligence, creativity, and the construction of knowledge in the science classroom. In *International perspectives on science education for the gifted* (pp. 19-30). Routledge.

Taşkın, D. (2016). Üstün yetenekli tanısı konulmuş ve konulmamış öğrencilerin matematikte yaratıcılıklarının incelenmesi: Bir özel durum çalışması. [Unpublished doctoral dissertation]. *Karadeniz Teknik University, Trabzon*.

Tekin Dede, A. (2015). Matematik derslerinde öğrencilerin modelleme yeterliklerinin geliştirilmesi: Bir eylem araştırması [Unpublished doctoral dissertation]. *Dokuz Eylül University, Izmir*.

Tekin Dede, A., & Yılmaz, S. (2015). 6. Sınıf öğrencilerinin bilişsel modelleme yeterlikleri nasıl geliştirilebilir. *International Journal of New Trends in Arts, Sports & Science Education*, *4*(1), 49-63.

Terman, L. M. (1925). *Genetic studies of genius: Vol. 1. Mental and physical traits of a thousand gifted children*. Stanford, CA: Stanford University Press.

Tomlinson, C.A. (1999). "Mapping a Route Toward Differentiated Instruction. *Educational Leadership* 57, 12–17.

Tomlinson, C. A. (Ed.). (2005). *Differentiated instruction*. Lawrence Erlbaum Associates.

Tomlinson, C. A. (2014). *The differentiated classroom: Responding to the needs of all learners*. Ascd.

Tomlinson, C., & Allan, S. D. (2000). Understanding differentiated instruction: Building a foundation for leadership. *Leadership for differentiating schools & classrooms*, 1-15.

Tomlinson, C. A., Brighton, C., Hertberg, H., Callahan, C. M., Moon, T. R., Brimijoin, K., ... & Reynolds, T. (2003). Differentiating instruction in response to student readiness, interest, and learning profile in academically diverse classrooms: A review of literature. *Journal for the Education of the Gifted*, *27*(2-3), 119-145.

Tomlinson, C. A., Kaplan, S. N., Renzulli, J. S., Purcell, J. H., Leppien, J. H., Burns, D. E., ... & Imbeau, M. B. (2008). *The parallel curriculum: A design to develop learner potential and challenge advanced learners*. Corwin Press.

Torrance, E. P. (1974). *Torrance tests of creative thinking*. Bensenville: Scholastic Testing Service.

Turner, R. (2007). Modelling and applications in PISA. In W. Blum, P. L. Galbraith, H. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education* (The 14th ICMI study, pp. 433–440). New York: Springer.

Türkman, B. (2020). The evolution of the term of giftedness & theories to explain gifted characteristics. *Journal of Gifted Education and Creativity, 7*(1), 17-24.

VanTassel-Baska, J. (2003). Selecting instructional strategies for gifted learners. *Focus on Exceptional Children, 36*(3), 1-12.

VanTassel-Baska, J. (2005). Domain-specific giftedness. *Conceptions of giftedness*, 358-376.

Van Tassel-Baska, J., & Brown, E. F. (2007). Toward best practice: An analysis of the efficacy of curriculum models in gifted education, *Gifted Child Quarterly*, *51*,342-358.

VanTassel-Baska, J., & Hubbard, G. F. (2016). Classroom-based strategies for advanced learners in rural settings. *Journal of Advanced Academics*, *27*(4), 285-310.

VanTassel-Baska, J., & Stambaugh, T. (2006). Project Athena: A pathway to advanced literacy development for children of poverty. *Gifted Child Today, 29*(2), 58-63.

Vogelaar, B., & Resing, W. C. (2018). Changes over time and transfer of analogy-problem solving of gifted and non-gifted children in a dynamic testing setting. *Educational Psychology*, *38*(7), 898-914.

Vorhölter, K. (2018). Conceptualization and measuring of metacognitive modelling competencies: Empirical verification of theoretical assumptions. *ZDM*, *50*(1), 343-354.

Vygotsky, LS (1978) *Mind in society: The development of higher psychological processes*. Harvard University Press, Cambridge, MA.

Webber, M. (2011). Look to the past, stand tall in the present: The integral nature of positive racial-ethnic identity for the academic success of Māori students. In W. Vialle (Ed.), *Giftedness from an indigenous perspective* (pp. 100–110). Unanderra, NSW: University of Wollongong Printery.

Welch, M. & Lim, H. (2000). The strategic thinking of novice designers' discontinuity between theory and practice. *Journal of Technology Studies*, *26*(2) 34-44

Wellisch, M., & Brown, J. (2013). Many faces of a gifted personality: Characteristics along a complex gifted spectrum. *Talent Development & Excellence*, *5*(2), 43-58.

Wessels, H. M. (2014). Levels of mathematical creativity in model-eliciting activities. *Journal of Mathematical Modelling and Application*, *1*(9), 22-40.

Wheeler, L. B., Navy, S. L., Maeng, J. L., & Whitworth, B. A. (2019). Development and validation of the classroom observation protocol for engineering design (COPED). *Journal of Research in Science Teaching*, *56*(9), 1285-1305.

Winebrenner, S., & Brulles, D. (2012). *Teaching Gifted Kids in Today's Classroom Professional Development Multimedia Package*. Free Spirit Publishing.

Winner, E. (2000). The origins and ends of giftedness. *American psychologist*, *55*(1), 159.

Wood, T., Merkel, G., & Uerkwitz, J. (1996). Criar um ambiente na aula para falar sobre a matemática. *Educação e Matemática*, (40), 39-43.

Yıldırım, B., & Altun, Y. (2015). STEM eğitim ve mühendislik uygulamalarının fen bilgisi laboratuar dersindeki etkilerinin incelenmesi. *El-Cezeri Journal of Science and Engineering*, *2*(2).

Yıldırım, A. , & Şimşek, H. (2005*). Sosyal bilimlerde nitel araştırma yöntemleri.* Ankara: Seçkin Yayıncılık.

Yildirim, U. (2019). *Altıncı sınıf öğrencilerinin matematiksel modelleme becerilerinin incelenmesi* [Unpublished master's thesis]. Erzincan Binali Yıldırım University.

Yildirim, T. P., Shuman, L., & Besterfield-Sacre, M.,.(2010). Model eliciting activities: assessing engineering student problem solving and skill integration processes. *International Journal of Engineering Education*, *26*(4), 831-845.

Yin, R. K. (2009). *Case study research: Design and methods* (4th ed.). Thousand Oaks, California: Sage Publications.

Yoon, S. Y., & Mann, E. L. (2017). Exploring the spatial ability of undergraduate students: association with gender, STEM majors, and gifted program membership. *Gifted Child Quarterly, 61*(4), 313-327.

Zawojewski, J. S., Hjalmarson, M. A., Bowman, K. J., & Lesh, R. (2008). A modeling perspective on learning and teaching in engineering education. In *Models and modeling in engineering education* (pp. 1-15). Brill Sense.

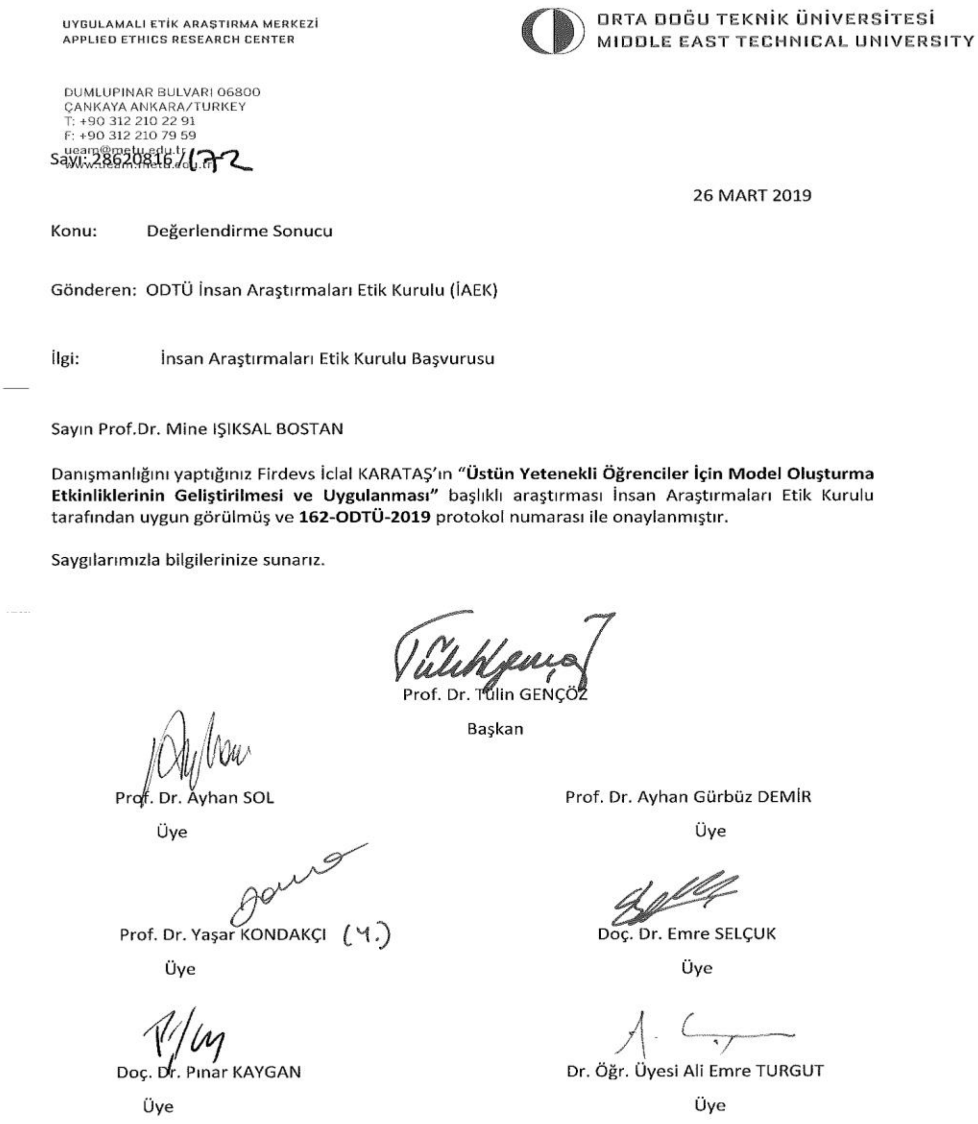
Zawojewski, J. , Lesh, R. (1999). O'Hare airport problem. Unpublished manuscript.

Zawojewski, J. S., Lesh, R., & English, L. (2003). A models and modeling perspective on the role of small group learning activities. *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*, 337-358.

Zbiek, R. M., & Conner, A. (2006). Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educational studies in mathematics, 63*(1), 89-112.

APPENDICES

A. APPROVAL OF THE METU HUMAN SUBJECTS ETHICS COMMITTEE



UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816 /172

26 MART 2019

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof.Dr. Mine IŞIKSAL BOSTAN

Danışmanlığını yaptığınız Firdevs İclal KARATAŞ'ın **"Üstün Yetenekli Öğrenciler İçin Model Oluşturma Etkinliklerinin Geliştirilmesi ve Uygulanması"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **162-ODTÜ-2019** protokol numarası ile onaylanmıştır.

Saygılarımızla bilgilerinize sunarız.

Prof. Dr. Tülin GENÇÖZ
Başkan

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR
Üye

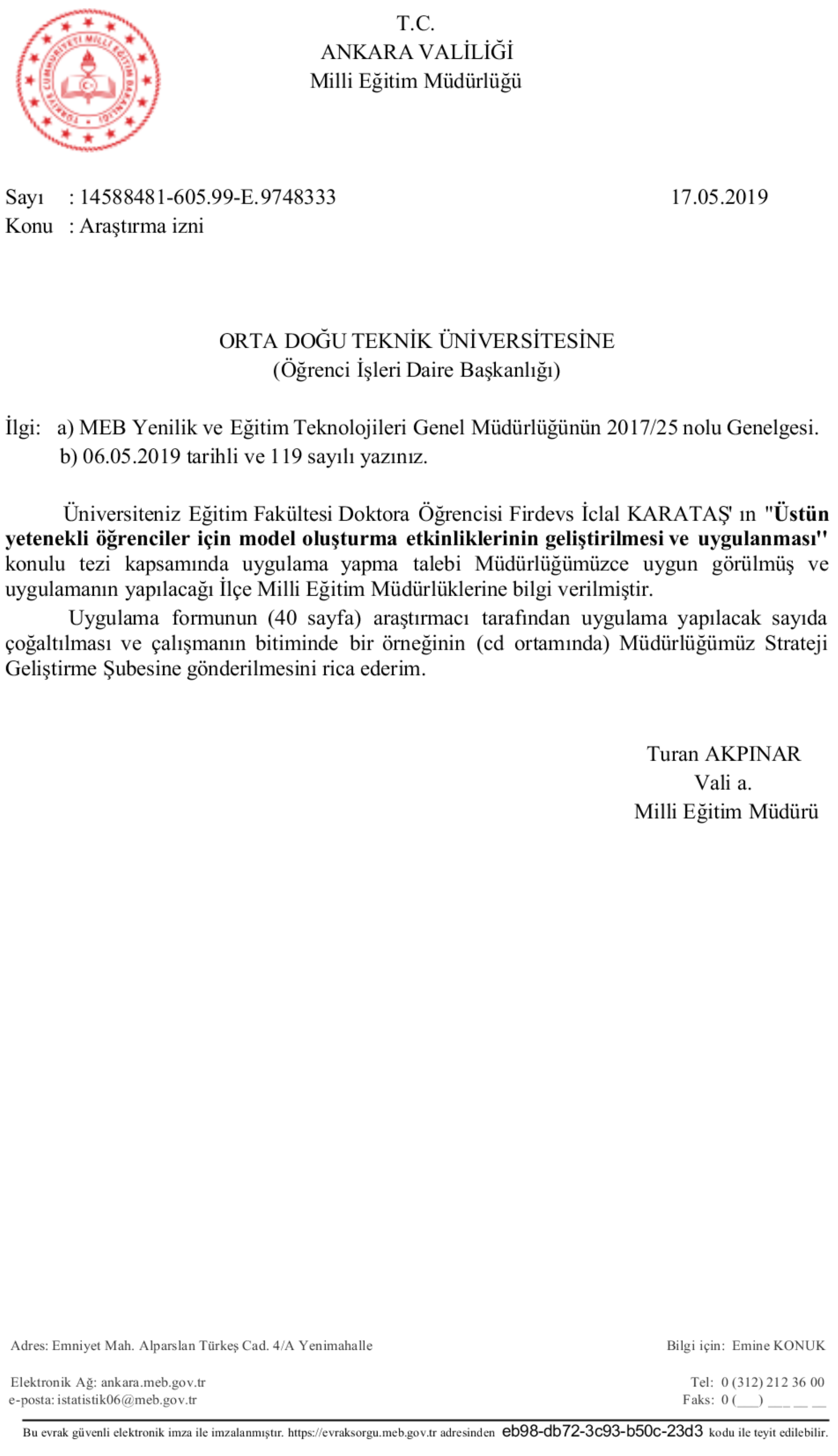
Prof. Dr. Yaşar KONDAKÇI (Y.)
Üye

Doç. Dr. Emre SELÇUK
Üye

Doç. Dr. Pınar KAYGAN
Üye

Dr. Öğr. Üyesi Ali Emre TURGUT
Üye

B. OFFICIAL PERMISSIONS OBTAINED FROM THE MINISTRY OF NATIONAL EDUCATION



T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

Sayı : 14588481-605.99-E.9748333 17.05.2019
Konu : Araştırma izni

ORTA DOĞU TEKNİK ÜNİVERSİTESİNE
(Öğrenci İşleri Daire Başkanlığı)

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2017/25 nolu Genelgesi.
b) 06.05.2019 tarihli ve 119 sayılı yazınız.

Üniversiteniz Eğitim Fakültesi Doktora Öğrencisi Firdevs İclal KARATAŞ' ın "**Üstün yetenekli öğrenciler için model oluşturma etkinliklerinin geliştirilmesi ve uygulanması"** konulu tezi kapsamında uygulama yapma talebi Müdürlüğümüzce uygun görülmüş ve uygulamanın yapılacağı İlçe Milli Eğitim Müdürlüklerine bilgi verilmiştir.

Uygulama formunun (40 sayfa) araştırmacı tarafından uygulama yapılacak sayıda çoğaltılması ve çalışmanın bitiminde bir örneğinin (cd ortamında) Müdürlüğümüz Strateji Geliştirme Şubesine gönderilmesini rica ederim.

Turan AKPINAR
Vali a.
Milli Eğitim Müdürü

Adres: Emniyet Mah. Alparslan Türkeş Cad. 4/A Yenimahalle Bilgi için: Emine KONUK

Elektronik Ağ: ankara.meb.gov.tr Tel: 0 (312) 212 36 00
e-posta: istatistik06@meb.gov.tr Faks: 0 (___) ___ __ __

C. INFORMED CONSENT FORM

ARAŞTIRMAYA GÖNÜLLÜ KATILIM FORMU

Bu araştırma, ODTÜ Matematik Eğitimi Bölümü doktora öğrencisi Firdevs İclal KARATAŞ ve ODTÜ Matematik Eğitimi Bölümü öğretim üyesi Prof. Dr. Mine IŞIKSAL BOSTAN tarafından doktora tezi kapsamında yürütülen bir çalışmadır. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Çalışmanın Amacı Nedir? Araştırmanın amacı sizler için model oluşturma etkinlikleri tasarlamak ve uygulamaktır. Araştırmaya katılmayı kabul ederseniz, sizden beklenen, hazırladığımız model oluşturma etkinlikleriyle çalışmanızdır. Bu çalışmaya katılım ortalama olarak 90 dakika sürmektedir.

Bize Nasıl Yardımcı Olmanızı İsteyeceğiz? Araştırmaya katılmayı kabul ederseniz, sizden 3 ila 4 kişiden oluşan bir çalışma grubunda grup arkadaşlarınızla birlikte model oluşturma etkinlikleriyle çalışmanız beklenmektedir. Yaklaşık olarak 1 buçuk saat sürmesi beklenen bu çalışmada sizlerin etkinlikteki sorunlara çözüm üretmeniz beklenmektedir. Çalışma süreciniz daha sonra içerik analizi ile değerlendirilmek üzere ses ve video kaydına alınacaktır. Sizlerle etkinlik öncesinde ve sonrasında bireysel mülakatlarda sorulan sorulara cevap vermeniz beklenmektedir.

Sizden Topladığımız Bilgileri Nasıl Kullanacağız? Araştırmaya katılımınız tamamen gönüllülük temelinde olmalıdır. Ankette, sizden kimlik veya kurum belirleyici hiçbir bilgi istenmemektedir. Cevaplarınız tamamıyla gizli tutulacak, sadece araştırmacılar tarafından değerlendirilecektir. Katılımcılardan elde edilecek bilgiler toplu halde değerlendirilecek ve bilimsel yayımlarda kullanılacaktır. Sağladığınız veriler gönüllü katılım formlarında toplanan kimlik bilgileri ile eşleştirilmeyecektir.

Katılımınızla ilgili bilmeniz gerekenler: Çalışma, genel olarak kişisel rahatsızlık verecek sorular veya uygulamalar içermemektedir. Katılım sırasında sorulardan ya da herhangi başka bir nedenden ötürü kendinizi rahatsız hissederseniz cevaplama işini yarıda bırakıp çıkmakta serbestsiniz. Böyle bir durumda çalışmayı uygulayan kişiye, çalışmadan çıkmak istediğinizi söylemek yeterli olacaktır. Çalışma sonunda, bu araştırmayla ilgili sorularınız cevaplanacaktır.

Araştırmayla ilgili daha fazla bilgi almak isterseniz: Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Araştırma hakkında daha fazla bilgi almak için ODTÜ Matematik Eğitimi Bölümü doktora öğrencisi Firdevs İclal Karataş (E-posta: iclal.karatas@metu.edu.tr) ile iletişim kurabilirsiniz.

Yukarıdaki bilgileri okudum ve bu çalışmaya tamamen gönüllü olarak katılıyorum.

(Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim Soyad Tarih İmza

----/----/-----

D. PARENTAL CONSENT FORM

VELİ ONAY FORMU

Sevgili Anne/Baba,

Bu çalışma ODTÜ Matematik Eğitimi Bölümü doktora öğrencisi Firdevs İclal Karataş ve ODTÜ Matematik Eğitimi Bölümü öğretim üyesi Prof. Dr. Mine IŞIKSAL BOSTAN tarafından doktora tezi kapsamında yürütülmektedir.

Bu çalışmanın amacı nedir? Çalışmanın amacı, üstün yetenekli öğrenciler için tasarlanan model oluşturma etkinliklerinin prensiplerini tartışmak ve öğrencilerin bu etkinliklerle çalışırken matematiksel modelleme yeterliliklerini belirlemektir.

Çocuğunuzun katılımcı olarak ne yapmasını istiyoruz?: Bu amaç doğrultusunda, çocuğunuzdan model oluşturma etkinlikleriyle çalışmasını isteyeceğiz ve cevaplarını/davranışlarını (ses kaydı, görüntü kaydı, not ederek, yazılı) biçiminde toplayacağız. Sizden çocuğunuzun katılımcı olmasıyla ilgili izin istediğimiz gibi, çalışmaya başlamadan çocuğunuzdan da sözlü olarak katılımıyla ilgili rızası mutlaka alınacak.

Çocuğunuzdan alınan bilgiler ne amaçla ve nasıl kullanılacak?: Çocuğunuzdan alacağımız cevaplar tamamen gizli tutulacak ve sadece araştırmacılar tarafından değerlendirilecektir. Elde edilecek bilgiler sadece bilimsel amaçla (yayın, konferans sunumu, vb.) kullanılacak, çocuğunuzun ya da sizin ismi ve kimlik bilgileriniz, hiçbir şekilde kimseyle paylaşılmayacaktır.

Çocuğunuz ya da siz çalışmayı yarıda kesmek isterseniz ne yapmalısınız?: Katılım sırasında sorulan sorulardan ya da herhangi bir uygulama ile ilgili başka bir nedenden ötürü çocuğunuz kendisini rahatsız hissettiğini belirtirse, ya da kendi belirtmese de araştırmacı çocuğun rahatsız olduğunu öngörürse, çalışmaya sorular tamamlanmadan ve derhal son verilecektir. Şayet siz çocuğunuzun rahatsız olduğunu hissederseniz, böyle bir durumda çalışmadan sorumlu kişiye çocuğunuzun çalışmadan ayrılmasını istediğinizi söylemeniz yeterli olacaktır.

Bu çalışmayla ilgili daha fazla bilgi almak isterseniz: Çalışmaya katılımınızın sonrasında, bu çalışmayla ilgili sorularınız yazılı biçimde cevaplandırılacaktır. Çalışma hakkında daha fazla bilgi almak için ODTÜ Matematik Eğitimi Bölümü Doktora Öğrencisi Firdevs İclal Karataş ile (e-posta: i.karatas@metu.edu.tr) ile iletişim kurabilirsiniz. Bu çalışmaya katılımınız için şimdiden teşekkür ederiz.

Yukarıdaki bilgileri okudum ve çocuğumun bu çalışmada yer almasını onaylıyorum (Lütfen alttaki iki seçenekten birini işaretleyiniz.

Evet onaylıyorum___   ***Hayır, onaylamıyorum___***

Annenin/Babanın adı-soyadı: ______________   Bugünün Tarihi:________________

Çocuğun adı soyadı ve doğum tarihi:________________

(Formu doldurup imzaladıktan sonra araştırmacıya ulaştırınız).

E. OBSERVATION PROTOCOL

GÖZLEM FORMU

Tarih: **Etkinlik Adı:**

Grup Adı:

Amaç

Bu gözlemin amacı, üstün yetenekli öğrencilerin grup halinde mühendislik temelli modelleme etkinlikleri (MOE'ler) ile çalışırken segiledikleri modelleme yeterliliklerini incelemektir.

Gözlem Soruları

Aşağıdaki sorular gözlem sürecine kılavuzluk etmek için kullanılmıştır.

1) Öğrenciler mühendislik temelli MOE'ler ile ilk karşılaştıklarında nasıl bir strateji uyguluyor?
2) Öğrenciler daha sonraki süreçlerde stratejilerini değiştiriyor mu? Neden?
3) Öğrenciler mühendislik temelli MOE'ler ile çalışırken fen, matematik veya mühendislik kavramlarından hangilerini kullanıyorlar? Nasıl?
4) Öğrenciler hangi durumlarda stratejilerini değiştiriyor?
5) Öğrencilerin grup içindeki iletişimi nasıl gerçekleşiyor?
6) Öğrenciler hangi durumlarda zorluk yaşıyor?
7) Öğrenciler araştırmacıya ne tür sorular yöneltiyor?
8) Öğrenciler modellerini nasıl değerlendiriyor?
9) Her bir grup üyesinin mühendislik temelli MOE üzerinde çalışırken üstlendiği rol nedir? (kayıt tutan, lider,, izleyen, strateji geliştiren vb.)
10) Öğrenciler mühendislik temelli MOE ile çalışırken göreve bağlılıkları nasıl değişiyor?

Gözlem Boyutları

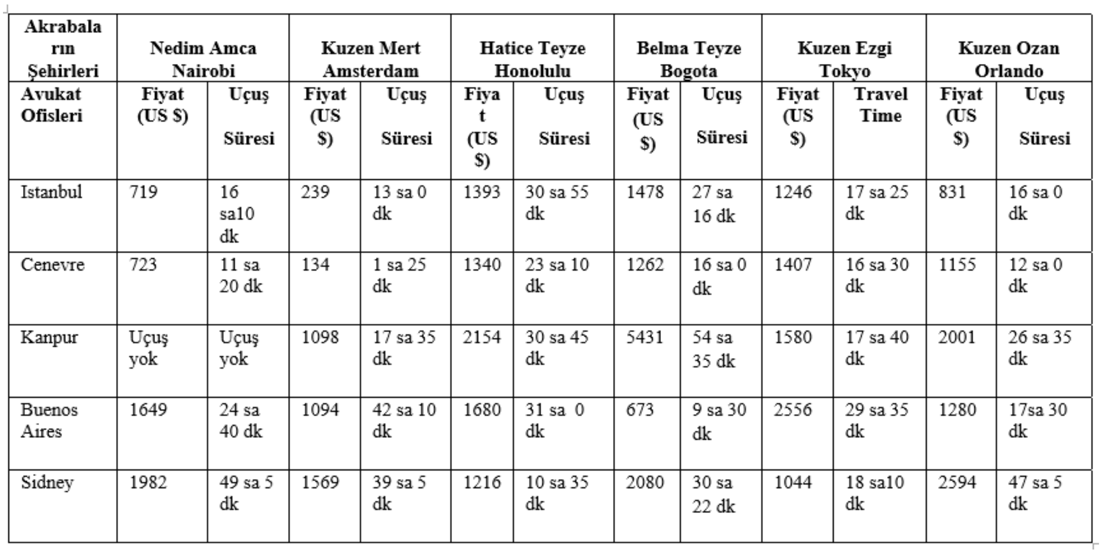
Öğrencilerin mühendislik temelli MOE'leri ile çalışırken sergiledikleri bilişsel modelleme yeterliliklerini belirlemek için aşağıda belirtilen noktalar hakkında gözlem yapılacaktır.

1) Etkinliklerin bağlamı (süresi, öğrencilerin göreve bağlılığını sürdürme durumları, gerekli materyaller, çizimleri, planları)
2) Öğrenciler fen, matematik ve mühendislik bilgilerini çözüm süreçlerine nasıl yansıtıyor? (Farklı uygulama örnekleri)
3) Üstün yetenekli öğrenciler kişisel özelliklerini çözüm modelleme süreçlerine nasıl yansıtıyor? (Beklenmedik öğrenci davranışları)
4) Gruplar içindeki etkileşim modelleme sürecini nasıl etkiliyor? (İlgi çekici tartışmalardan kısa notlar)

Notlar:

F. ENGINEERING-BASED MEAS

Dr. Ahmet'in Vasiyeti



Akrabaların Şehirleri	Nedim Amca Nairobi		Kuzen Mert Amsterdam		Hatice Teyze Honolulu		Belma Teyze Bogota		Kuzen Ezgi Tokyo		Kuzen Ozan Orlando	
Avukat Ofisleri	**Fiyat (US $)**	**Uçuş Süresi**	**Fiyat (US $)**	**Uçuş Süresi**	**Fiyat (US $)**	**Uçuş Süresi**	**Fiyat (US $)**	**Uçuş Süresi**	**Fiyat (US $)**	**Travel Time**	**Fiyat (US $)**	**Uçuş Süresi**
Istanbul	719	16 sa10 dk	239	13 sa 0 dk	1393	30 sa 55 dk	1478	27 sa 16 dk	1246	17 sa 25 dk	831	16 sa 0 dk
Cenevre	723	11 sa 20 dk	134	1 sa 25 dk	1340	23 sa 10 dk	1262	16 sa 0 dk	1407	16 sa 30 dk	1155	12 sa 0 dk
Kanpur	Uçuş yok	Uçuş yok	1098	17 sa 35 dk	2154	30 sa 45 dk	5431	54 sa 35 dk	1580	17 sa 40 dk	2001	26 sa 35 dk
Buenos Aires	1649	24 sa 40 dk	1094	42 sa 10 dk	1680	31 sa 0 dk	673	9 sa 30 dk	2556	29 sa 35 dk	1280	17sa 30 dk
Sidney	1982	49 sa 5 dk	1569	39 sa 5 dk	1216	10 sa 35 dk	2080	30 sa 22 dk	1044	18 sa10 dk	2594	47 sa 5 dk

Hava Durumu ve Coğrafi Veriler
Av. Bora Aydın (Avukat Ofisleri)

Şehir	En yüksek sıcaklık (Nisan)	En düşük sıcaklık (Nisan)	Ortalama yağış miktarı (in.)	Yükseklik(ft)	Ortalama kar yağış miktarı (in.)	Kirlilik derecesi (1-5)
Istanbul	62	46	1.9	121	0	3
Cenevre	56	38	2.4	1210	1	2
Kanpur	102	70	1	429	0	5
Buenos Aires	71	58	3.5	53	10	1
Sidney	73	59	5.2	244		1

Akrabalar için Hava Durumu ve Coğrafi Veriler

Şehir	En yüksek sıcaklık (Nisan)	En düşük sıcaklık (Nisan)	Ortalama yağış miktarı (in.)	Yükseklik(ft)	Ortalama kar yağış miktarı (in.)
Orlando	80	58	2.7	98	0
Tokyo	64	50	4.9	59	13
Bogota	66	46	4.4	8612	0
Honolulu	83	68	1.18	1280	0
Amsterdam	55	38	2.1	3	1
Nairobi	77	56	9.5	5672	0

G. ENGINEERING-BASED MEAS

Köprü Yapımı

Veri Seti 1

Malzemeler	Her bir madde içinkullabilecek max malzeme miktarı	Malzeme Maliyeti
Çöp şiş	200	3.75 TL /100 çubuk
Pipet	200	3.5 TL /100 çubuk
Dil çubuğu	300	9.75 TL /100 çubuk
Kürdan	1000	TL /200 kürdan

Kullanılan yapıştırıcı	Malzeme Maliyeti
Hızlı Yapıştırıcı	15 TL
Silikon	20 TL
Tutkal	10 TL

Kafes, kiriş ve kemer köprü çeşitlerinden hangisini kullandığınızı, köprünün taşıyabileceği yük miktarını, hangi yapıştırıcıyı ve malzemeyi kullandığınızı yazınız

Kafes köprü: düz bir düzende düzenlenmiş, genellikle demir veya çelik olmak üzere birçok bağlı parçadan yapılır. Bu tip köprüler tipik olarak inşa edilmesi en ekonomik olanlardır.

Kemer Köprü:

Bu köprünün taşlarını yerleştirmek için yerçekimi ve kemerin iki yarısını itmek için bir kemer taşı kullanılır.

Kiriş köprüsü, en basit köprü türüdür. Geçmişte onlar, bir dere boyunca bir kütük şeklini almış olabilirler ama bugün, büyük kutu çelik kiriş köprülerine daha aşinayız. Çok çeşitli kiriş köprüleri vardır.

<u>Kullanılan malzemelerin tanımı</u>

Pipet: Özellikle soğuk içecekleri içerken kullanılan, genellikle plastikten yapılmış ince tüp.

Çöp şiş: Ahşaptan yapılan sivri uçlu ince çubuk

Dil çubuğu: Dil çubuğu genellikle kayın ağacından yapılır. Kayın ağacı, ortalama ağırlığa sahip sıkı dokulu bir kerestedir. Kırılmaya karşı son derece dayanıklı olduğu bilinmektedir.

Kürdanlar: Kürdanlar genellikle daha yumuşak ve daha hoş kokulu ağaç olan kayın ağacı, portakal ağacı ve ıhlamur ağacından yapılır. Kürdanlar, kestane, balsam, akçaağaç, titrek kavak, söğüt ve beyaz ağaç gibi ağaçlardan da yapılabilir.

Veri Seti 2

Malzemeler	Her bir madde içinkullabilecek max malzeme miktarı	Malzeme Maliyeti
Çöp şiş	200	3.75 TL /100 çubuk
Pipet	200	3.5 TL /100 çubuk
Dil çubuğu	300	9.75 TL /100 çubuk
Kürdan	1000	3 TL /200 kürdan
Spagetti	1 paket	1.5 TL

H. ENGINEERING-BASED MEAS

Mars Lunarcrete 1

Regolit ve Bağlayıcı Çözelti Veri Seti 1

Regolit ve Bağlayıcı Çözelti	Yoğunluk(g/cm^3)	Rüzgar Direnci (km/h)	Ufalabilirlik (yıkılmadan önce ağırlık miktarı)	Fiyat (milyon)
A	9320	248		$10.9m
B	5560	170		$4.1m
C	6200	229		$4.4m
D	4780	208		$7.1m

Yoğunluk: Bir nesnenin kütlesi hacmine bölünür. $\frac{Kütle}{Hacim}$ g/cm^3

Rüzgâr Direnci: Rüzgârın bir nesneye karşı kuvveti.
Ufalabilirlik: Bir nesnenin daha küçük parçalar haline gelmesi
Fiyat: Malzemeyi uzaya gönderme fiyatı yaklaşık $25/kg olarak hesaplanır.

H. ENGINEERING-BASED MEAS

Mars Lunarcrete 2

Regolit + bağlayıcı çözeltileri:
Sürecin bu kısmı için bu tarifleri test edebilir ve gerekli ayarlamaları yapabilirsiniz. Her grup en az 250 ml çözeltiye ihtiyaç duyacaktır.

Regolit tarifi (7:2)

- 500 gr un
- 200 gr tuz
- 250 mL su

Regolit tarifi (3:1)

- 400 gr un
- 200 gr tuz
- 250 mL su

Regolit tarifi (4:3)

- 800 gr un
- 200 gr tuz
- 930 mL su

Regolit tarifi (2:1)

- 400 gr un
- 250 mL su

Öneriler:

- Yapıştırıcı yapılarınıza güç katabilir. Tutkalın katılara eklenmeden önce seyreltilmesi, karıştırmaya yardımcı olacak ve çözeltinin yapışkanlığını azaltacaktır.
- Baharat eklemek renk ve koku ekleyebilir. Mars ışıltısını vermek için kakao tozu kullanılabilir.
- • Hamurunuzu gıda boyasıyla renklendirmek istiyorsanız, içindekileri karıştırmadan önce sıvıya ekleyin.
- Kum ilginç bir seçimdir burada denemek isteyebileceğiniz birkaç tarif var:
    - 400 gr kum
    - 300 gr mısır unu
    - 375 ml su

    - 400 gr kum
    - 250 ml Yapıştırıcı

I. CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Karataş-Aydın, Firdevs İclal
Nationality: Turkish (TC)
Date and Place of Birth: 07 March 1989, Giresun
Marital Status: Married
Phone: +90 545 664 82 81
email: iclal.karatas@giresun.edu.tr

EDUCATION

Degree	**Institution**	**Year of Graduation**
MS	Boğaziçi University Secondary Science and Mathematics	2014
BS	METU Elementary Mathematics Education	2012
High School	Beşikdüzü Anatolion Teacher High School, Trabzon	2006

WORK EXPERIENCE

Year	**Place**	**Enrollment**
2020- Present	Giresun University	Deputy Quality Coordinator
2017-2020	Beştepe College	Academic Coordinator
2015-2017	Bilfen Çayyolu Schools	Mathematics Teacher
2014-2015	Türkiye Gifted and Talented School	Mathematics Teacher
2013-2014	Sainte Pulcherie Foundation Yeni Nesil 2000 Schools	Mathematics Teacher
2012-2013	Ted Zonguldak College	Mathematics Teacher

FOREIGN LANGUAGES

Advanced English, Beginner German

PUBLICATIONS

A. Conference Proceedings

A1. Karataş-Aydın, F.I., Işıksal Bostan, M., Tekin Sitrava, R., & Yemen Karpuzcu, S. (2022, July 4-6). Middle School Mathematics Teachers' Professional Noticing: The Case of Representation of Fractions on the Number Line [Paper Presentation]. *14th annual International Conference on Education and New Learning Technologies (EDULEARN22),* Palma de Mallorca, Spain.

A2. Karataş-Aydın, F.I. & Işıksal Bostan, M (2021). Engineering based modelling experiences of gifted students: An example of eco-friendly car.*13th International Conference on Education and New Learning Technologies (EDULEARN21),* 5-6 July 2021, pp. 6733-6737.

A3. Karataş-Aydın, F.I., Baran-Sarac, A.B., Işıksal- Bostan, M., Tekin- Sitrava, R., & Yemen-Karpuzcu, S. (2021, Ekim). *Öğretmenler Öğrenci Düşüncelerine Ne Derece Karşılık Verebiliyor? Tam Sayılarla Çarpma İşlemi Modelleme Örneği*. 5. Uluslararası Türk Bilgisayar ve Matematik Eğitimi Sempozyumu (TÜRKBİLMAT-5), Alanya, Türkiye, 28-30 Ekim, 2021.

A4. Karatas, F.I.& Isiksal-Bostan, M. (2019). Gifted Students' Mathematical Modelling Experience from Socio-Cultural Perspective, *Proceedings of International Congress of Gifted and Talented Education* (IGATE), 1-3 November 2019, pp.187-190.

A5. Karatas, F.I.& Isiksal-Bostan, M. (2019). Mathematically Gifted Students' Reflections on Using History of Mathematics in Mathematics Classroom. *Proceedings of the 11th International Conference on Mathematical Creativity and Giftedness (MCG 11)*,22-24 August 2019, pp.191-196.

A6. Karatas, F.I. (2018). *The needs of gifted and talented students for developing an effective mathematics program*, 15th Asian Gifted and Talented Conference, Bangkok, 20-24 August.

A7. Karatas, F.I. & Isıksal, M. (2018). *Üstün Yetenekli Öğrencilerin Matematik Dersinde Matematik Tarihini Kullanmaya İlişkin Görüşleri,* Üstün Yeteneklilerin Eğitimi Kongresi, 4-6 Mayıs, 2018.

A8. Karatas, F.I. & Salioglu-Karadeniz, M. (2018). *Investigation of Written Arguments of Pre-service Teachers about Fractions*. International Conference on Mathematics and Mathematics Education (ICMME-2018), Ordu University, Ordu, 27-29 June 2018, pp.604-605

A9. Alper, A., Caner, K. & Karataş, F.I. (2017). *STEM Öğretmenler için Sitem mi?* Fatih Projesi Eğitim Teknolojileri Zirvesi, Ankara, 17-18 Kasım 2018, pp.228-232.

A10. Karatas, F.I., Soyak, O. & Alp, A. (2017). The Investigation of Mathematical Problem Solving Processes of Fifth Grade Students. *CERME 10 – Tenth Congress of the European Society for Research in Mathematics Education,* Feb 2017, Dublin, Ireland.

A11. Karataş, F.I& Ubuz, B. (2016). *Liselere giriş sınavı matematik sorularının bilişsel istem seviyelerinin incelenmesi.* Sözlü bildiri. 12. Ulusal Fen ve Matematik Eğitimi Kongresi. 28-30 Eylül 2016. Karadeniz Teknik Üniversitesi, Trabzon.

A12. Karatas, F.I. & Tutak- Aslan F. (2016). *Examining Turkish Secondary Mathematics Teachers' Technological Pedagogical Content Knowledge.* ICME 13-13th International Congress on Mathematical Education. Hamburg, Germany, July 24-31,2016.

A13. Karatas, F.I. & Tutak- Aslan F. (2015). An examination of secondary mathematics teachers' technological pedagogical content knowledge. Konrad Krainer; Naďa Vondrová. *CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education*, Feb 2015, Prague, Czech

Republic. pp.2361-2366, Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education.

A14. Karatas, F.I. (2014). A Model for the Relationship of Elementary Teachers' Technological Self-efficacy and Student Achievement. In M. Searson & M. Ochoa (Eds.), *Proceedings of Society for Information Technology & Teacher Education International Conference 2014* (pp. 895-902). Chesapeake, VA: Association for the Advancement of Computing in Education (AACE).

A15. Karatas, F. I. & Tutak, F.A. *Adaptation of Technological Pedagogical Content Knowledge (TPACK) and Technology Integration Self- Efficacy Scale (TISE) into Turkish*. International Conference on Education in Mathematics, Science and Technology (ICEMST), 16-18 May 2014, Konya, Turkey.

A16. Karataş, F. I. (2014). *Üstün Zekâlı ve Yetenekli Çocukların Eğitimde Üniversite İşbirliği Uygulaması*. Sözlü Bildiri. 2.Zekâ ve Yetenek Kongresi.29-30 Kasım 2014. Ortadoğu Teknik Üniversitesi, Ankara.

A17. Güneysu, A., Karataş, İ., & Aşık, O. (2013). Attitudes of Children Towards Dancing Robot Nao: A Kindergarten Observation. *5th International Conference on Social Robotics.* Bristol, UK, October 27-29, 2013.

B. Journal Articles

B1. Karatas-Aydin, F. I., & Isiksal-Bostan, M. (2022). Through Their Eyes: Gifted Students' Views on Integrating History of Mathematics Embedded Videos into Mathematics Classrooms. *SAGE Open*, *12*(2), 1-15.

B2. Karataş-Aydın, F.I. (2021). The Needs of Gifted Middle School Students for Developing an Effective Mathematics Program. *Journal of Individual Differences in Education*, *3*(2), 30-43.

B3. Karatas, F.I & Tutak- Aslan F. (2017). Lise matematik öğretmenlerinin teknolojik pedagojik alan bilgileri ve teknolojiyi bütünleştirme öz-yeterlilikleri. *Mustafa Kemal Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, *14* (37).

C. Projects

Project Title: Investigation of Middle School Mathematics Teachers' Professional Noticing Skills of Students' Mathematical Thinking and Transferring of these Experiences into Learning Environment
Project length: 24 months (2020-2022)
Funding institution: The Scientific and Technological Research Council of Turkey (1003)
Role in the project: Scholarship Holder
Project coordinator: Mine Işıksal-Bostan, Professor of Mathematics Education

Project Title: Mathematical Modeling and Applications in Scientific Process
Project length: 7 days (2021)
Funding institution: The Scientific and Technological Research Council of Turkey (2237)
Role in the project: Participant
Project coordinator: Osman Raşit Işık, Associate Professor of Mathematics Education

HOBBIES

Tennis, Badminton, Riding a bicycle, Swimming

J. TURKISH SUMMARY / TÜRKÇE ÖZET

ÜSTÜN YETENEKLİ İLKÖĞRETİM ÖĞRENCİLERİNİN MODELLEME YETERLİLİKLERİNİN MÜHENDİSLİK TEMELLİ MODELLEME ETKİNLİKLERİ YOLUYLA İNCELENMESİ

GİRİŞ

Üstün yeteneklilik kavramının araştırmacılar arasında yaygın olarak kabul gören bir tanımı bulunmamaktadır. Önerilen tanımlar ise araştırmacının uzmanlık alanına, araştırmanın odağına ve zaman içindeki araştırma eğilimlerine göre değişiklik göstermektir (Türkman, 2020). Bazı araştırmacılar bir çocuğun üstün yetenekliliğinin standart zekâ testinden alınan puanlara göre belirlenmesi gerektiğini savunurken (Jensen, 1980; Terman, 1925), diğer araştırmacılar ise bu tür testlerle birtakım özelliklerin belirlenemeyeceğini iddia etmektedir (Gagné, 2015; Renzulli, 2016; Stenberg, 2018). Böylece, üstün yeteneklilik terimi zamanla zekâ temelli tanımlamalardan çok boyutlu ele alınan bir kavrama dönüşmüştür (Winner, 2000).

Üstün yetenekliliğin çağdaş tanımları ve kavramlaştırılması ışığında, potansiyel olarak üstün yetenekli çocukların belirli özellikler sergilemesi beklenmektedir. Bu özellikler arasında hızlı öğrenme, daha uzun yoğunlaşma süresi, olağanüstü hafıza, karmaşık kavramları anlama yeteneği ve gelişmiş gözlem becerileri bulunur (Harrison, 2004). Bununla birlikte, üstün yetenekli öğrenciler sınıf ortamına farklı öğrenme özelliklerini yansıtmaktadır. Örneğin, bu öğrenciler problemleri hızlı bir şekilde çözebilirler ve farklı bağlamlara aktarılan soyut fikirleri inceleyebilirler (Gross vd., 2001). Ayrıca yeni etkinliklere katılma konusunda güçlü bir merak, duyarlılık ve isteklilik gösterebilirler (Davis ve Rimm, 2004). Bu nedenle, üstün yetenekli öğrencilere yönelik sunulan bir program sadece zorlayıcı ve esnek olmamalı, aynı zamanda üst düzey düşünme becerileri (van Tassel-Baska, 2003) ile merak, yaratıcılık, öngörü, sabır ve hayal gücü gibi kişisel özelliklerin gelişimini de teşvik etmelidir (Özyaprak, 2016; Türkman, 2020).

Üstün yetenekli öğrencilerin belirli özelliklere ve özgün becerilere sahip olmaları nedeniyle müfredatta farklılaştırmaya ihtiyaç duyulmaktadır (Assouline ve Lupkowski-Shoplik, 2021; Özdemir ve Işıksal-Bostan, 2021; Winebrenner ve Brulles, 2012). Alanyazındaki araştırmalar, bu öğrencilerin akademik ihtiyaçlarını karşılamak için özel öğretim fırsatlarına ihtiyaç duyduklarını göstermektedir (Borland, 2009; Johnson, 2000; Matthews ve Foster, 2006). Bunun yanı sıra, çalışmalarda, öğretmenlerin yetersiz ve nadiren farklılaştırma stratejilerini kullanmaları nedeniyle, okullarda, özellikle ilköğretim düzeyinde, üstün yetenekli öğrencilerin pek zorlanmadıklarını ortaya koyulmuştur (Reis vd., 2004; Tomlinson vd., 2003). Sonuç olarak, üstün yetenekli öğrencilerin çoğunluğu sınıfta öğrenme isteğini kaybetmekte ve sıkılmaktadır (Diezmann ve Watters, 2000). Bu nedenle, üstün yetenekli öğrencilerin özgün ihtiyaçlarını karşılamak için öğretmenler, kolay bir şekilde uygulanabilen, öğrencinin öğrenmesini olumlu yönde etkileyen ve öğrencinin sınıf düzeyi için müfredat öğretim standartlarıyla bağlantılı farklılaştırma yöntemlerini kullanmalıdır (Tomlinson, 2014).

Bazı ülkelerde, üstün yetenekli öğrencilerin özel ihtiyaçlarını dikkate alan köklü programlar bulunmaktadır (Heuser vd., 2017). Türkiye, üstün yetenekliler eğitimine gösterilen önem açısından diğer ülkelerin hızına yetişememiştir. Cumhuriyetin kurulduğu 1923 yılından bu yana üstün yetenekli öğrenciler özel eğitim alanında en çok ihmal edilen alanlardan biri olmuştur (Ataman, 1998). Türk ulusal müfredatı büyük ölçüde -sadece olmasa da- beklendik davranışlar sergileyen öğrencilerin yetenek ve öğrenme kapasitelerine göre geliştirildiğinden, üstün yetenekli öğrencilerin eğitim ihtiyaçlarını karşılayamamıştır (Levent ve Bakioğlu, 2013). Ancak bu öğrencilerin özellikleri ve ihtiyaçları sınıf arkadaşlarından farklı eğitim programları gerektirdiğinden (Tomlinson, 2014), onlara özgü bazı özel eğitim programları hazırlanmıştır (Akgül, 2021). Böyle bir eğitim programı sağlamak için 1995 yılında Milli Eğitim Bakanlığı tarafından Bilim ve Sanat Merkezleri (BİLSEM) kurulmuştur. Bu merkezler aracılığıyla daha fazla üstün yetenekli öğrenciye ulaşılmaya çalışılmıştır. BİLSEM, üstün yetenekli öğrencilerin örgün kurumlardaki eğitimlerini kesintiye uğratmamak için okul sonrası faaliyetler yürütmektedir (MEB, 2012). Bu nedenle BİLSEM, üstün yetenekli öğrencilere ilgileri doğrultusunda eğitim imkanları sunmakta ve mevcut kapasitelerini en üst düzeyde kullanmalarını sağlamaktadır (MEB, 2012).

BİLSEM'in üstün yetenekli öğrenciler için uygun bir eğitim programı sunma girişimine rağmen, öğrencilerin ihtiyaçlarını tam anlamıyla karşılamada yetersiz kaldığı görülmektedir. Kurnaz (2014), son beş yıla ait çalıştay ve toplantı raporları ile Türkiye'nin çeşitli bölgelerinden otuz iki BİLSEM yöneticisinin görüşlerini dikkate alarak BİLSEM'in mevcut durumunu, sorunlarını ve sorunlarına yönelik önerilerini ortaya koymak amacıyla bir çalışma gerçekleştirmiştir. Sözü edilen çalışmada, BİLSEM'deki eğitim faaliyetleri için bir öğretim programı çerçevesinin olmamasının büyük bir eksiklik oluşturduğu ifade edilmiştir. Ayrıca, BİLSEM'de gerçekleştirilen etkinliklerin öğrencilerin örgün olarak devam ettikleri okullarında uygulanan müfredatla örtüşmediği ve öğrencilerin ilgisini çekmekte yetersiz kaldığı da belirtilmiştir. Bu önemli geri bildirim doğrultusunda, 2016-2017 döneminde MEB bir öğretim programı çerçevesi hazırlamış ve bunu BİLSEM'lerde esnek bir şekilde uygulayarak sorunlu program bileşenlerinin bazılarını çözmeye çalışmıştır. BİLSEM'ler arasındaki farklı uygulamalar ortadan kaldırılmış, temel beceriler belirlenmiş, öğretmen ve öğrencilerin ne yapacakları konusundaki kafa karışıklıkları giderilmeye çalışılmıştır. Çetin ve Doğan (2018) gerçekleştirilen bu güncellemeleri araştırmış, öğretim programı çerçevesinin kapsamı ve uygulanmasındaki sorunları belirlemiştir. Araştırmada, öğretim programı çerçevesinde yer alan etkinliklerin sırasının, süresinin ve düzeyinin uygun olmadığı, bilimsel hatalar içerdiği, öğrencilerin ilgisini çekmediği ve okullarda uygulanan müfredatla örtüşmediği sonucuna ulaşılmıştır. Ayrıca bu konuların öğretmenler için bazı sıkıntılar oluşturduğu da tespit edilmiştir. Daha yakın tarihli bir çalışmada Bildiren ve Citil (2022), Türkiye'de üstün yeteneklilerin eğitiminde 1923-2020 dönemini tarihsel bir perspektiften araştırmış ve ulusal düzeyde üstün yeteneklilerin eğitimindeki çağdaş uygulamaları değerlendirmiştir. Araştırmacılar, BİLSEM'de görev alan öğretmenlerin görüşlerinin de üstün yetenekli öğrencilere yönelik uygulanacak programların, bu öğrencilerin özel ihtiyaçlarına göre farklılaştırılması gerektiği yönünde olduğunu vurgulamıştır. Tüm bu çalışmalardan elde edilen bilgilerden yola çıkarak Türkiye'den seçilen bir örneklem olan BİLSEM'de üstün yetenekli öğrencilerin her zaman kapsamlı bir eğitim almadıklarını gösterebilecek bazı sorunlar olduğu sonucuna varılabilir. Bu nedenle BİLSEM'de üstün yetenekli öğrencilerin ihtiyaçlarını karşılamaya yönelik sorunları hedef olarak alan uygun etkinliklerin geliştirilmesi ve uygulanması gerekmektedir.

Bilim, teknoloji, mühendislik ve matematik (BİLTEMM) eğitimi, üstün yetenekli öğrencilerin ihtiyaçlarını karşılamaya yönelik etkili bir öğrenme ortamı sunarak, onların potansiyellerini ortaya koymalarını sağlayan bir eğitim yaklaşımı olarak önerilmektedir (Şen, 2018; Yoon ve Mann, 2017). Müşteri odaklı gerçek yaşam durumlarını ve mühendislik tasarım süreçlerini açık uçlu problem çözmeyle bütünleştiren model oluşturma etkinlikleri (MOE'ler) (Maiorca ve Stohlmann, 2014), BİLTEMM uygulamaları için bir araç olarak hizmet etme potansiyeline sahiptir (Baker ve Galenti, 2017; Kertil ve Güler, 2016). MOE'ler, öğrencilerin kendi fikirlerini daha açık bir şekilde ifade etmelerini ve düşünme becerilerini geliştirmelerini sağlayarak problem çözme deneyimlerini artırır (Liljedahl vd., 2016). Ayrıca MOE'ler öğrencilerin etkinliklerle çalışırken matematiksel akıl yürütmelerini, açıklamalarını ve gerekçelendirmelerini ortaya çıkarmak için de kullanılabilir (Doerr ve Lesh, 2011; Hamilton vd., 2008; Mentzer vd., 2014). Öğrenciler, gerçek yaşam senaryolarını matematiksel olarak açıklamanın zorluğundan dolayı MOE'leri genellikle zor bulurlar. Ancak matematiksel modelleme, öğrencilerin matematiksel olarak bilimsel bilgi ve becerilerini geliştirmenin yanı sıra onların dünyayı anlamlandırmalarına da yardımcı olur (Groshong, 2018). Bu tür etkinlikler, öğrencilerin sınıfa getirdikleri farklı öğrenme stillerine ve güçlü yönlerine hitap ederek sınıf deneyimlerini geliştirir (English, 2013; Hamilton vd., 2008). Kısacası, MOE'lerin sahip olduğu özellikler, bu etkinlikleri üstün yetenekli öğrencilerin eğitimi için önemli bir araç haline getirmektedir. Üstün yetenekli öğrenciler, daha az tekrara ihtiyaç duyarlar ve daha üst düzey düşünme gerektiren etkinlikleri tercih ederler (Gross vd., 2001). Bu nedenle, MOE'ler üstün yetenekli öğrencilerin hem zorluk taleplerini hem de akademik ihtiyaçlarını karşılama potansiyeline sahiptir.

Öğrencilerin öğrendikleri matematiğin gerçek yaşam durumlarında nasıl uygulandığını görmeleri önemlidir (English, 2011). MOE'ler, hem bir müşteriyi hem de bir kullanıcıyı bir araya getirerek ve böylece matematik ve mühendislik arasında bir köprü görevi görür, aynı zamanda gerçek dünya sorunlarına yeni bir boyut ekleyerek buna izin verir (Mann vd., 2011). Bu sorunlar matematiksel olmasına rağmen, öğrenciler mühendislik tasarım sürecine doğrudan dâhil olurlar (Cunningham ve Hester, 2007). Daha doğrusu, müşterinin gereksinimleri hakkında bilgi ararlar, çeşitli model önerileri için beyin fırtınası yaparlar, grup üyeleriyle işbirliği içerisinde bir strateji geliştirirler, müşteri tarafından sunulan verileri kullanarak ilk modellerini oluştururlar. Modeli test ederler ve daha sonra ek ama benzer verileri kullanarak modellerini gözden geçirirler (Mann vd., 2011). Bir model oluşturma etkinliği (MOE)'ye başarılı bir çözüm üretmek için öğrencilerin etkili sorular sorması, verileri elde etmesi, incelemesi, yorumlaması ve sonuçlarını sunması gerekir. Mühendislik tasarım sürecindeki problemleri çözmek için de benzer beceriler gereklidir. Bu nedenle, matematiksel modelleme sürecinin ve mühendislik tasarım sürecinin temel özellikleri olan yinelemeli yapı ve model ortaya çıkarma MOE'yi, mühendislikle bütünleştirebilme için etkili araçlar haline getirir (Diefes-Dux vd., 2004; Lyon ve Magana, 2021). Özel olarak, mühendislik temelli MOE'ler, yapısal olarak anlamlı bir ürün oluşturmaya çalışırken öğrencilerden mevcut düşünce biçimlerini yinelemelerinin, test etmelerinin, iyileştirmelerinin veya gözden geçirmelerinin istendiği özgün mühendislik durumları olarak tanımlanır (English & Mousoulides, 2011). Bu etkinlikler sayesinde öğrenciler, matematiksel ve bilimsel kavramları gerçek dünyadaki mühendislik problemlerine uygulama ve ayrıca problemlerin doğasını açıklama ve tahmin etme fırsatı bulurlar (English & Mousoulides, 2011). Bu nedenle mühendislik temelli MOE'ler, üstün yetenekli öğrenciler için farklı derecelerde karmaşıklık, genişlik ve anlama derinliği olan zorlu görevler sağlar (Dailey, 2017). Ayrıca üstün yetenekli öğrencilerin ve başarılı mühendislerin üst düzey düşünme, problem çözme becerileri ve merak gibi ortak özellikleri (Mann ve diğerleri, 2011), mühendislik temelli MOE'lerin çözümünde önemli bir rol oynamaktadır. Dolayısıyla mühendislik temelli MOE'ler, üstün yetenekli öğrencilerin ihtiyaçlarını karşılamak için bir araç olarak kullanılabilir.

Modelleme, alan bilgisinin yanı sıra çeşitli yetkinlik ve becerilere ihtiyaç duyması nedeniyle üstün yetenekli öğrenciler için bilişsel olarak zorlayıcı bir etkinlik olabilir. Mühendislik temelli MOE'ler, öğrencilerin kavramsal anlamaları ve gerçek dünya problemlerini çözme çabaları sırasında geliştirdikleri süreçleri ortaya çıkarmanın bir yolu olarak kullanılabilir (Lesh ve Doerr, 2003). Buna göre modelleme yeterlilikleri, modelleme sürecini uygun şekilde tamamlamak için gerekli beceri ve yetenekler olarak tanımlanır ve birey bu sürece katılmaya istekli olmalıdır (Maaß, 2006). Modellemenin bilişsel perspektifi, öğrencilerin modelleme sürecine dâhil olduklarında zihinlerinde neler olduğunu açıklamak ve analiz etmek için çok önemlidir (Blum, 2011). Biccard ve Wessels' e (2011) göre bilişsel modelleme yeterlilikleri, tüm modelleme sürecini kapsamaktadır. Bu tanımlara göre modelleme yeterliliği, modelleme sürecinin tanımı ile ilişkilendirilir (Jensen, 2007). Döngünün aşamaları yeterlilikler açısıdan incelenir, değerlendirmeler için ölçüt olarak kullanılabilir ve bilişsel modelleme yeterlilikleri olarak adlandırılabilir.

Matematiksel modelleme bakış açıları uygulama alanlarına göre farklılık göstermekte ve matematiksel modellemenin farklı yönlerini öne çıkaran mevcut tanımlara göre değişmekle birlikte (Hıdıroğlu ve Bukova-Güzel, 2016; Kaiser ve Sriraman, 2006), ana süreçlerin temel özellikleri belirlenmiştir. Bununla birlikte, özellikle bilişsel yaklaşım kullanılarak matematiksel modelleme için sürecin alt adımlarının belirli özellikleri ayrıntılı olarak tartışılmamıştır (Blomhøj ve Jensen, 2003; Galbraith ve Stillman, 2006). Matematiksel modelleme; matematik öğretiminde çok önemli bir rol oynamakla birlikte, matematik derslerinde gerçek modelleme problemleri nadiren kullanılmaktadır (Blum, 2011; Turner, 2007). Alanyazında yeterliliklerin tanımlanmış olmasına ve bu yeterliliklerin belirli bir ölçü ve tespiti yapılmış olmasına rağmen, bu alanda daha fazla araştırma yapılması gerekmektedir (Maaß, 2006). Üstün yetenekli öğrencilerle ilgili olarak ise, önceki bilgilerini problem durumunun özgün koşullarına göre uyarlayarak kullanabildikleri (Pativisan, 2006), problem çözerken yaşıtlarına göre daha esnek ve yaratıcı düşünebildikleri belirtilmiştir (Bayazıt ve Koçyiğit, 2017). Ayrıca, alışılmış yaklaşımların dışında yeni çözümler ve stratejiler de geliştirebilmektedirler (English, 2007b). Bu nedenle, üstün yetenekli öğrencilerin modelleme sürecinde bu kendilerine has özellikleri nasıl yansıttığını belirlemek kritik öneme sahiptir. Bu görüşle tutarlı olarak, öğretmenlerin, bu öğrencilerin yeterliliklerini ortaya çıkarmak için onların önceki bilgilerini günlük problemlerine aktarmalarına yardımcı olmayı amaçlayan etkinlikleri nasıl tasarlayacaklarını ve uygulayacaklarını bilmeleri önemlidir. Bu bağlamda, mevcut çalışma, ilköğretimde üstün yetenekli öğrencilerin mühendislik temelli MOE'lere katıldıklarında grup olarak bilişsel modelleme yeterliliklerini analiz etmektedir.

Çalışmanın Amacı ve Araştırma Soruları

Bu tez çalışmasının amacı, ilköğretimde üstün yetenekli öğrencilerin mühendislik temelli model oluşturma etkinliklerine katıldıklarında sergiledikleri bilişsel modelleme yeterliliklerini araştırmaktır. Bu nedenle, bu çalışma aşağıdaki araştırma sorularını ele almaktadır:

1. İlköğretimde üstün yetenekli öğrenciler, mühendislik temelli model oluşturma etkinlikleriyle çalışırken bilişsel modelleme yeterliliklerini (içselleştirme/yorumlama/yapılandırma/sembolleştirme/uyarlama/düzenleme/genelleme) ne ölçüde sergilemektedirler?

Çalışmanın Önemi

Bu tez çalışmasının hem alanyazınına hem de eğitim pratiğine katkısı olacağı düşünülmektedir. Bu çalışmanın alanyazınına birincil katkısı üstün yeteneklilik üzerinedir. Üstün yetenekli öğrencilerin kendilerine özgü ihtiyaçlarını karşılamak için çağdaş bir üstün yeteneklilik anlayışına ve özel öğretim uygulamalarına bulgu sağlayabilir. Örneğin, Paralel Müfredat Modeli (Tomlinson vd., 2008) ve Üçlü Zenginleştirme Modeli (Renzulli, 1977) gibi farklı üstün yetenekliler eğitimi müfredat modelleri tarafından sağlanan bir çerçeve, bu çalışmada üstün yetenekli öğrencilerin benzersiz ilgi alanlarına ve hazır bulunuşluklarına göre uyarlanmış, zorlu ve anlamlı öğrenme deneyimlerinin oluşturulmasında kullanılmıştır. 21. yüzyılda üstün yetenekli öğrencilerin ihtiyaçlarına göre öğretimi farklılaştırmak için mühendislik tasarım süreci, öğrencilere benzer bir öğrenme deneyimi sağlamakta ve bu nedenle öğretim yaklaşımı olarak önerilmektedir (Dailey, 2017; Mann ve Mann, 2021). Zaman içinde test edilmiş bu temel yaklaşımların ötesinde, mevcut çalışmada aynı zamanda üstün yetenekli öğrencilerin 21. yüzyıl becerilerini edinmelerine odaklanan çağdaş teorilerden (Gardner, 2011; Renzulli, 2021; Sternberg, 2019) ilham alınmıştır. Buna ek olarak, 21. yüzyılın ihtiyaçlarını karşılamak üzere üstün yeteneklilik anlayışı için yeni bir paradigma önerilmiştir; buna göre, bu öğrencilerin öğrenme deneyimleri, yeterlilikleri, göreve bağlılıkları ve başkalarıyla ilgilenmeleri yoluyla insanların refahına katkıda bulunmalıdır (Chowkase, 2022). Bu nedenle, mühendislik tasarım sürecinin belirtilen doğası, üstün yetenekli öğrencilere bu üç yolu uygulamak ve bunları araştırma ile doğrulamak için uygun öğrenme ortamları sağlayabilir. Mühendislik tasarım süreci, alanyazında üstün yetenekli öğrenciler için önerilen bir yaklaşım olsa da (Dailey, 2017; Mann vd., 2011; Mann ve Mann, 2021), üstün yetenekli öğrencilerin mühendislik tasarım süreciyle ilgili deneyimlerine ilişkin ampirik çalışmalar sınırlıdır, ancak son zamanlarda alanyazınında yer bulmaktadır (Han ve Şim, 2019; Sen, 2018; Sen vd., 2021). Bu bağlamda, bu çalışmanın bulguları, üstün yetenekli öğrencilerin mühendislik tasarım sürecine dâhil olduklarında yaşadıkları deneyimleri araştırarak üstün yetenekliler eğitimi alanyazınına katkıda bulunabilir.

Bu çalışmanın üstün yeteneklilik ile ilgili alanyazınına bir başka katkısı, ilköğretimde üstün yetenekli öğrenciler için matematiksel modelleme süreci ile mühendislik tasarım süreci arasındaki bağlantıları belirlemeye çalışmasıdır. Groshong (2018), öğretme ve öğrenme uygulamalarında ilgili süreçlerdeki benzerliklerin belirlenmesinin önemini vurgulamaktadır. Bu paralellikler, genellikle disiplinlerarası bilgi ve becerilere dayanan durumları modellemek için faydalı olabilir. Matematiksel modelleme döngüsü ile mühendislik tasarım süreci arasındaki güçlü ilişki, her iki sürecin de yinelemeli yapısını vurgulamaktadır (Zawojewski vd., 2008). Bununla birlikte, modelleme deneyimlerini tasarlama ve bu tür süreçlerde yer almanın önemi (English, 2017) özellikle ilköğretim (Dorie vd., 2014; Portsmore vd., 2012) ve üstün yetenekli öğrenciler (Mann vd., 2011) için göz ardı edilmiştir. Dolayısıyla, bu çalışma, üstün yetenekli olarak tanımlanan belirli bir öğrenci grubu için matematiksel modelleme süreci ile mühendislik tasarım süreci arasındaki paralelliği ortaya çıkarmada ampirik kanıtlar sağlayabilir.

Bu çalışma üstün yeteneklilik ile ilgili alanyazına olan olası katkısının yanı sıra matematiksel modelleme ve mühendislik çalışmalarını birleştirerek de alanyazına katkı sağlayabilir. Ulusal Matematik Öğretmenleri Konseyi (NCTM, 2000), okul öncesinden liseye kadar matematiksel modellemenin müfredata dâhil edilmesi gerektiğini vurgulamaktadır. Bu görüşe uygun olarak, dünyadaki birçok ülke matematik müfredatlarını matematiksel modellemeyi içerecek şekilde revize etmiştir (CCSI, 2014; Eğitim Bakanlığı Singapur, 2007). Benzer şekilde, revize edilen Türk ilköğretim matematik müfredatı (MEB, 2013; 2018) matematiksel modelleme yaklaşımına dayanmaktadır. Ancak Doğan ve arkadaşlarının (2019) gerçekleştirdiği çalışma, matematiksel modellemenin Türkiye'de mevcut ilköğretim matematik ders kitaplarına yansımadığı ve kullanılan modellerin sadece somut ve görsel yapılarla sınırlı olduğunu ortaya koymaktadır. Ayrıca, ilköğretim matematik öğretmenlerinin matematiksel modellemenin sınıfta uygulanmasına yönelik bilgi ve materyal eksikliği de alanyazında belirtilmektedir (Dedebaş, 2017; Işık ve Mercan, 2015). Bu nedenle ilköğretim öğrencilerine yönelik modelleme konusunda yapılacak çalışmalar gerekli ve önemlidir. Özellikle MOE'ler, matematiği gerçek yaşam durumları ile ilişkilendirerek matematik ve mühendislik arasındaki boşluğu kapatma ve gerçek yaşamda karşılaşılan zorlukların başka bir yönünü geliştirme potansiyeline sahiptir (Diefex-Dux vd., 2008). Bu nedenle, MOE'lerin açık uçlu yapısı, öğrencilerin yeterince zor bir düzeyde bir çözüm oluşturmayı düşünmelerine olanak sağlar. Bu tür özellikler, MOE'lerin hem mühendislik tasarım süreci için gerekli yeterlilikleri hem de üstün yetenekli öğrencilerin ihtiyaçlarını karşılamasını sağlar (Mann vd., 2011). Öğrencilerin yaratıcılığını geliştirmek, onları ileri mühendislik bilgi ve tekniklerini kullanmaya motive etmek ve bu öğrencilerde kavramsal düşünmeyi teşvik etmek için temel olarak MOE'lerin lisans mühendislik eğitiminde kullanılmasına odaklanan çalışmalar olmasına rağmen (Moore ve Diefex-Dux, 2004; Moore vd., 2014; Yıldırım vd., 2010), mühendisliği ilköğretim sınıflarına entegre etmek için MOE'lerin kullanımına ilişkin sınırlı sayıda çalışma bulunmaktadır (Cunningham ve Hester, 2007; English, 2007; English ve Mousoulides, 2011; Sen, 2018). Bu bağlamda, mevcut çalışma, ilköğretimde üstün yetenekli öğrencilerin mühendislik temelli MOE'lerle çalışırken yaşadıkları deneyimlerini sunarak alanyazına önemli bir katkı sağlayabilir.

Ayrıca matematiksel modelleme ve mühendislik eğitimi alanyazınını birleştirerek her iki alana da önemli katkılar sağlamaktadır. Bu çalışmanın bir diğer önemli katkısı, üstün yetenekli öğrencilerin matematiksel modelleme yeterlilikleri ile mühendislik yeterliliklerini birleştirerek bulgulara dayalı öğretim uygulamaları sağlaması olabilir. Bilim camiası, bu çağda küresel vatandaş yetiştirilmesindeki ihtiyaçları ele almak için, karmaşık gerçek yaşam problemleriyle başa çıkmada bireylerin modelleme yeterliliklerini geliştirmeye vurgu yapmıştır (Biccard, 2010; Blomhøj ve Jensen, 2003; Ludwig ve Xu, 2010; Maaß, 2006; Şahin ve Erarslan, 2017). Uluslararası çalışmalar incelendiğinde, farklı yaş gruplarından öğrencilerin modelleme yeterliliklerini belirlemeye ve geliştirmeye yönelik çalışmaların olduğu görülmektedir (Biccard, 2010; Biccard ve Wessels, 2011; Blomhøj ve Jensen, 2003; Kaiser, 2007; Ludwig ve Reit, 2012; Maaß, 2006). Ulusal çalışmalar incelendiğinde ise öğrencilerin modelleme yeterliliklerini inceleyen çalışmaların arttığı görülmektedir (Hıdıroğlu ve Bukova Güzel, 2016; Kabar ve İnan, 2018; Kocayayla, 2019; Şahin ve Erarslan, 2017; Yıldırım, 2019). Ancak Aztekin ve Şener (2015) tarafından yürütülen bir meta-sentez çalışmasında, matematiksel modelleme çalışmalarının katılımcılarının genellikle öğretmen adayları olduğunu ve öğrenciler üzerinde yapılan çalışmaların daha çok lise öğrencilerini kapsadığını belirtilmiştir. Bu çalışmalar, genel olarak MOE'lerde öğrencilerin modelleme yeterliliklerini geliştirdiğini (Biccard, 2010; Blomhoj ve Jensen, 2003; Kaiser, 2007; Maaß, 2006), öğrenciler yaş olarak ne kadar büyükse, o kadar başarılı olduklarını göstermiştir (Henning ve Keune, 2007; Ludwig ve Xu, 2010). Ayrıca, varsayımda bulunma (Yıldırım, 2019) ile doğrulama ve gerekçelendirme yeterliliklerinde öğrencilerin güçlük yaşadıkları da ortaya koyulmuştur (Kabar ve İnan, 2018; Kaiser, 2007; Maaß, 2006; Tekin-Dede ve Yılmaz, 2015). Ayrıca, Koç (2020) son yirmi yılda yapılan modelleme tezlerini inceleyerek benzer bulgulara ulaşmıştır. Bu alanyazın taraması, ilkokul öğrencileriyle yürütülen az sayıda çalışmayı ortaya çıkarmıştır. Alanyazında bazı yeterliliklerin belirlenmiş olmasına ve bu yeterliliklerin bir kısmının ölçülebilmesine rağmen, bu alanda yapılması gereken çalışmalar halen devam etmektedir. Maaß'a (2006) göre, modelleme yeterlilikleri ve bu yeterliliklere eşlik eden sınırlamalar hakkında az sayıda kapsamlı araştırma vardır. Yine de, sınırlı olduğu kabul edilen çalışmaların yararlı bileşenleri belirlenmiş ve matematiksel modelleme yeterlilikleri ile mühendislik yeterlilikleri arasındaki ilişki araştırılmıştır (de Villiers, 2018; Huffman, 2015). De Villiers (2018) tarafından yapılan çalışmada MOE aracılığıyla matematiksel modelleme yeterlilikleri ile birlikte gelişebilecek önemli mühendislik yeterlilikleri eşleştirilmiştir. Bu bağlamda mevcut çalışma, üstün yetenekli ilköğretim öğrencilerinin modelleme yeterliliklerini ve hangi yeterliliklerin ne düzeyde ortaya çıktığını belirleyerek bu boşluğun giderilmesine önemli katkılar sağlayabilir. Bu çalışmada kullanılan çerçevenin çeşitli ortamlarda ve çeşitli öğrenci gruplarıyla doğrulanması sonucunda, ilköğretimde üstün yetenekli öğrenciler için çerçeveyi zenginleştirme ve uyarlama potansiyeline sahiptir. Disiplinlerarası bir yapıya sahip olan bu çalışma, matematik eğitiminin yanı sıra fen, mühendislik ve üstün yetenekliler eğitiminde alanyazına önemli katkılar sağlayabilir.

Bu çalışmanın diğer bir katkısı, öğrenciler, öğretmenler ve eğitim kaynakları tasarımcıları açısından eğitim ortamlarına olası katkılarının bulunmasıdır. Pratik olarak, matematiksel modelleme, mühendisliği bütünleştirmek ve 21. yüzyılın zorluklarını matematik eğitimine dâhil etmek için bir araçtır (English, 2017). Bu bağlantılar, iki ana nedenden dolayı kurulabilir: (1) öğrenciler, matematiksel modellemede gerçek dünya problemleri için disiplinler arası bilgiyi kullanırlar ve (2) bir modelleme sürecini yürütmek için gereken yeterlilikler, 21. yüzyıl becerileri ile önemli ölçüde örtüşür (Maass vd., 2019). Matematiksel modelleme deneyimleri, önemli müdahale ve yönlendirici desteklerle grup içi ve gruplar arası işbirliklerinin oluşmasına imkân sağlar (English & Mousoulides, 2011). Mevcut araştırma, bu müdahalelerin her birinin, öğrencilerin modellerini oluşturma ve güçlendirme konusundaki ilerlemeleri üzerindeki etkililiğini izlemeyi de amaçlamaktadır. Bu nedenle, bu çalışmanın bulguları, ilköğretimde üstün yetenekli öğrencilerin matematiksel modelleme sürecine ilişkin gelecekteki araştırmalar için faydalı olabilir. Yeterliliklerinin belirlenmesi, öğretmenlere mühendislik temelli MOE'lerin geliştirilmesinde ve ayrıca öğrencilerin ilerlemesine daha fazla destek sağlanmasında yardımcı olabilir. Dolayısıyla bu çalışmanın bulguları, öğrencilerin BİLSEM'lerde ve gelecekteki sınıflarda modelleme yeterliliklerini geliştirmelerini kolaylaştıran öğretim yöntemleri ve materyallerinin geliştirilmesinde kullanılabilir. Ek olarak, bu çalışmanın bulguları ilköğretimde üstün yetenekli öğrenciler için daha etkili ve destekleyici eğitsel veya özel programların geliştirilmesini desteklediği için müfredat tasarım çalışmalarına katkıda bulunabilir.

YÖNTEM

Araştırma Deseni

Bu çalışma üstün yetenekli ilköğretim öğrencilerinin grup olarak mühendislik temelli MOE'lerle çalışırken sergiledikleri bilişsel modelleme yeterliliklerini belirlemeyi hedeflediği için araştırma deseni olarak durum çalışması kullanılmıştır. Durum sayısı, araştırmacının ilgisi ve araştırmacının amacı gibi özelliklere bağlı olarak durum çalışmasının çeşitli sınıflandırmaları yapılmaktadır (Merriam, 2009; Stake, 2005; Yin, 2009). Stake (2005) durum çalışmasını gerçek, araçsal ve kolektif olmak üzere üç kategoriye ayırmaktadır. Gerçek durum çalışmaları bir durumu daha iyi anlamak için gerçekleştirilen çalışmalardır. Araçsal durum çalışmalarında bir konu hakkında derinlemesine bilgi edinmek veya belirli bi duruma ışık tutmak amaçlanmıştır yani durumun kendisi ikincil önemdedir. Diğer yandan, kolektif durum çalışmaları, araçsal durum çalışmaları ile aynı amaca sahiptir, ancak birden fazla durumu içerir.

Kolektif durum çalışmasının kullanılması, araç olarak bir dizi bireysel durum çalışmasının derinlemesine araştırılmasını gerektirir. Bu çalışmada, ilköğretimde üstün yetenekli öğrencilere yönelik etkinliklerin kullanımından ziyade, öğrencilerin modelleme yeterliliklerini belirlemeyi amaçlandığından kolektif durum çalışması yaklaşımı kullanılmıştır. Buna göre, mühendislik temelli MOE'ler araçsal değere sahip olabilir. Araştırmacı, benzer vakalardaki bulguları analiz etmek için benzer özelliklere sahip durumları seçebilir (Mills vd., 2009). Bu çalışma kapsamında, Köprü İnşaatı, Mars Lunarcrete ve Dr. Ahmet'in Vasiyeti olmak üzere mühendislik temelli üç MOE'nin her biri, benzer özelliklere sahip araçsal bir durum olarak ele alınmış ve bütüncül olarak değerlendirilmiştir. İlköğretimde üstün yetenekli öğrencilerin tasarım sürecine dahil olurken modelleme yeterliliklerini araştırmak için bu özel etkinlikler bir araç olarak kullanılmıştır. Bu nedenle, bu çalışmada durumlar "başka bir şeyi anlamamıza yardımcı olmak" için ikincil ve destekleyici bir rol oynamaktadır (Stake, 1994, s.237).

Katılımcılar

Bu çalışmanın katılımcılarını, Ankara'da bir BİLSEM'de bireysel yetenekleri fark ettirme programına devam eden 19 üstün yetenekli ilköğretim öğrencisi oluşturmaktadır. Araştırmacı, tasarım süreçleri ile ilgili detaylı bilgiyi üstün yetenekli öğrenci grubundan mühendislik temelli MOE'lerle çalışırken elde etmeyi hedeflediğinden amaçlı örneklem kullanmıştır. Bu çalışma, BİLSEM'de yürütülen yaz okulu programları kapsamında gerçekleştirilmiştir. Bu çalışmanın amacı ve kapsamı hakkında öncelikle yöneticiler, öğretmenler, veliler ve öğrenciler bilgilendirilmiştir. Daha sonra bu çalışmaya katılmaya gönüllü olan öğrenciler yaz okulunun bu programına başvurmuşlardır. Çalışmanın katılımcı grupları veli onayı alındıktan sonra oluşturulmuştur. BİLSEM'deki yöneticiler yaz okulu programındaki tercih edilen programlara göre katılımcıları bitirdikleri sınıf seviyelerine göre iki gruba ayırmıştır. Katılımcıların kimliğinin gizli kalması için her katılımcıya bir numara ve her gruba bir harf verilmiştir. Katılımcılara ilişkin demografik bilgiler Tablo 3.1'de sunulmuştur.

Tablo 3.1. Katılımcılara ilişkin demografik bilgiler

Grup	Katılımcı	Cinsiyet	Okul türü	Sınıf seviyesi	Yaş	BİLSEM'e devam süresi (yıl)
Grup A	Öğrenci 1	Erkek	Devlet	5	10	1
	Öğrenci 2	Kız	Özel	5	10	2
	Öğrenci 3	Erkek	Özel	6	12	2

Tablo 3.1. (devam)

Grup B	Öğrenci 4	Kız	Özel	6	11	2
	Öğrenci 5	Kız	Devlet	5	10	2
	Öğrenci 6	Erkek	Özel	5	10	3
Grup C	Öğrenci 7	Kız	Özel	6	11	3
	Öğrenci 8	Erkek	Devlet	5	10	2
	Öğrenci 9	Erkek	Devlet	6	11	2
Grup D	Öğrenci10	Kız	Özel	6	10	3
	Öğrenci 11	Erkek	Devlet	5	10	1
	Öğrenci 12	Erkek	Devlet	5	10	1
Grup E	Öğrenci 13	Erkek	Devlet	4	9	1
	Öğrenci 14	Erkek	Özel	4	9	1
	Öğrenci 15	Erkek	Devlet	4	9	1
Grup F	Öğrenci 16	Kız	Özel	4	10	2
	Öğrenci 17	Erkek	Devlet	4	9	2
	Öğrenci 18	Erkek	Devlet	4	9	2
	Öğrenci 19	Erkek	Devlet	4	9	1

Veri Toplama Araçları

Bu çalışmanın veri toplama araçlarını üstün yetenekli ilköğretim öğrencilerinin modelleme deneyimleri ve grup olarak katılımcılarla gerçekleştirilen görüşmelerin video kayıtları, çalışma yaprakları, araştırmacının alan notları ve öğretmenin gözlem notları oluşturmaktadır. Araştırma gerçekleştirilirken tüm uygulama süreci video veya ses kayıt cihazı ile kaydedilmiştir.

Mühendislik temelli MOE'ler

İlköğretimde üstün yetenekli öğrencilere yönelik etkinlikler hazırlanırken bu öğrencilerin kendine has özelliklerine hitap eden farklılaştırılmış etkinliklerin özellikleri belirlenmiştir (Özdemir, 2016). Sonuç olarak mühendislik temelli MOE'lerin çok disiplinli, açık uçlu, karmaşık ve özgün olmaları nedeniyle uygun etkinlikler olduğu belirlenmiştir (Lesh ve Doerr, 2003). Açık bir şekilde ifade etmek gerekirse, bu tür etkinlikler, üstün yetenekli öğrencilerin öğretim ihtiyaçlarını karşılama noktasında zorluk içerir, ilgi çekicidir ve üst düzey düşünme becerisi gerektirir (Özdemir, 2016).

Buna göre, mevcut çalışmada kullanılan üç mühendislik temelli MOE'nin tümü, matematik ve mühendislik eğitiminde MOE'leri tasarlamak için kullanılan altı temel ilkeye (Diesfes-Dux vd., 2008) ve bu özel grubun, yani ilköğretimde üstün yetenekli öğrencilerin ön bilgilerine dayalı olarak geliştirilmiştir. Bu çalışma kapsamında Köprü Yapımı, Dr. Ahmet'in Vasiyeti ve Mars Lunarcrete etkinliklerinden yararlanılmıştır. Üç farklı mühendislik temelli MOE'nin kullanılmasının amacı, katılımcıları inşaat, mimari ve veri madenciliği mühendisliği gibi çeşitli mühendislik alanlarına hazırlamak için tasarlanmış çeşitli etkinliklerde bilişsel modelleme yeterliliklerinin analizini yapmaktır. Başka bir deyişle, Köprü Yapımı ve Mars Lunarcrete etkinlikleri, inşaat ve mimar mühendisliğini kullanarak bir prototip oluşturmayı içerir. Birincisi günlük hayatta sıkça karşılaşılan bir durum iken ikincisi uzayla ilgilidir. Öte yandan Dr. Ahmet'in Vasiyeti etkinliği ürün değil, prosedür geliştirmeyi gerektirir. Bu nedenle araştırmacı, farklı mühendislik temelli MOE formlarıyla çalışırken ilköğretimde üstün yetenekli öğrencilerin bilişsel modelleme yeterliliklerini araştırmayı amaçlamaktadır.

Odak Grup Görüşmesi

İlköğretimde üstün yetenekli öğrenciler grup olarak mühendislik temelli MOE'lerle çalışırken grup olarak modelleme yeterliliklerini araştırmak amaçlandığı için bu çalışmada odak grup görüşmesi yapılmıştır. Odak grup görüşmesi, odak gruba katılan bireylerle belirli bir konu hakkında görüşmeler yaparak veri toplama süreci olarak ifade edilmektedir (Merriam, 2009). Odak grup görüşmesi, görüşülen kişiler arasındaki etkileşimin en iyi bilgiyi vermesi ve birbirleriyle işbirliği yapması muhtemel olduğunda kullanılabilecek yararlı bir araçtır (Frankel ve Wallen, 2006). Süreç boyunca her oturumun sonunda araştırmacı öğrencilerle odak grup görüşmesi yapmış ve öğrencilerden her bölümde ne yaptıklarını açıklamalarını istemiştir. Araştırmacı tasarım sürecinde düşüncelerini ortaya çıkarmak için neden ve nasıl sorularını yöneltmiştir. Köprü Yapımı etkinliğinin ilk bölümünün sonunda gruplara sorulan bazı örnek sorular şu şekildedir. Örneğin; a) Çiziminizde köprünüzü ne yapmayı planladığınızı anlatır mısınız? (b) Ne tür bir köprü inşa edeceksiniz? (c) Hangi malzemeleri kullanacaksınız? (d) Köprüyü inşa ederken düşündüğünüz en önemli şey neydi? (e) Hangi şekilleri kullanacaksınız ve neden? (f) Köprünüzü nasıl yeterince güçlü hale getireceksiniz? (g) Köprünüzün maliyeti ne olacak? şeklindeki sorular gruba yöneltilmiştir.

Araştırmacının Alan Notları

Mevcut çalışmada, araştırmacı katılımcı-gözlemci olarak hareket etmiş ve her bölümden hemen sonra gözlemlerini uygun şekilde belgelemiştir. Özellikle, ilköğretimde üstün yetenekli öğrencilerin grup olarak mühendislik temelli MOE'lerle çalışırken ilginç bir tartışma, beklenmedik bir durum ve katılımcıların kendine özgü davranışları olduğunda araştırmacı tanımlayıcı notlar almıştır.

Gözlem Notları

Katılımcı olmayan gözlemci, katılımcılar tarafından yürütülen etkinliklere katılmadan araştırma ortamında bulunan ve notlar alan gözlemcidir (Creswell, 2012; Merriam, 2009). Mevcut çalışmada, BİLSEM'deki fen bilgisi öğretmeni katılımcı olmayan gözlemci olarak görev almıştır. Gözlemlerine dayalı olarak ekte verilen gözlem formu doğrultusunda bazı notlar almış ve araştırmacı ile paylaşmıştır.

Veri Toplama Süreci

Bu çalışmada veri toplama süreci, etkinliklerin hazırlanmasını, pilot çalışmayı ve asıl uygulamayı içermektedir. Pilot çalışmanın uygulanmasına başlamadan önce ODTÜ Etik Kurulu'ndan ve Milli Eğitim Bakanlığı'ndan ilgili yasal izinler alınmıştır. Etkinliklerin bağlamı, ilgili alanyazın incelenerek araştırmacı tarafından geliştirilmiştir. Pilot uygulama öncesinde etkinliklerin taslakları BİLSEM'deki fen, matematik ve teknoloji öğretmenlerine sunulmuş ve geri bildirimleri alınmıştır. Uzman görüşü alma sürecinde fen ve matematik derslerinin olduğu zamanlarda araştırmacı araştırma ortamında bulunmuş, öğrenci ve öğretmenlerin araştırmacıyı ortamda kabul etmesi ve yabancılaşmaması için öğretmen ve öğrencilerle zaman geçirmiştir. BİLSEM'deki öğretmenlere ek olarak, fen, teknoloji ve matematik eğitimi ve mühendis gibi çeşitli alanlardan uzman görüşü alınarak etkinliklerin son hali, BİLSEM'de bireysel yetenekleri fark ettirme programında matematik ve fen derslerine devam eden üstün yetenekli öğrencilere uygulanmıştır. Bu yaklaşım sayesinde araştırmacı, üstün yetenekli öğrencilerin asıl çalışma öncesindeki davranışları hakkında fikir sahibi olmanın yanı sıra eğitim süreçlerini de izleyebilmiştir. Bu süreçte öğrencilerin doğal davranışlarını etkilememek için kayıt yapılmadan sınıflarda tutulan video kamera, ses kayıt cihazı gibi materyaller kullanılmıştır. Pilot çalışmanın analizinin ardından etkinliklerin son hali BİLSEM'deki yaz okulunda uygulanmıştır. Son olarak, asıl çalışmadan elde edilen veriler analiz edilmiş ve raporlanmıştır.

Verilerin Analizi

Bu çalışma, üstün yetenekli ilköğretim öğrencilerinin mühendislik temelli MOE'ler ile çalışırken bilişsel modelleme yeterliliklerini araştırmayı ve grup olarak bilişsel modelleme yeterliliklerini ve ilgili alt modelleme yeterliliklerini ne ölçüde sergilediklerini belirlemeyi amaçlamaktadır. Buna bağlı olarak, elde edilen veriler, de Villiers (2018) tarafından önerilen bilişsel yeterlilik sınıflandırmasının adapte edilmiş şekline dayalı olarak analiz edilmiştir. Yedi bilişsel modelleme yeterliliği; içselleştirme, yorumlama, yapılandırma, sembollleştirme, uyarlama, düzenleme ve genellemeyi içermektedir. Ayrıca, gruplardaki üstün yetenekli ilköğretim öğrencilerinin mühendislik temelli MOE'ler (Köprü Yapımı, Mars Lunarcrete ve Dr. Ahmet'in Vasiyeti) ile çalışırken bu tür bilişsel modelleme yeterliliklerini ne ölçüde sergiledikleri, de Villiers (2018) tarafından önerilen grup modelleme yeterliliği gözlem kılavuzunun uyarlanmış şekline dayanarak analiz edilmiştir. Özetle, ilgili matematiksel modelleme ve mühendislik alt yeterliliklerini tanımlayan bilişsel modelleme yeterlilik çerçevesi, her bir bilişsel modelleme yeterliliğinin ve ilgili alt modelleme yeterliliğinin ayrıntılarını açıklamak için kullanılmıştır. Ayrıca her bir düzeyin detayını sıfırdan üçe kadar olan puanlarla tanımlayan bilişsel modelleme yeterlilik dereceli puanlama anahtarı, grupların bilişsel modelleme yeterlik düzeylerini belirlemek için kullanılmıştır.

Kodlama sürecinde iki yaygın analiz yöntemi vardır: (a) araştırmacıların kavramsal bilgilerine dayalı olarak kodlar oluşturdukları açık kodlama yaklaşımı; (b) araştırmacıların olgulara veya alanyazındaki bulgulara dayalı olarak geliştirilen kodları kullandığı önceden belirlenmiş kodların kullanılması (Creswell, 2012). İlgili alanyazının gözden geçirilmesinden sonra, de Villiers (2018) tarafından sağlanan çerçeve ve dereceli puanlama anahtarını kullanarak mühendislik temelli MOE'lerle çalışırken üstün yetenekli öğrencilerin modelleme yeterliklerinin analizi yapılmıştır. Bu çalışmadan elde edilen veriler doğrultusunda bahsedilen çerçeve ve dereceli puanlama anahtarı uyarlanarak verilerin analizinde kullanılmıştır.

BULGULAR VE TARTIŞMA

Bu bölümde araştırma sorularına yanıt verecek şekilde mevcut çalışmadan elde edilen bulgular, yedi bilişsel modelleme yeterliliği ekseninde sunulmaktadır. Diğer bir deyişle, üstün yetenekli ilköğretim öğrencilerinin mühendislik temelli MOE'ler üzerinde grup olarak çalışırken sergiledikleri modelleme yeterlilikleri ve bu modelleme yeterliliklerini ne derecede gösterdikleri incelenmekte ve tartışılmaktadır. Üç mühendislik temelli MOE'den elde edilen verilere dayanarak, yedi bilişsel modelleme yeterliliği ve her bir bilişsel modelleme yeterliliğine ilişkin alt modelleme yeterlilikleriyle birlikte ele alınmıştır. Daha açık bir ifadeyle, her bir bilişsel modelleme yeterliliği kendi alt modelleme yeterlilikleri ve bunların mühendislik ve matematiksel modelleme alt yeterlilikleri ile ilgili göstergeleri üzerinden incelenmiştir.

İlk bilişsel modelleme yeterliliği olan içselleştirme; problemi anlama, ilgili bilgileri toplama, durumu basitleştirme, etik değerlendirme ve esneklik ve yenilik alt modelleme yeterlilikleri bağlamında kategorize edilmiştir. De Villiers (2018) tarafından önerilen çerçeveye ek olarak, mevcut çalışmanın verilerinden etik değerlendirme ve esneklik ve yenilik alt modelleme yeterlilikleri ortaya çıkmıştır. Problemi anlama alt modelleme yeterliliğine ilişkin bulgulara göre, tüm gruplardaki üstün yetenekli ilköğretim öğrencileri, içselleştirme alt yeterliliğine ilişkin örnek göstergeler sergilemişlerdir. Öğrencilerin hepsi açık bir şekilde ana konuları tanımlayıp özetlemiş ve neden problem olduklarını açık bir şekilde açıklamıştır. Böylece, üstün yetenekli ilköğretim öğrencileri için problemi anlamanın ne basit ne de doğrudan bir yeterlilik olmadığı ortaya çıkmıştır. Yukarıda tartışılan birinci alt modelleme yeterliliğine benzer şekilde, tüm gruplardaki ilköğretimde üstün yetenekli öğrenciler, ikinci içselleştirme alt modelleme yeterliliği, ilgili bilgileri toplama konusunda da örnek göstergeler sergilemişlerdir. Özel olarak, mühendislik temelli MOE'lerle çalışırken, kolayca görünmeyen gizli veya örtük bilgileri ortaya çıkarmışlar ve problem durumu için ilgili tüm bilgileri kullanmışlardır. İlköğretimde üstün yetenekli öğrenciler tüm gruplarda içselleştirmenin bir diğer alt modelleme yeterliliği olan durumu basitleştirmede çoklu temsiller kullanarak örnek göstergeler sergilemişlerdir. Yukarıda bahsedilen içselleştirme alt yeterliliklerinden farklı olarak, mevcut çalışmanın veri analizinden etik değerlendirme modelleme alt yeterliliği ortaya çıkmıştır. Tüm gruplar, etik değerlendirme alt modelleme yeterliliğini örnek seviyede sergilemiştir. Daha net bir şekilde ifade etmek gerekirse, tüm gruplar temel etik sorunu, ilgili ve etkilenen tarafları ve ayrıca mühendisliğin insanları ve yerleri nasıl etkilediğini açıkça belirleyerek örnek seviyede etik değerlendirme modelleme alt yeterliliği sergilemiştir. Bu çalışmanın bulgularından ortaya çıkan bir diğer içselleştirme modelleme alt yeterliliği ise esneklik ve yeniliktir. İçselleştirmenin diğer alt modelleme yeterliliklerinden farklı olarak, farklı mühendislik temelli MOE'lerde gruplar farklı derecelerde esneklik ve yenilik göstermiştir. Dr. Ahmet'in Vasiyeti etkinliğindeki her iki grup ve Mars Lunarcrete etkinliğindeki C grubu, bu alt modelleme yeterliliğini yeterli düzeyde göstermiştir. Bu çalışmadaki altı grubun tümü, verilere büyük ölçüde esneklik sunmuştur. Bununla birlikte, örnek düzeyde esneklik ve yenilik sergileyen gruplar, yeni ve benzersiz çözümler geliştirmek için özgün fikirler de ortaya koyarken, yeterli düzeyde esneklik ve yenilik sergileyen gruplarda bu gösterge bulunamamıştır.

İkinci bilişsel modelleme yeterliliği olan yorumlama; varsayımda bulunma, özelliklerin belirlenmesi ve koşulların ve sınırlılıkların belirlenmesi şeklinde alt modelleme yeterlilikleri bağlamında incelenmiştir. Yorumlama modelleme yeterliğinde bir önceki bölümde olduğu gibi yeni bir alt modelleme yeterliği ortaya çıkmamış olsa da, alt modelleme yeterliklerine ait olan özelliklerin belirlenmesi, koşul ve sınırlılıkların belirlenmesine ait örnek düzey göstergeleri çalışmadan elde edilen verilere göre genişletilmiştir. Bu çalışmanın bulgularına göre, altı grubun tamamı örnek bir düzeyde varsayımda bulunma alt modelleme yeterliliğini sergilemiştir. Başka bir deyişle, gruplar halindeki üstün yetenekli öğrenciler yenilikçi ve anlayışlı varsayımlar sergilemiş ve varsayımların sonuçları için açık ve tutarlı bir değerlendirme göstermişlerdir. Yorumlama yeterliliğinin bir diğer alt modelleme yeterliliğine bakıldığında, bu çalışmadaki tüm gruplar, özelliklerin belirlenmesi alt modelleme yeterliğini örnek derecede göstermiştir. Özellikle, ilköğretimde üstün yetenekli öğrenciler, problemdeki önemli miktarları, değişkenleri ve ayrıca pratik, ekonomik, sosyal, çevresel, kalite güvencesi ve güvenlik faktörlerini göz önünde bulundurarak problemle nasıl ilişki kurduklarını fark etmiştir. Tüm gruplar, koşulların ve sınırlılıkların belirlenmesini yani yorumlamanın son alt modelleme yeterliliğini de örnek bir düzeyde göstermiştir. Bir başka ifadeyle, ilköğretimde üstün yetenekli öğrenciler, insanların, malzemelerin, donanımların, araçların ve finansmanın verimli kullanımı ve etkileşimi açısından açık koşullar ve sınırlılıkları gözeterek açıklama yapmışlardır.

Üçüncü bilişsel modelleme yeterliliği olan yapılandırma, yenilikçi planlama ve tasarım ile ilişki kurma alt modelleme yeterlilikleri bağlamında incelenmiştir. Yenilikçi planlama ve tasarım modelleme alt yeterliliğinden elde edilen bulgular, mühendislik temelli farklı MOE'lerdeki grupların bu alt modelleme yeterliliğinde değişen seviyelerde yeterlilik sergilediğini göstermektedir. Dr. Ahmet'in Vasiyeti etkinliğindeki her iki grup alt modelleme yeterliliğini yeterli düzeyde sergilerken, Köprü Yapımı ve Mars Lunarcrete etkinliklerindeki diğer gruplar, yenilikçi planlama ve tasarım modelleme alt yeterliliğini örnek düzeyde göstermiştir. Bir başka ifadeyle, diğer gruplar, sorunu açıklamak ve E ve F grupları tarafından oluşturulan doğru bir modeli tamamlamak için çoklu temsillerle durumsal modeller kurmak için yenilikçi planlama ve tasarım kullanmıştır. Bu farklığın nedeni Dr. Ahmet'in Vasiyeti etkinliğinde, öğrencilerin prototipler yerine prosedürü oluşturmaları olabilir. Dolayısıyla, E ve F gruplarındaki üstün yetenekli ilköğretim öğrencileri durumsal modellerini oluşturmadan önce problem durumunu ayrıntılı olarak tartıştıkları için beklenmedik durumların meydana gelme olasılığı azalmış olabilir. Yapılandırma yeterliliğinin ilişki kurma modelleme alt yeterliliği ile ilgili olarak, tüm gruplar bu alt modelleme yeterliliğini örnek düzeyde göstermiştir. Daha açık bir şekilde ifade etmek gerekirse, ilköğretimde üstün yetenekli öğrenciler gruplar halinde, karşılıklı bağımlılık, etkileşimler ve faktörlerin göreceli önemini göz önünde bulundurarak problemlerin çözümü için genel bir kural, formül, strateji, model veya prototip oluşturmuştur. Başka bir deyişle, bu çalışmanın bulguları, mühendislik temelli MOE'lerle çalıştıklarında, tüm grupların temel değişkenler arasında ilişkiler belirlediğini ve kurduğunu göstermiştir.

Dördüncü bilişsel modelleme yeterliliği olan sembolleştirme; uygun sembolleri seçme, sembolleri kullanma, problemlere metodik yaklaşma ve disiplinler arası bilgiyi uygulama alt modelleme yeterlilikleri altında kategorize edilmiştir. Ayrıca, de Villiers (2018) tarafından önerilen çerçeve, bu çalışma sonucunda disiplinler arası bilgiyi uygulama yeni modelleme alt yeterliliğini de içerecek şekilde güncellenmiştir. Bu çalışmanın bulguları, tüm grupların uygun sembolleri seçme modelleme alt yeterliliğini örnek bir düzeyde sergilediklerini göstermiştir. Bu, ilköğretimde üstün yetenekli öğrencilerin uygun bir çözüme götürecek matematiksel, bilimsel veya mühendislik araçlarını seçtikleri anlamına gelmektedir. Ayrıca, mühendislik temelli MOE'lerin disiplinler arası doğası nedeniyle öğrencilerin sadece uygun matematiksel araçları değil, aynı zamanda bilimsel ve mühendislik araçlarını da seçtiklerini göstermektedir. Öte yandan, tüm gruplar sembolleri kullanma alt modelleme yeterliliğinde örnek teşkil edecek düzeyde sergilememiştir. Köprü Yapımı ve Dr. Ahmet'in vasiyeti etkinliklerindeki gruplardan biri sembolleri kullanma alt modelleme yeterliliğini yeterli düzeyde sergilemiştir. Modellerinde kullanılan sembolleri doğru bir şekilde açıklamışlar ve tanımlamışlardır. Bununla birlikte, sembolleri kullanma alt modelleme yeterliliğini örnek düzeyde sergileyen gruplar, problemle çalışmak için olası alternatif yöntemler de sunmuşlardır. Sembolleştirmenin diğer bir modelleme alt yeterliliği olan problemlere metodik yaklaşmanın örnek göstergeleri tüm gruplar tarafından sergilenmiştir. Yani, akıl yürütmelerinin tüm yönleri tamamen doğruydu ve durumun yapısını tatmin edici bir çözüme dönüştürmüşlerdir. Özellikle ilginç bir bulgu, disiplinler arası bilgiyi uygulama gibi yeni ortaya çıkan sembolleştirme modelleme alt yeterliliğidir. Problemleri çözmek için disiplinler arası bilginin uygulanması ile tüm gruplar tarafından üstlenilen derinlemesine araştırmalar, bu alt modelleme yeterliliğinin örnek bir göstergesini yansıtmaktadır. Altı grubun tümünde öğrenciler, mühendislik problemlerine çözümler geliştirme ve gerekçelendirme girişimlerinde mevcut teknolojileri, mühendisliğin farklı bileşenlerini, birime dayalı olan ve olmayan fen ve matematik içeriklerini tartışmıştır.

Beşinci olarak, uyarlama modelleme yeterliliği; de Villiers tarafından önerilen iyileştirme ve test etme, açıklama, zarif bir çözüm üretme alt yeterliliklerinin yanı sıra uyarlanabilme ve aktarılabilme ile yaratıcı yaklaşım modelleme alt yeterlilikleri üzerinden incelenmiştir. Öncelikle, bu çalışmanın bulguları, tüm grupların, iyileştirme ve test etme modelleme alt yeterliliğini örnek bir düzeyde sergilediğini göstermiştir. Diğer bir ifadeyle, tüm gruplar problemlerin altında yatan yapıyı diğer benzer problemlerle ilişkilendirmiştir. Bazı gruplar modelin parçalarını iyileştirmeyi tercih ederken, diğer gruplar geliştirdikleri çözümlerin yeni duruma uymaması durumunda tüm modelleme sürecinden tekrar geçmiştir. Örneğin, Dr. Ahmet'in Vasiyeti etkinliğine dâhil olan grupların hiçbiri yeni durum için ilk ve son tercihlerini değiştirmemiştir, ancak diğer seçenekleri değiştirmiştir. Öte yandan, Köprü Yapımı etkinliğinde B grubu, yeni malzeme kullanarak yeni bir durum için tamamen yeni bir köprü inşa etmeye karar vermiştir. Sonuç olarak, bu çalışmadaki tüm gruplar, modellerini tamamen veya kısmen iyileştirerek değişen koşullarla ilgilenmiştir. Açıklama alt modelleme yeterliliğine bakıldığında, farklı mühendislik temelli MOE'lerdeki tüm grupların örnek bir açıklama düzeyi sergilediği görülmektedir. Başka bir şekilde söylemek gerekirse, öğrenciler modellerini uyarlarken akıl yürütmelerine ilişkin derinlemesine açıklamalar yapmışlardır. Bulgular, ilköğretimde üstün yetenekli öğrencilerin sadece problem durumunu etkileyebilecek miktarları ve değişkenleri tanımakla kalmayıp, aynı zamanda şaşırtıcı bir şekilde ilgili ve etkilenen tarafları ve beklentilerini de belirlediklerini göstermektedir. Özellikle, MOE'lere katılım sırasında mühendislik faaliyetinin çevresel etkilerini de belirlemişlerdir. Uyarlamanın başka bir alt yeterliliği olarak zarif bir çözüm üretme modelleme alt yeterliliğinde, altı grubun tümü örnek düzeyde modelleme alt yeterliliğini sergilemiştir. Yani çözümlere götüren zarif yaklaşımlar kullanmışlardır. Bununla birlikte, çeşitli gruplar, çeşitli yaklaşımlarda alt modelleme yeterliliğinin örnek göstergelerini sergilemiştir. Bazı gruplar en az malzeme ve çabayla en zarif çözümü ararken, diğer gruplar bunu uygun yöntem ve malzemelerle gerçekleştirmiştir. Buna bağlı olarak, öğrenciler mühendislik etkinliğinin olumsuz etkilerini azaltmak ve istenen çözümü en üst düzeye çıkarmak için önlemler önermişlerdir. Yeni ortaya çıkan uyarlama modelleme yeterliliğinin, uyarlanabilme ve aktarılabilme alt yeterliliği mevcut çalışmanın en çarpıcı bulgularından biridir. Tüm gruplar, yeni duruma kolayca uyum sağlayarak ve önceki bilgilerini okul içinde veya dışında aktararak bu alt modelleme yeterliliğinin örnek göstergelerini sergilemişlerdir. Bu çalışmanın bir diğer çarpıcı bulgusu, yeni bir alt modelleme yeterliliği olan, yaratıcı yaklaşımın ortaya çıkmasıdır. Çeşitli mühendislik temelli MOE'lerle çalışan gruplar, bu alt yeterliliği farklı seviyelerde sergilemiştir. Her mühendislik temelli MOE'deki gruplardan biri yaratıcı yaklaşım alt modelleme yeterliliğinin örnek bir göstergesini sergilerken, diğer grup aynı şekilde yeterli düzeyde bir gösterge sergilemiştir. Örnek düzeyinde olanlar, sorunları çözmek için yaratıcı bir yaklaşım sergilemiş ve bu yaklaşımın altında yatan nedenleri de açıklamışlardır.

Altıncı bilişsel modelleme yeterliliği olan düzenleme; modelleme alt yeterlikleri olan değerlendirme ve yargılama, yansıtma ve detaylandırma açısından incelenmiştir. Bu çalışmanın bulguları, de Villiers (2018) tarafından önerilen çerçeveye ek olarak detaylandırma alt modelleme yeterliliğinin ortaya çıkmasına neden olmuştur. Genel olarak, bu çalışmanın bulguları, altı grubun hepsinin örnek bir değerlendirme ve yargılama alt modelleme yeterliliği sergilediğini ortaya koymuştur. Bir başka ifadeyle, gruplar gerçek yaşam durumları veya önceki içeriklerle yapılan açık bağlantıların yanı sıra kapsamlı analiz, sentez ve değerlendirme sunmuştur. Diğer yandan, bu çalışmanın bulguları, Dr. Ahmet'in Vasiyeti etkinliği dışındaki grupların, yansıtma alt modelleme yeterliliğini örnek düzeyde sergilediğini göstermiştir. Öte yandan, bu etkinlikteki her iki grup da yeterli düzeyde yansıtma modelleme alt yeterliliği sergilemiştir. Daha açık bir şekilde ifade etmek gerekirse, tüm gruplar kendi düşüncelerindeki güçlü ve zayıf yönleri belirlediler, ancak diğer bakış açılarını karşılaştırırken sorunla ilgili alternatif bakış açılarını da fark etmişlerdir. Ayrıca örnek göstergeler sergileyen gruplar, bunları alternatif bakış açıları bağlamında da değerlendirmişlerdir. Farklı gruplardaki öğrenciler, tüm süreç boyunca mühendislik temelli MOE'ler ile çalışırken, çözümleri ve modelleri üzerinde kritik kontroller yapmıştır. Mevcut çalışmanın bulgularına göre, tüm gruplar kendi deneyimlerini ve düşüncelerini yansıtmışlardır. Yansıtma alt modelleme yeterliliğine benzer şekilde, Dr. Ahmet'in vasiyeti etkinliğindeki iki grup, detaylandırma alt modelleme yeterliliğini yeterli derecede gösterirken, diğer dört grup örnek teşkil edecek düzeyde göstermiştir. Alt modelleme yeterliliğini yeterli düzeyde sergileyen gruplar, fikirlerinin açıklanmasında önemli ayrıntılar sağlayarak sürecin teknik genişliğini ve derinliğini sunmuştur. Öte yandan, örnek düzeyde alt modelleme yeterliliği sergileyen diğer gruplar, fikirlerinin açıklanmasında daha fazla ayrıntıya yer vermiştir.

Son bölümde ise genelleme bilişsel modelleme yeterliliği, benzer ilişki kurma, genel veya bağımsız akıl yürütme ve kullanımı kolay model oluşturma alt modelleme yeterlikleri açısından incelenmiştir. Benzer ilişki kurma modelleme alt yeterliliğine ilişkin olarak, altı grubun tamamı bu alt yeterliliğin örnek göstergelerini sergilemiştir. Buna dayanarak, üstün yetenekli öğrenciler genellemelerini gerçek yaşam ve diğer disiplinlerle bağlantılı olarak oluşturmuştur. Bunun ötesinde, bulgular, ilköğretimde üstün yetenekli öğrencilerin karşılıklı bağımlılığı, etkileşimleri ve faktörlerin göreceli önemini, de Villiers (2018) tarafından önerilen benzer ilişki kurma modelleme alt yeterliliğinin örnek göstergelerinden farklı olarak düşündüklerini ortaya koymuştur. Genelleme modelleme yeterliliğinin başka bir alt yeterliliğine devam edildiğinde, altı grubun tamamı örnek bir genel veya bağımsız akıl yürütme alt modelleme yeterliliği sergilemiştir. Özellikle, belirli bir durumda genellemeleri kolayca belirleme yeteneğini göstermiş ve ayrıca bir çözümü kanıtlamak için tümdengelim yoluyla akıl yürütme kullanmışlardır. Son olarak, farklı mühendislik temelli MOE'lerdeki gruplar, kullanımı kolay model oluşturma yeterliliğini farklı seviyelerde göstermiştir. Mars Lunarcrete etkinliğinde D grubu ve Dr. Ahmet'in Vasiyeti etkinliğinde F grubu dışında, diğer gruplar bu alt modelleme yeterliliğini yeterli düzeyde sergilemiştir. D ve F grubu modelleri diğer durumlara kolayca uyarlanabilirken ve tahminleri doğruyken, diğer grupların modelleri diğer durumlar için bir model olarak kullanmak için küçük basitleştirmelere ihtiyaç duymuştur.

Uygulamalar için Öneriler

Bu çalışmanın bulguları, üstün yetenekli öğrenciler için öğretimi farklılaştırmada mühendislik temelli MOE'lerin etkili araçlar olarak uygulanabileceğini ortaya koymaktadır. Mühendislik temelli MOE'lerin karmaşık, disiplinler arası ve esnek doğası, bu etkinlikleri üstün yetenekli öğrencilerin genellikle yaşadıkları can sıkıntısı, zorluk eksikliği ve motivasyon gibi sorunların üstesinden gelmek için önemli bir araç haline getirebilir (Mann vd., 2011). Bu çalışmada, üstün yetenekli öğrenciler, beklenmedik durumlarla karşı karşıya kalsalar bile süreç boyunca görev bağlılığı göstererek modellerini geliştirmek için kararlı adımlar atmışlardır. Mevcut çalışmanın bulguları, mühendislik temelli MOE'lerin ilköğretimde üstün yetenekli öğrencilerin modellerini inşa etmek ve güçlendirmek için işbirlikçi bir öğrenme ortamına katılmalarını sağladığını ortaya koymaktadır. Bu bağlamda, öğrencilere kendi bakış açılarını grup üyelerinin bakış açılarıyla birleştirerek ve böylece kendilerinin ve başkalarının bakış açılarını da yansıtarak görüşlerini dışa vurma fırsatları verilebilir.

Bu çalışmanın bir başka sonucu da üstün yetenekli ilköğretim öğrencilerinin öğretmenleri ile ilgilidir. İlgili alanyazın, hem heterojen hem de homojen sınıflardaki üstün yetenekli öğrencilerin öğretmenlerinin üstün yetenekli öğrencilere özgü ihtiyaçlarını karşılamak için uygun farklılaştırma stratejilerine ihtiyaç duyduklarını göstermektedir (Bildiren ve Citil, 2021; Reis vd., 2004). Özellikle, etkinliğin yapısı ve etkili öğretim stratejilerinin kullanılması üstün yetenekli öğrencilere eğitim verilirken büyük önem taşımaktadır (Diezmann ve Watters, 2000). BİLSEM'lerde tam olarak yapılandırılmış bir öğretim programı çerçevesinin olmaması (Çetin ve Doğan, 2018) ve normal okullarda üstün yetenekli öğrencilerin ihtiyaçlarını karşılamak için izlenen özel öğretim programlarının eksikliği göz önüne alındığında, mevcut çalışma üstün yetenekli öğrencilerin ve öğretmenlerinin görüşleri doğrultusunda uygulanan ve revize edilen örnek mühendislik temelli MOE'ler sunmaktadır. Dolayısıyla öğretmenler bu tür etkinlikleri sınıflarına uyarlayarak uygulayabilir veya mühendislik temelli MOE'lerin özelliklerini ve üstün yetenekli öğrencilerin öğretim ihtiyaçlarını göz önünde bulundurarak benzer etkinlikler geliştirebilirler.

Mevcut çalışmanın bir diğer sonucu, eğitim kaynak tasarımcıları ile ilgilidir. Bildiren ve Çitil (2021) tarafından belirtilen BİLSEM'de üstün yetenekli öğrencilerin ihtiyaçlarını karşılamaya yönelik etkinliklerin geliştirilmesi ve uygulanması ile ilgili sorunlar göz önüne alındığında, mevcut çalışmanın bulguları, mühendislik temelli MOE'lerin öğrencilere eğitim fırsatları sağladığını göstermektedir. Bu tür sorunları aşmak için, BİLSEM'de üstün yetenekli öğrencilere sunulan öğrenme ortamlarında bu öğrencilerin potansiyellerini en üst düzeye çıkarma gerekliliği doğrultusunda (MEB, 2012), bu çalışmanın bulguları, bireysel yetenekleri fark ettirme programına devam eden üstün yetenekli ilköğretim öğrencileri için mühendislik temelli MOE'lerin uygulanmasına ilişkin uygulama örnekleri sunmaktadır. Bu etkinlikler, bu programlara devam eden üstün yetenekli öğrencilerin yaratıcılıklarını ve güçlü oldukları alanları ortaya çıkarmada kullanılabilir.

Bireysel yetenekleri fark ettirme programlarında üstün yetenekli öğrencilerin özgün yetenek ve potansiyellerini gerçekleştirmelerine yardımcı olmak için yaratıcılığı teşvik eden etkinlikler geliştirilmekte ve uygulanmaktadır (MEB, 2012). Buna bağlı olarak, bu çalışmanın bulguları mühendislik temelli MOE'lerin öğrencilerin yaratıcılığını ortaya çıkardığı çıkarımında bulunmuştur. Alanyazında da belirtildiği gibi (Chamberlin vd., 2013; Chamberlin ve Moon, 2005), mühendislik tabanlı MOE'ler de farklı alanlarda yaratıcılığı tanımlamak için bir araç olarak kullanılabilir.

K. THESIS PERMISSION FORM / TEZ İZİN FORMU

(Please fill out this form on computer. Double click on the boxes to fill them)

ENSTİTÜ / INSTITUTE

Fen Bilimleri Enstitüsü / Graduate School of Natural and Applied Sciences ☐

Sosyal Bilimler Enstitüsü / Graduate School of Social Sciences ☒

Uygulamalı Matematik Enstitüsü / Graduate School of Applied Mathematics ☐

Enformatik Enstitüsü / Graduate School of Informatics ☐

Deniz Bilimleri Enstitüsü / Graduate School of Marine Sciences ☐

YAZARIN / AUTHOR

Soyadı / Surname : Karataş-Aydın
Adı / Name : Firdevs İclal
Bölümü / Department : İlköğretim / Elementary Education

TEZİN ADI / TITLE OF THE THESIS (**İngilizce** / English): AN INVESTIGATION OF ELEMENTARY GIFTED STUDENTS' MODELING COMPETENCIES THROUGH ENGINEERING-BASED MODEL ELICITING ACTIVITIES

TEZİN TÜRÜ / DEGREE: **Yüksek Lisans** / Master ☐ **Doktora** / PhD ☒

1. **Tezin tamamı dünya çapında erişime açılacaktır. /** Release the entire work immediately for access worldwide. ☐
2. **Tez iki yıl süreyle erişime kapalı olacaktır.** / Secure the entire work for patent and/or proprietary purposes for a period of **two years**. ***** ☐
3. **Tez altı ay süreyle erişime kapalı olacaktır.** / Secure the entire work for period of **six months**. ***** ☒

***** *Enstitü Yönetim Kurulu kararının basılı kopyası tezle birlikte kütüphaneye teslim edilecektir. / A copy of the decision of the Institute Administrative Committee will be delivered to the library together with the printed thesis.*

Yazarın imzası / Signature .... **Tarih** / Date ....
(Kütüphaneye teslim ettiğiniz tarih. Elle doldurulacaktır.)
(Library submission date. Please fill out by hand.)

Tezin son sayfasıdır. / This is the last page of the thesis/dissertation.